Problem Set 4: Scattering amplitudes in gauge theories

Discussion on Wednesday 09.04 13:45-14:30, HIT H 51
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Exercise 7 – The 6-gluon split-helicity NMHV amplitude

Determine the first non-trivial next-to-maximally-helicity-violating (NMHV) amplitude

$$A_6^{\text{tree}}(1^+, 2^+, 3^+, 4^-, 5^-, 6^-)$$

from the BCFW recursion relation and our knowledge of the MHV amplitudes. Consider a shift of the two helicity states 1^+ and 6^- and show that

$$\begin{aligned} A_6^{\text{tree}}(1^+,2^+,3^+,4^-,5^-,6^-) = {} & \frac{\langle 6|p_{12}|3]^3}{\langle 61\rangle\langle 12\rangle[34][45][5|p_{16}|2\rangle} \frac{1}{(p_6+p_1+p_2)^2} \\ & + \frac{\langle 4|p_{56}|1]^3}{\langle 23\rangle\langle 34\rangle[16][65][5|p_{16}|2\rangle} \frac{1}{(p_5+p_6+p_1)^2}\,, \end{aligned}$$

where $p_{ij} = p_i + p_j$.

Exercise 8 – Conformal algebra

Show that the representation of the conformal generators constructed in the lecture

$$\begin{aligned} p^{\alpha\dot\alpha} &= \lambda^\alpha \tilde\lambda^{\dot\alpha}\,, \qquad k_{\alpha\dot\alpha} = \partial_\alpha \partial_{\dot\alpha}\,. \\ m_{\alpha\beta} &= \lambda_{(\alpha}\partial_{\beta)} := \tfrac{1}{2}\left(\lambda_\alpha\partial_\beta + \lambda_\beta\partial_\alpha\right)\,, \qquad \overline{m}_{\dot\alpha\dot\beta} = \tilde\lambda_{(\dot\alpha}\partial_{\dot\beta)}\,, \\ d &= \tfrac{1}{2}\lambda^\alpha\partial_\alpha + \tfrac{1}{2}\tilde\lambda^{\dot\alpha}\partial_{\dot\alpha} + 1\,. \end{aligned}$$

indeed obeys the commutation relations of the conformal algebra

$$\begin{aligned} [d,p^{\alpha\dot\alpha}] &= p^{\alpha\dot\alpha}\,, \quad [d,k_{\alpha\dot\alpha}] = -k_{\alpha\dot\alpha}\,, \quad [d,m_{\alpha\beta}] = 0 = [d,\overline{m}_{\dot\alpha\dot\beta}]\,, \\ [k_{\alpha\dot\alpha}, p^{\beta\dot\beta}] &= \delta_\alpha^\beta\,\delta_{\dot\alpha}^{\dot\beta}\,d + m_\alpha{}^\beta\,\delta_{\dot\alpha}^{\dot\beta} + \overline{m}_{\dot\alpha}{}^{\dot\beta}\,\delta_\alpha^\beta\,, \end{aligned}$$

The helicity generator is given by $h = -\frac{1}{2}\lambda^\alpha\partial_\alpha + \frac{1}{2}\tilde\lambda^{\dot\alpha}\partial_{\dot\alpha}$. It commutes with all generators of the conformal algebra.