$$Z[\eta,\bar\eta,J] = N\int \mathcal{D}\bar\psi\,\mathcal{D}\psi\,\mathcal{D}\phi\; e^{iS_0 + iS_{se} + i\int d^4x\,(\lambda\bar\psi\psi\phi + J\phi + \bar\psi\eta + \bar\eta\psi)}$$

$$= N\, e^{iS_{int}}\; Z_0[\eta,\bar\eta,J]$$

(1)

$$e^{i^4[\,\longrightarrow\, + \frac{1}{2}\, ---\,]}$$

$$Z[\eta,\bar\eta,J] = N\Big[1 + i\lambda\int d^4x\, \frac{i\delta}{\delta\eta(x)}\frac{\delta}{i\delta\bar\eta(x)}\frac{\delta}{i\delta J(x)} +$$

$$\frac{(+i\lambda)^2}{2}\int d^4x\int d^4y\; \frac{i\delta}{\delta\eta(y)}\frac{\delta}{i\delta\bar\eta(y)}\frac{\delta}{i\delta J(y)}$$

$$\frac{i\delta}{\delta\eta(x)}\frac{i\delta}{i\delta\bar\eta(x)}\frac{\delta}{i\delta J(x)}\Big]\; e^{\cdots}$$

$$\frac{\delta}{\delta \xi(x)} = (i)\; \underset{x}{\bullet} - - - \bullet \; e^{[\;]}$$

$$\frac{\delta}{\delta \bar{\eta}(x)} = (i)^2 \left(\underset{x}{\bullet} - - - \bullet\right) \left(\underset{x}{} \longrightarrow \bullet\right) e^{[\;]}$$

$$\frac{\delta}{\delta \eta(x)} = (i)^2 \left(x - - - \bullet\right) \left[\underset{x}{} \longrightarrow \underset{x}{} + i \; \underset{x}{} \longrightarrow \quad \longrightarrow \underset{x}{} \right] e^{[\;]}$$

$$\frac{\delta}{\delta \xi(y)} = (i)^2 \left(x - - - y\right) \left[\underset{x}{\bigcirc} + i \; \bullet \longrightarrow \underset{x}{} \quad \underset{x}{} \longrightarrow \bullet \right] e^{[\;]}$$

$$+ (i)^3 \begin{matrix} x - - - \circ \\ y - - - \circ \end{matrix} \left[\underset{x}{\bigcirc} - i \; \bullet \longrightarrow \underset{x}{} \quad \underset{x}{} \longrightarrow \circ \right] e^{[\;]}$$

$$\frac{\delta}{\delta \bar{\eta}(y)} = \left[\begin{matrix} (i)^2 \; (x - - - y) \\ + (i)^3 \; \begin{matrix} x - - - \\ y - - - \end{matrix} \end{matrix} \right] \left[-i \; y \longrightarrow x \quad x \longrightarrow \circ \right.$$

$$+ i \; \underset{x}{\bigcirc} \quad \underset{y}{} \longrightarrow \circ \; - (i)^2 \; \bullet \longrightarrow \underset{x}{} \quad \underset{x}{} \longrightarrow \bullet$$

