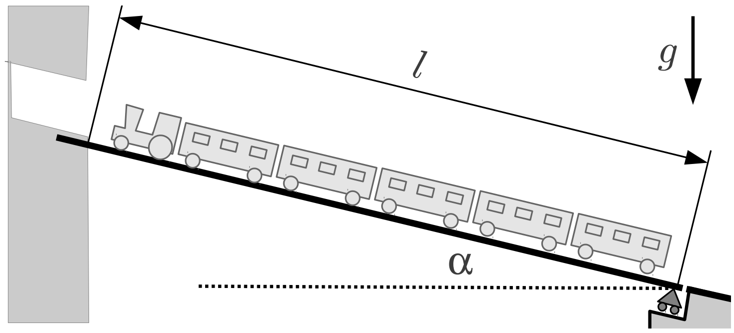
Exercise 1. *Beams*

This task is composed of several independent subtasks, all of them being a simple examples of modelling using beams as the basic structure element.

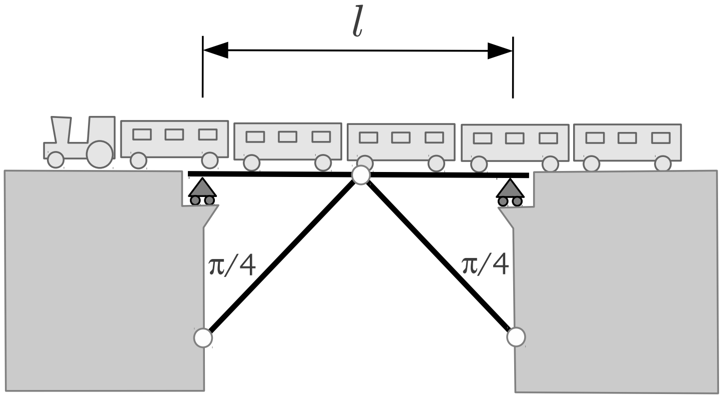
1.1. *Simple model for a skew bridge*

When travelling from Chur to Arose by RhB, the train has to get more than 1000 altitude meters. At one place it does roll over a skew bridge into a tunnel. The drawing below does show our model representation of the situation. Find the displacement field under the assumption that the load on the bridge is homogeneous with length density given by μ. You may further assume that the train moves at constant speed. The bridge deck is anchored at left and simply supported on the right.



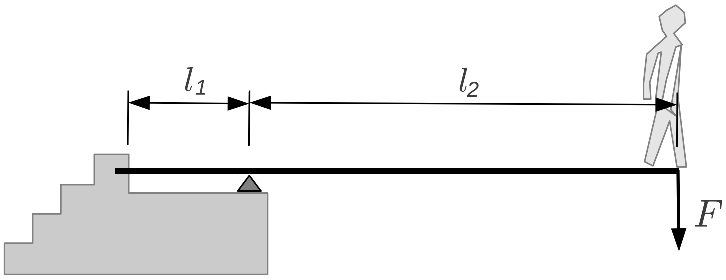
1.2. *Bridge with support*

For the bridge structure in the scheme below find the force by which the supporting beams hold the the bridge deck in the centre. Assume that the mass of the bridge construction is negligible with respect to the mass of the train. Further assume that the load caused by the train is homogeneous. The contacts displayed by empty circles enable change of direction freely, but they fix the beam end points.



1.3. *Springboard*

Although for a springboard it is not obvious that we may regard it as an elongated quasi 1-dimensional beam, we will approximate it like that. Find the elastic energy of the springboard for the situation displayed below. Neglect the mass of the springboard itself.

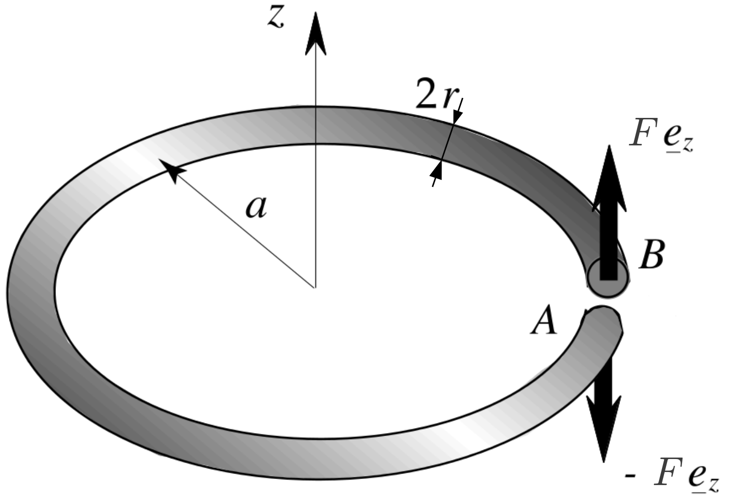


Exercise 2. ***Hollow cylindric bar***

In this task we are going to examine a hollow cylindric bar with inner radius $R - \delta/2$ and outer radius $R + \delta/2$; with thin wall, $\delta \ll R$. Use the von Mises criterion to find the torques $M_{y,z}$ (normal and axial) such that the material stays in the linear elastic regime. Treat the 2 cases separately. Follow the lecture slides. Express the moments as a function of the angle per length $\frac{\mathrm{d}\omega_{x,z}}{\mathrm{d}s}$.

Exercise 3. ***Cutted ring***

In this exercise we are going to examine an elongated object, a cutted ring, displaced out of the plane. Without any force applied it would lie in the plane $z = 0$. We apply forces $\pm Q\overline{e}_z$ at the ends. We are interested in the vertical displacement of the ends of the ring. The large radius of the ring is a and the small radius is r. Neglect the gravity.



Use the *Frenet* frame using the tangent ($\overline{t}$), normal ($\overline{n}$) and binormal ($\overline{b}$) unit vectors. Recall that relations for their derivatives,

$$\frac{\mathrm{d}\overline{t}}{\mathrm{d}s} = C\overline{n}, \qquad \frac{\mathrm{d}\overline{n}}{\mathrm{d}s} = -C\overline{t} + T\overline{b}, \qquad \frac{\mathrm{d}\overline{b}}{\mathrm{d}s} = -T\overline{n}, \tag{1}$$

with curvature C and torsion T. Find C and T for our problem.

Use the equilibrium equations

$$\frac{\mathrm{d}\overline{T}}{\mathrm{d}s} = \overline{0}, \qquad \frac{\mathrm{d}\overline{M}}{\mathrm{d}s} + \bar{t} \times \overline{T} = \overline{0}, \tag{2}$$

to find the force field $\overline{T}$ and torque field $\overline{M}$.

In order to find the displacement field you have to first get the angle field $\overline{\omega}$ and relative elongation field ϵ using the Hooke's law in corresponding form,

$$\frac{\mathrm{d}\overline{\omega}}{\mathrm{d}s} = \frac{M_t}{\mu J}\bar{t} + \frac{M_n}{EI}\overline{n} + \frac{M_b}{EI}\bar{b}, \qquad \epsilon = \frac{T_t}{ES}, \tag{3}$$

and afterwards the displacement

$$\frac{\mathrm{d}\overline{u}}{\mathrm{d}s} = \epsilon\bar{t} + \overline{\omega} \times \bar{t}. \tag{4}$$

Recall that for a full circular section you have $I = \frac{\pi r^4}{4}$ and $J = \frac{\pi r^4}{2}$.