Problem Set 3: Scattering amplitudes in gauge theories

Discussion on Wednesday 25.03 12:45-14:30, HIT H 51
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Exercise 6 – Non-trivial quark-gluon scattering

Show by using the color ordered Feynman rules and a suitable choice for the gluon-polarization reference vector $q_3^{\alpha\dot{\alpha}} = \mu_3^\alpha\,\tilde{\mu}_3^{\dot{\alpha}}$ that the first non-trivial $\bar{q}qgg$ scattering amplitude is given by

$$A^{\text{tree}}_{\bar{q}qg^2}(1^-_{\bar{q}}, 2^+_q, 3^-, 4^+) = \frac{\langle 13\rangle^3 \langle 23\rangle}{\langle 12\rangle \langle 23\rangle \langle 34\rangle \langle 41\rangle}\,.$$

Also convince yourself that the result for this amplitude has the correct helicity assignments as was done in class for the pure-gluon four point amplitude. That is compute

$$h_i\, A^{\text{tree}}_{\bar{q}qg^2}(1^-_{\bar{q}}, 2^+_q, 3^-, 4^+) =?$$

Exercise 7 – Three-point amplitudes from color-ordered Feynman rules

Prove the expressions for the three-point MHV amplitude

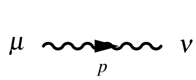
$$A_3^{MHV}(i^-, j^-) = \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\,, \quad [12] = [23] = [31] = 0\,,$$

and the three-point anti-MHV amplitude

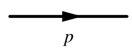
$$A_3^{\overline{MHV}}(i^+, j^+) = -\frac{[ij]^4}{[12][23][31]}\,, \quad \langle 12\rangle = \langle 23\rangle = \langle 31\rangle = 0\,,$$

by an explicit calculation using the color-ordered Feynman rules given in class (see also on the next page).

Propagators

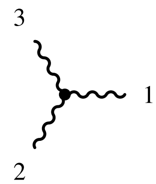


$$-\frac{i}{p^2+i0}\,\eta_{\mu\nu}$$

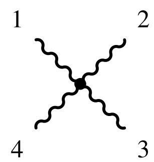


$$\frac{i\,\not{p}}{p^2+i0}\,\eta_{\mu\nu}$$

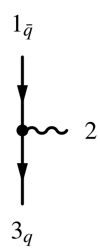
Color ordered vertices



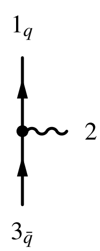
$$-g\,\frac{i}{\sqrt{2}}\Big[(\varepsilon_1\cdot\varepsilon_2)\,(p_{12}\cdot\varepsilon_3)+(\varepsilon_2\cdot\varepsilon_3)\,(p_{23}\cdot\varepsilon_1)$$
$$+(\varepsilon_3\cdot\varepsilon_1)\,(p_{31}\cdot\varepsilon_2)\Big] \qquad p_{ij}:=p_i-p_j$$



$$g^2\,\frac{i}{2}\Big[2(\varepsilon_1\cdot\varepsilon_3)\,(\varepsilon_2\cdot\varepsilon_4)-(\varepsilon_1\cdot\varepsilon_2)\,(\varepsilon_3\cdot\varepsilon_4)$$
$$-(\varepsilon_1\cdot\varepsilon_4)\,(\varepsilon_2\cdot\varepsilon_3)\Big]$$



$$+g\,\frac{i}{\sqrt{2}}\,[3|\,\not{\varepsilon}_2\,|1\rangle$$



$$-g\,\frac{i}{\sqrt{2}}\,[1|\,\not{\varepsilon}_2\,|3\rangle$$

Table 1.4: Color-ordered Feynman rules in a compact notation using dummy polarization vectors ε_i^μ and summy spinors λ_i respectively $\tilde{\lambda}_i$ to absorb the index structures. All momenta are outgoing, outgoing quark lines are represented by $|p]$ and $[p|$ states, whereas outgoing anti-quark lines are represented by $|p\rangle$ and $\langle p|$ states, compare the discussion in section 1.8.