Fibonacci anyons & Topological quantum Computers

By Christos Charalambous
1. Quantum Computer
   - Quantum Computing
   - Decoherence

2. “Any”-ons
   - Abelian anyons
   - Non-abelian anyons
   - Physical realization of non-abelian anyons
   - Fractional Quantum Hall effect
   - Geometric phase
   - Topological Quantum Computer

3. Computing using anyons
   - Braiding
   - Fusion
   - Recap: Fusion rules for minimal models
   - Fusion space
   - F & R matrices
   - The unitary B-matrix
   - Compatibility equations: Pentagon and Hexagon equations

4. Fibonacci model
   - Fibonacci/Yang Lee model
   - Chern-Simons theory
   - Recap of WZW models

5. Summary
1. Quantum Computer
   - Quantum Computing
   - Decoherence

2. “Any”-ons
   - Abelian anyons
   - Non-abelian anyons
   - Physical realization of non-abelian anyons
   - Fractional Quantum Hall effect
   - Geometric phase
   - Topological Quantum Computer

3. Computing using anyons
   - Braiding
   - Fusion
   - Recap: Fusion rules for minimal models
   - Fusion space
   - F & R matrices
   - The unitary B-matrix
   - Compatibility equations: Pentagon and Hexagon equations

4. Fibonacci model
   - Fibonacci/Yang Lee model
   - Chern-Simons theory
   - Recap of WZW models

5. Summary
Quantum computation

- Classical computer: limited computational power

- Interference  →  Quantum physics could speed up processes

- Qubit:  \[ |\psi> = \alpha |0> + \beta |1> \]  where  \[ a^2 + \beta^2 = 1, \ a, \beta \in \mathbb{C} \]
  - Hilbert spaces:  \[ \psi_1 \in H_1, \ \psi_2 \in H_2 \]
    \[ \rightarrow \psi_{12} = \sum_{i,j=\{1,2\}} a_{ij} \psi_i \otimes \psi_j \in H_1 \otimes H_2 \]
  - classical: \( m \) bits  \( \rightarrow 2^m \) states
  - quantum: \( m \) qubits  \( \rightarrow 2^m \) basis states

- Entanglement: speeding up classical algorithms
Quantum computation

- **Quantum Circuits:** unitary operators that act on a Hilbert space, generated by \( n \) qubits, whose states encode the information we want to process. Quantum Circuits are composed of elementary Quantum gates.

- **Universality:** existence of a universal set of quantum gates, the elements of which can perform any unitary evolution in SU(\( N \)) with arbitrary accuracy

Need:

1. Single qubit rotation gates that can span SU(2):
   \[ |\psi\rangle \rightarrow U |\psi\rangle \]
2. A Two-qubit entangling gate:
   \[ U \in SU(2) \]

\[ CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \]

Examples of quantum algorithms:
- Deutsch algorithm
- Shor’s factoring algorithm
Quantum computation

Deutsch Algorithm

- Boolean Function $F$: Constant ($f(0)=f(1)$) or Balanced ($f(0)\neq f(1)$)?

Single qubit rotation gate called Hadamard:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

- Requires only 1 measurement to answer while classically it takes 2 evaluations of $F$

Decoherence

Very easy for errors to appear in the system due to interactions with the environment:

Examples:

1. Bit flips: $|0\rangle \rightarrow |1\rangle$, $|1\rangle \rightarrow |0\rangle$

2. Phase flips: $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) \rightarrow \frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$
Goal: encode information in an environment independent way

Idea: Topological properties are insensitive to local perturbations
1. Quantum Computer
   - Quantum Computing
   - Decoherence

2. “Any”-ons
   - Abelian anyons
   - Non-abelian anyons
   - Physical realization of non-abelian anyons
   - Fractional Quantum Hall effect
   - Geometric phase
   - Topological Quantum Computer

3. Computing using anyons
   - Braiding
   - Fusion
   - Recap: Fusion rules for minimal models
   - Fusion space
   - F & R matrices
   - The unitary B-matrix
   - Compatibility equations: Pentagon and Hexagon equations

4. Fibonacci model
   - Fibonacci/Yang Lee model
   - Chern-Simons theory
   - Recap of WZW models

5. Summary
Abelian anyons

Exchanging:
\[ |\psi_A > |\psi_B > \rightarrow e^{i\theta} |\psi_B > |\psi_A > \]

Winding:
\[ |\psi_A > |\psi_B > \rightarrow (e^{i\theta})^2 |\psi_A > |\psi_B > \]

In 3D:
\[ (e^{i\theta})^2 = I \]