$$\left. \bullet \longrightarrow \underset{y}{} \right] e$$

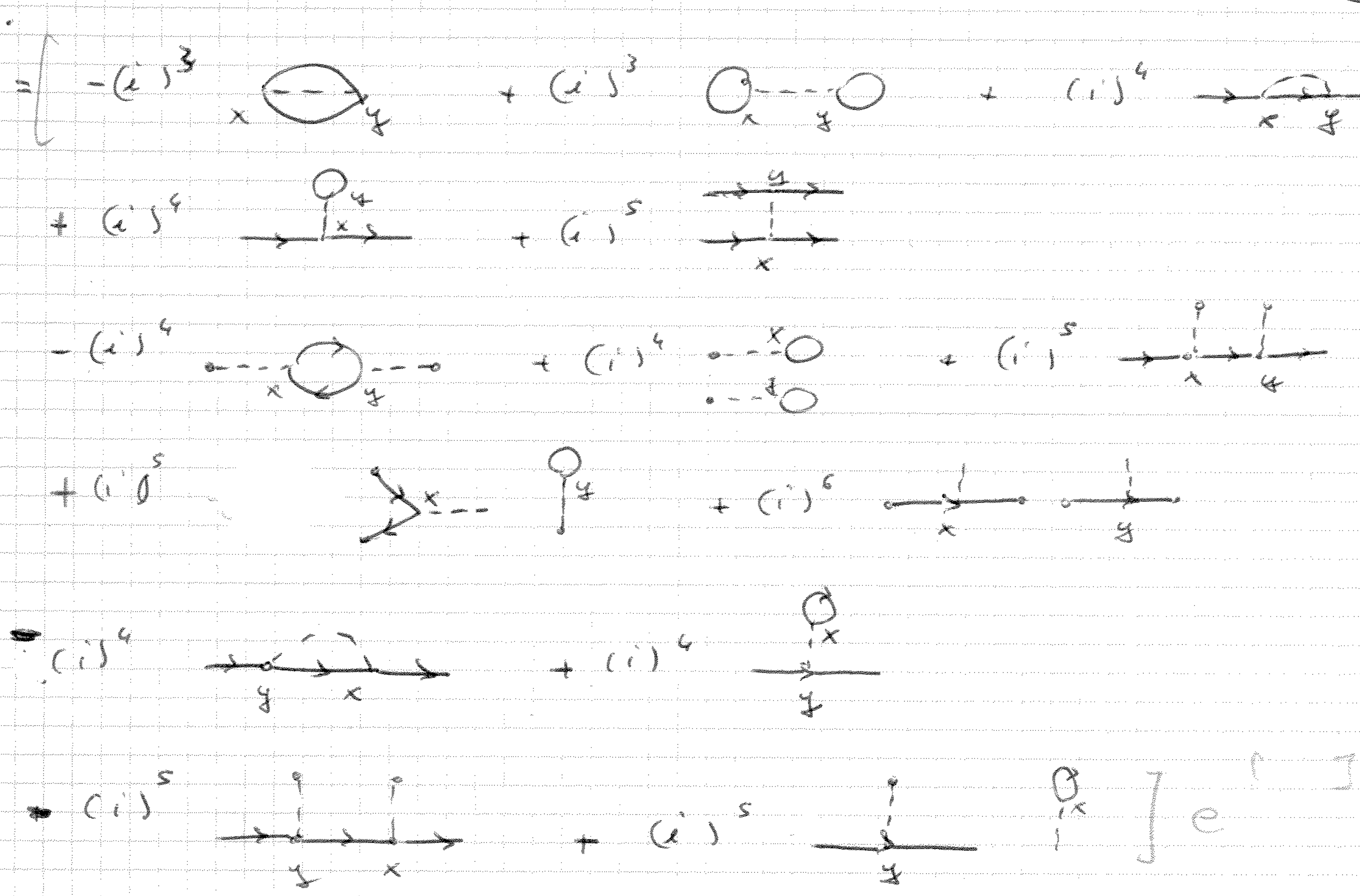
$$\frac{\delta}{\delta \eta(y)} = \left[(i)^2 \; (x - - - y) + (i)^3 \begin{matrix} x - - - \circ \\ y - - - \circ \end{matrix} \right] \times$$

$$\left[-i \; \longrightarrow_{y} \; {}_{x} \quad x \longrightarrow {}_{y} \; + i \; \underset{x}{\bigcirc} \quad \underset{y}{\bigcirc} \right.$$

$$+ (i)^2 \; \bullet \longrightarrow \underset{x}{} \quad x \longrightarrow y \quad \underset{y}{\circ} \longrightarrow \circ \; + (i)^2 \; \circ \longrightarrow \underset{x}{} \quad x \longrightarrow \bullet \quad \underset{y}{\bigcirc}$$

$$\left. + (i)^3 \; \circ \longrightarrow \underset{x}{} \quad \underset{x}{} \longrightarrow \bullet \quad \underset{y}{\circ} \longrightarrow \circ \quad \bullet \longrightarrow \underset{y}{} \right] e^{[\;]}$$

$$+ (i)^2 \; \underset{y}{} \longrightarrow \underset{x}{} \quad \underset{x}{} \longrightarrow \bullet \quad \longrightarrow \underset{y}{} \qquad - (i)^2 \; \underset{x}{\bigcirc} \quad \underset{y}{} \longrightarrow \circ \quad \circ \longrightarrow \underset{y}{}$$

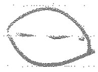


<u>ou</u>

Normalization : Vacuum diags :

$$Z[0,0,0] = 1 = N \int D\bar{\psi}\, D\psi\, D\phi\; e^{iS_0 + iS_{sc} + i\lambda \int d^4x\, \bar{\psi}\psi\phi}$$

$$\Rightarrow \frac{1}{N} = e^{i\lambda \int d^4x\, \frac{i\delta}{\delta\eta(x)} \frac{\delta}{i\delta\bar{\eta}(x)} \frac{\delta}{i\delta J(x)}}\, Z_0[\bar{\eta},\eta,J]\Big|_{J=\eta=\bar{\eta}=0}$$

$$= 1 + \frac{(i\lambda)^2}{2!}\Big[-i^3$$  $$+\, i^3$$  $$\Big] + O(\lambda^3)$$

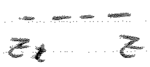
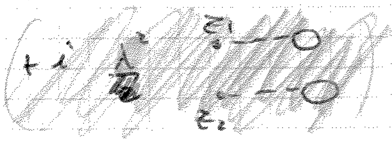
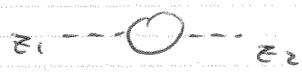
Scalar prop.

