Seventh Exercise Sheet due to 17. April

Exercise 1 (Decoherence of a two level system) *A two level atom in an external magnetic field pointing in the z-direction is often described by a Lindblad equation (used for example in NMR)*

$$\begin{aligned}\dot{\rho}(t) &= \mathcal{L}\rho(t) \\ &= -i[H,\rho] + \sum_{\alpha=1}^{3} 2\Gamma_\alpha \rho \Gamma_\alpha^* - \Gamma_\alpha^* \Gamma_\alpha \rho - \rho \Gamma_\alpha^* \Gamma_\alpha,\end{aligned}$$

where[1] $H = B\sigma_z$, $\Gamma_1 = \sqrt{a}\sigma_z$, $\Gamma_2 = \sqrt{b_+}\sigma_+$, $\Gamma_3 = \sqrt{b_-}\sigma_-$. *This is the most general Lindblad equation which is rotationally symmetric around the axis of the magnetic field, z-axis (try to think about it little bit).*

Take a simple case $b_+ = b_- =: b$ and $B = 0$. Prove that solutions of the Lindblad equation have a form $\rho(t) = \frac{1+n(t)\cdot\sigma}{2}$, where the Bloch vector $n(t)$ solves equations

$$\begin{aligned}\dot{n}_z(t) &= -T_\parallel^{-1} n_z(t), \\ \dot{n}_y(t) &= -T_\perp^{-1} n_y(t), \\ \dot{n}_x(t) &= -T_\perp^{-1} n_x(t).\end{aligned}$$

The relaxation times $T_\parallel$, $T_\perp$ depend on a and b. Show that an inequality

$$T_\parallel \geq 2T_\perp$$

always holds true. Sketch all results on the Bloch sphere.

[1]Recall $\sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$