→ Boson: \( \theta = 2\pi + 2\pi n \) \[ |\psi_{AB} > \rightarrow |\psi_{BA} > \]

→ Fermion: \( \theta = \pi + 2\pi n \) \[ |\psi_{AB} > \rightarrow -|\psi_{BA} > \]

Move to 2D:

Any \( \theta \rightarrow \) “Any”-ons

Exchanging=braiding
Non-abelian anyons

- Degenerate state space \( \{\psi_i\}, \ i = 1, \ldots, d \) then:

\[
\begin{pmatrix}
\psi_1 \\
\psi_2 \\
\vdots \\
\psi_d
\end{pmatrix}
\rightarrow
\begin{pmatrix}
\psi'_1 \\
\psi'_2 \\
\vdots \\
\psi'_d
\end{pmatrix}
= 
\begin{pmatrix}
U_{11} & \cdots & U_{1d} \\
\vdots & \ddots & \vdots \\
U_{d1} & \cdots & U_{dd}
\end{pmatrix}
\begin{pmatrix}
\psi_1 \\
\psi_2 \\
\vdots \\
\psi_d
\end{pmatrix}
\]

→ Candidate space to store and process quantum information
Physical realization of Non-abelian anyons:

1. Degenerate ground state
2. Finite energy gap $\Delta E$ for ground state
3. Adiabaticity
4. Anyons being far apart
5. All local operators have vanishing correlation functions apart from identity
Trapped $e^-$ gas

Conductance: $\sigma = \nu \frac{e^2}{h}$ where $\nu$ a fractional number

→ Fractional charge

Abelian anyons for $\nu = \frac{1}{3}$

Laughlin states: Trial wavefunctions

Expect → fractional statistics = anyonic statistics
Geometric phase

Time evolution of a state:

\[ |\psi(T)\rangle = \exp(i\gamma_n(C)) \exp \left\{ -\frac{i2\pi}{\hbar} \int_0^T dt \, E_n(R(t)) \right\} |\psi(0)\rangle \]

where \( \gamma_n(C) \) is Berry’s geometric phase:

\[ \gamma_n(C) = i \oint_C <n, R(t) | \nabla_R n, R(t) | dR = \oint_C A_\mu dR^\mu = \oint_S \frac{1}{2} F_{\mu\nu} dR^\mu \wedge dR^\nu \]
Geometric phase

Vector potential gauge transformation:

\[ A_\mu \rightarrow A_\mu - \partial_\mu a_n \]

Vector field:

\[ (F_{\mu\nu})^{if} := (\partial_\mu A_\nu - \partial_\nu A_\mu)^{if} \]

→ invariant under gauge transformations

If \((F_{\mu\nu})^{if} \neq 0\) i.e. not diffeomorphic invariant as well:

→ Case 1: Non-degenerate state space → Abelian geometric phase, U(1)

Example:
Case 2: Degenerate state space \((F_{\mu \nu})^i f\) a matrix:

Example:

\[ A B C \neq C B A \]

Transformations of the state space are elements of SU(2)

For N degenerate state space transformations are elements of SU(N)
In general:
\[ |\psi(C)\rangle = \Gamma_A(C) |\psi(0)\rangle \]
\[ \Gamma_A(C) = \exp(i\gamma_n(C)) \]

the following properties hold:

(A) \[ \Gamma_A(C_2 \cdot C_1) = \Gamma_A(C_2)\Gamma_A(C_1) \]

(B) \[ \Gamma_A(C_0) = I \]

(C) \[ \Gamma_A(C^{-1}) = \Gamma^{-1}_A(C) \]

(D) \[ \Gamma_A(C \circ f) = \Gamma_A(C) \]

(A)+(B)+(C) \rightarrow Forms a Group
Geometric phase

• Relate parametric space to anyons coordinates
• Assume vector field $F_{\mu\nu}$ is confined to anyons position

Hence:

non-abelian geometric phases evolutions

evolutions in a system of non-abelian anyons
Quasiparticle worldlines forming braids carry out unitary transformations on a Hilbert space of $n$ anyons.

This Hilbert space is exponentially large and its states cannot be distinguished by local measurements.

→ candidate model for fault-tolerant quantum computing
1997 A. Kitaev: System of non-abelian anyons with suitable properties can efficiently simulate a quantum circuit.

2000 Freedman, Kitaev and Wang: System of anyons can be simulated by a quantum circuit.

Equivalence of the two views of the system, i.e. between an anyonic computational model (e.g. a Topological quantum computer) and a quantum circuit.