$$\frac{\delta^2}{i\delta J(z_1)\, i\delta J(z_2)} \; iW[\;] \Big|_{\bar\eta = \eta = J = 0}$$

$$= \frac{\delta}{i\delta J(z_1)} \; \frac{1}{Z[\;]} \; \frac{\delta}{i\delta J(z_2)} \; Z[\;] \Big|_{J=0}$$

$$= \frac{1}{(i)^2}\Big[- \frac{1}{Z^2[\;]} \; \frac{\delta}{\delta J(z_1)} Z[J] \; \frac{\delta}{\delta J(z_2)} Z[J] +$$

$$+ \frac{1}{Z[J]} \; \frac{\delta^2}{\delta J \, \delta J} \; Z[J] \Big] \Big|_{J=0}$$

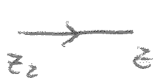
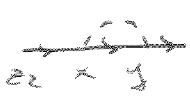
$= -i$   $+$ 

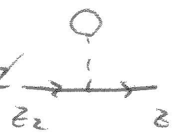
Fermion prop.

$$i \frac{\delta}{\delta \eta(z_1)} \; \frac{\delta}{i\delta \bar\eta(z_2)} \; iW[\;] \Big|_{\bar\eta = \eta = J = 0}$$

$$= \frac{\delta}{\delta \eta(z_1)} \; \frac{1}{Z[\;]} \; \frac{\delta}{\delta \bar\eta(z_2)} \; Z[\;] \Big|_{=0}$$

$$= -\frac{1}{Z^2[\;]} \; \frac{\delta Z[\;]}{\delta \eta(z_1)} \; \frac{\delta}{\delta \bar\eta(z_2)} \; Z[\;] + \frac{1}{Z[\;]} \; \frac{\delta^2}{\delta\eta\,\delta\bar\eta} \; Z[\;] \Big|_{=0}$$

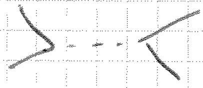
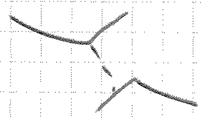
$= i$  $+$ 

$+\cancel{i}$  $\frac{1}{\cancel{i}}$

Scattering 4 fermions:

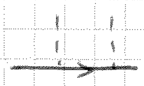
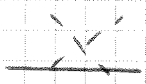
3

$$\frac{\delta}{\delta\psi(z_4)}\,\frac{\delta}{\delta\bar\psi(z_3)}\,\frac{\delta}{\delta\psi(z_2)}\,\frac{\delta}{\delta\bar\psi(z_1)}\; iW[\;]\Big|_{J=0}$$

$= i$ ( $-$ )

Scattering 2 f → 2 sc

$$\frac{1}{(i)^2}\,\frac{\delta}{\delta\psi}\,\frac{\delta}{\delta\bar\psi}\,\frac{\delta}{\delta J}\,\frac{\delta}{\delta J}\; iW\Big|_{J=0}$$

$= -i$ ( $+$ )

Ex. 2

Under a global transformation, we have

$$\psi' = e^{i\alpha}\psi$$

$$J^\mu = \frac{\delta \mathcal{L}}{\delta(\partial_\mu \psi)}\delta\psi = \bar{\psi}\gamma^\mu\psi$$

So that the conserved charge is

$$Q = \int d^3x \; J^0(x) = \int d^3x \; \bar{\psi}\gamma^0\psi = \int d^3x \; \psi^+\psi$$

ex. 3

a) Infinitesimally we have

$$U(\theta) = 1 + ig\,\theta_a T^a$$

$$U(\theta)^{-1} = 1 - ig\,\theta_a (T^a)^+$$

From Unitarity we get

$$U^+U = 1 \simeq (1 - ig(T^a)^+\theta_a)(1 + ig\,T^b\theta_b) \simeq 1 + O(\theta^2)$$

so that $(T^a)^+ = T^a$

Computing $(U')^{-1}U^{-1}U'U =$

$$= \left(1 - ig\,T^a\theta_a - \frac{1}{2}g^2(T^a)^2(\theta^a)^2\right)\left(1 - ig\,T^b\theta^b - \frac{1}{2}g^2(T^b)^2\right.$$

$$\cdot\left(1 + ig\,T^a\theta_a - \frac{1}{2}g^2(T^a)^2(\theta^a)^2\right)\left(1 + ig\,T^b\theta^b - \frac{1}{2}g^2(T^b)^2\theta_b^2\right.$$

$$= 1 - g^2[T^a, T^b]\theta^a\theta^b + O(\theta^3)$$

since $U, U' \in SU(N)$ are chosen to be indipendent elements, this result must be a non-trivial element of $SU(N)$ for $a \neq b$. Therefore, we can write

$$= 1 + i g^2 f^{abc} \theta^a \theta^b T^c + O(\theta^3)$$

b) To show that f^{abc} are real, consider

$$([T^a, T^b])^\dagger = -i (f^{abc})^\dagger (T^c)^\dagger$$

$$-[T^a, T^b] = -i (f^{abc})^\dagger T^c$$

$$[T^a, T^b] = i (f^{abc})^\dagger T^c$$

$$\Downarrow$$

$$(f^{abc})^\dagger = f^{abc}$$