Problem Set 5: Scattering amplitudes in gauge theories

Discussion on Wednesday 16.04 13:45-14:30, HIT H 51
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Exercise 8 – Fermionic Delta Functions

The integration over anti-commuting or Grassmann odd variables is defined by

$$\int d\theta 1 = 0\,, \qquad \int d\theta\,\theta = 1\,,$$

where θ is an anti-commuting coordinate. I.e. integration is identical to differentiation here.

1) Show that $\delta(\theta) = \theta$ by integrating the fermionic δ-function against a test-function $F(\theta)$.

2) Prove the following relations for the helicity spinors λ and μ

$$\delta^{(2)}(\lambda^\alpha\, a + \mu^\alpha\, b) = \begin{cases} \dfrac{\delta(a)\,\delta(b)}{|\langle\lambda\mu\rangle|} & \text{for } a,b \text{ Grassmann even (commuting)} \\ \delta(a)\,\delta(b)\,\langle\lambda\mu\rangle & \text{for } a,b \text{ Grassmann odd (anti-commuting)} \end{cases}$$

3) Use this to show that

$$\delta^{(8)}\left(\sum_{i=1}^{3} \tilde{\lambda}_i^{\dot\alpha}\,\bar{\eta}_{i\,A}\right) = [12]^4\,\delta^{(4)}\left(\bar{\eta}_{1,A} - \frac{[23]}{[12]}\,\bar{\eta}_{3\,A}\right)\,\delta^{(4)}\left(\bar{\eta}_{2,A} - \frac{[31]}{[12]}\,\bar{\eta}_{3\,A}\right) \qquad (1)$$

where $\bar{\eta}_{iA}$ are anti-commuting variables. In fact they are the complex conjugates of the η_{iA} that were introduced in class.

Exercise 9 – 3-point Superamplitudes

1) Argue that for MHV 3-particle kinematics, i.e. $[ij] = 0$ but $\langle ij\rangle \neq 0\ \forall i,j \in \{1,2,3\}$, together with the conditions of p and q invariance and local helicity $h_i = 1$ the only possible form of the 3-point MHV amplitude is

$$\mathbb{A}_3^{MHV} = \frac{\delta^{(4)}(\sum \lambda_i \tilde{\lambda}_i)\,\delta^{(8)}(\sum \lambda_i^\alpha \eta_i^A)}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\,.$$

2) This entails the complex conjugated relation

$$\overline{\mathbb{A}_3^{MHV}} = (\mathbb{A}_3^{MHV})^* = -\frac{\delta^{(4)}(\sum \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum \lambda_i^\alpha \bar{\eta}_A)}{[12][23][31]}\,,$$

using $\langle ij\rangle^* = -[ij]$. In order to transform this back into the $\{\lambda,\tilde{\lambda},\eta\}$ original on-shell superspace we need to perform a Fourier transformation of the anti-commuting $\bar{\eta}$ in the sense of

$$\Phi(\eta) = \int d^4\bar{\eta}\, e^{i\eta^A \bar{\eta}_A}\, \bar{\Phi}(\bar{\eta})\,.$$

Show that under this Fourier transformation using the result (1) one obtains the anti-MHV 3-point superamplitude

$$\mathbb{A}_3^{\overline{MHV}} = -\frac{\delta^{(4)}(\sum \lambda_i \tilde{\lambda}_i)\, \delta^{(4)}(\eta_1^A[23] + \eta_2^A[31] + \eta_3^A[12])}{[12][23][31]}\,.$$