Is there an anyonic computational model that can simulate a quantum circuit that exhibits universality?
1. Quantum Computer
   - Quantum Computing
   - Decoherence
2. “Any”-ons
   - Abelian anyons
   - Non-abelian anyons
   - Physical realization of non-abelian anyons
   - Fractional Quantum Hall effect
   - Geometric phase
   - Topological Quantum Computer
3. Computing using anyons
   - Braiding
   - Fusion
   - Recap: Fusion rules for minimal models
   - Fusion space
   - F & R matrices
   - The unitary B-matrix
   - Compatibility equations: Pentagon and Hexagon equations
4. Fibonacci model
   - Fibonacci/Yang Lee model
   - Chern-Simons theory
   - Recap of WZW models
5. Summary
Braiding

\[ \sigma_i := \text{exchange of } ith \text{ and } (i + 1)th \text{ anyon} \]

- World lines cannot cross
- \( \text{braids } \sigma_i \) (particle histories) in distinct topological classes
- 1:1 correspondence of the topological classes with distinct elements of a Braid set

1. Any braid can be obtained by multiplying elementary braids
2. The inverse of any braid exists
3. Existence of Vacuum
4. Associativity for disjoint \( \sigma_i \)

\[ \rho(\sigma_i) \] representations of the Braid group = the unitary transformations

Braid group
Braiding

Defining relations of Braid group for an anyonic model:

1. Exchanges of disjoint particles commute:
   \[ \sigma_j \sigma_k = \sigma_k \sigma_j \quad |j - k| \geq 2 \]

2. Yang-Baxter relation:
   \[ \sigma_j \sigma_{j+1} \sigma_j = \sigma_{j+1} \sigma_j \sigma_{j+1} \quad j = 1, 2, \ldots, n - 2 \]
Fusion

- **Fusion**: The process of bringing two particles together

- A non abelian anyonic model is defined starting from the superselection sector:
  
  Finite set of particles that are linked by the following fusion rules, and their charges are conserved under local operations

Fusion algebra:

\[ a \times b = \sum_c N^c_{ab} c \]

where \( N^c_{ab} \) can be matrices

Abelian \( N^c_{ab} = 1 \)

Non-abelian \( \sum_c N^c_{ab} > 1 \)

- **Associativity**: \[ \sum_e N^e_{ab} N^c_{de} = \sum_e N^e_{bd} N^c_{ea} \]
- **Commutativity**: \[ N^c_{ab} = N^c_{ba} \]
Recap: Fusion rules for minimal models

- Same fusion algebra:
  \[ \phi_i \times \phi_j = \sum_k N^k_{ij} \phi_k \]

- Commutativity also holds:
  \[ N^k_{ij} = N^k_{ji} \]

- Associativity:
  \[ \sum_l N^l_{jk} N^m_{il} = \sum_l N^l_{ij} N^m_{lk} \]

→ Minimal models can be mapped to anyonic models
Fusion spaces

- Fusion spaces $V^c_{ab}$: are subspaces of the space of all possible fusion outcomes which are also Hilbert spaces:

$$V_a \otimes V_b = \bigoplus c N^c_{ab} V^c_{ab}$$

The logical states $|0\rangle$ & $|1\rangle$ will be encoded in one of these fusion spaces.

- Relation of dimensionality of fusion spaces and quantum dimension:

$$d_a d_b = \sum c N^c_{ab} d_c$$

- Simplest non-abelian example: fusion rule for Spin-$\frac{1}{2}$ particles

$$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$$

(i.e. $2 \times 2 = 1 + 3$)
Fusion space basis (fusion trees)

Fibonacci anyons fusion rule:

\[ \tau \otimes \tau = 1 \oplus \tau \]

1 = vacuum
\[ \tau = \text{non-abelian anyon} \]

The Fusion (Anyonic) Hilbert space:

- Fusion trees are orthogonal basis elements of a Hilbert space
- If the initial and final states are fixed then the dimension of this space depends on the number of in-between outcomes. For the above example the dimension is 2.

\[
\left| (\tau \tau \rightarrow 1)\tau \rightarrow \tau > \right| \quad \text{and} \quad \left| (\tau \tau \rightarrow \tau)\tau \rightarrow \tau > \right|
\]
The order of the fusion should not be relevant (associativity):

\[
\begin{align*}
\tau & \quad \tau & \quad \tau \\
1,\tau & \quad \tau & \quad \tau
\end{align*}
\]

Therefore there exists a matrix \( F \) that transforms one basis to the other:

\[
\begin{align*}
&= F
\end{align*}
\]

The R-matrix

Exchange of \( a \) & \( b \) before fusion = self-rotation of outcome \( c \) after fusion:

\[
R_{a,b} = R_{c,b,a}
\]

therefore just a phase factor is obtained. For many in between outcomes \( R = \) diagonal matrix:
Consider superposition of multiple fusion outcomes, and consider the braiding of two particles that do not have a direct fusion outcome. Exchanges result in a non-diagonal matrix $R$.

