Ninth Exercise Sheet due to 15. May

Exercise 1 (Weak coupling for general interactions) *In this exercise you should convince yourself that the derivation of weak coupling limit for a general interaction*

$$H_{int} = \sum_\alpha F_\alpha \otimes \Phi_\alpha$$

does not provide any additional conceptual difficulty. Decompose all operators F_α *into eigenprojections of* $-i[H_S, \cdot]$ *as*

$$F_\alpha = \sum_\omega A_\alpha(\omega), \quad e^{itH_S} F_\alpha e^{-itH_S} = \sum_\omega A_\alpha(\omega) e^{-it\omega}.$$

Define a matrix

$$h_{\alpha\beta}(x) = \mathrm{Tr}(G \exp(iH_B x) \Phi_\alpha \exp(-iH_B x) \Phi_\beta).$$

Show that the limiting weak coupling Lindbladian can be expressed in terms of $\hat{h}_{\alpha\beta}(\omega)$, $A_\alpha(\omega)$*;* $\hat{}$ *stands for the Fourier transform.*

Show that the limit operator has the correct Lindblad form provided $\hat{h}_{\alpha\beta}(\cdot)$ *is a positive matrix. Prove that this is indeed so using the Bochner theorem.*

Exercise 2 (Rotating wave approximation) *If you still have some energy left you can apply the first exercise to a Hamiltonian*[1]

$$H = \frac{\omega_0}{2}\sigma_z + \int_0^{cut\,off} \omega a_\omega^* a_\omega \mathrm{d}\omega + \lambda \int_0^\infty h(\omega)(\sigma_+ a_\omega + \sigma_- a_\omega^*).$$

This is the Hamilotnian of Exercise 8.a in the rotating wave approximation.

[1] If you are unsure about the Hamiltonian take a look at ex sheet.8. Also note that h in the Hamiltonian is a given function not the h appearing in the first exercise.