By applying $F$ matrices on the $R$ matrices, we can change to a basis where the anyons do have a direct fusion outcome:

$$B = FRF^{-1} = \rho(\sigma_i)$$

Example:

$F$ & $R$ fully describe all the processes we can do in an anyonic model of computation.
Compatibility equations: Pentagon & Hexagon equations

Fusion is associative \(\rightarrow\) pentagon equation must hold:

\[
(F_{5_{12c}}^d)^a (F_{5_{a34}}^c)^b = \sum_e (F_{234}^d)^c (F_{1e4}^5)^d (F_{123}^b)^e
\]

Associativity of fusion + allow braiding \(\rightarrow\) Hexagon equation

\[
\sum_b (F_{231}^4)^c R_{1b}^4 (F_{123}^4)^b = R_{13}^c (F_{213}^4)^c R_{12}^a
\]

These two equations encode all the constraints we can impose on F & R
1. Quantum Computer
   - Quantum Computing
   - Decoherence
2. “Any”-ons
   - Abelian anyons
   - Non-abelian anyons
   - Physical realization of non-abelian anyons
   - Fractional Quantum Hall effect
   - Geometric phase
   - Topological Quantum Computer
3. Computing using anyons
   - Braiding
   - Fusion
   - Recap: Fusion rules for minimal models
   - Fusion space
   - F & R matrices
   - The unitary B-matrix
   - Compatibility equations: Pentagon and Hexagon equations
4. Fibonacci model
   - Fibonacci/Yang Lee model
   - Chern-Simons theory
   - Recap of WZW models
5. Summary
Fibonacci anyonic model

- Simple and rich structure
  \[ \tau \otimes \tau = 1 \oplus \tau \]

- Encoding of logical states (the naive way):

  \[ |0\rangle = |(\bullet, \bullet)_1\rangle = \begin{array}{c}
    \bullet
  \end{array} \begin{array}{c}
    \bullet
  \end{array}_1 = \frac{T}{T} \begin{array}{c}
    \bullet
  \end{array} \begin{array}{c}
    \bullet
  \end{array} \]

  \[ |1\rangle = |(\bullet, \bullet)_\tau\rangle = \begin{array}{c}
    \bullet
  \end{array} \begin{array}{c}
    \bullet
  \end{array}_\tau = \frac{T}{T} \begin{array}{c}
    \bullet
  \end{array} \begin{array}{c}
    \bullet
  \end{array} \]

13/05/13  Fibonacci anyons & Topological Quantum Computers  Christos Charalambous
Fibonacci anyonic model

- Simple and rich structure

\[ \tau \otimes \tau = 1 \oplus \tau \]

- Encoding of logical states:

\[ |0\rangle = |((\bullet, \bullet)_1, \bullet)_{\tau} \rangle = \begin{array}{c}
\text{\bullet} \\
\text{\bullet} \\
1 \\
\end{array}_{\tau} = \begin{array}{c}
1 \\
\text{\tau} \\
\text{\tau} \\
\end{array} \]

\[ |1\rangle = |((\bullet, \bullet)_{\tau}, \bullet)_{\tau} \rangle = \begin{array}{c}
\text{\bullet} \\
\text{\bullet} \\
\text{\tau} \\
\end{array}_{\tau} = \begin{array}{c}
\text{\tau} \\
\text{\tau} \\
\text{\tau} \\
\end{array} \]

\[ |N\rangle = |((\bullet, \bullet)_{\tau}, \bullet)_1 \rangle = \begin{array}{c}
\text{\bullet} \\
\text{\bullet} \\
\tau \\
\end{array}_{\tau} = \begin{array}{c}
\text{\tau} \\
\text{\tau} \\
\text{\tau} \\
\end{array} \]

The last state is not a problem as we will see right now:
Fibonacci anyonic model

From fusion rules and pentagon equation for Fibonacci model:

\[ F^{\tau \tau \tau} = F^{1 \tau \tau} = F^{\tau 1 \tau} = F^{\tau \tau 1} = 1 \]

\[ F^{\tau \tau \tau} = \begin{pmatrix} 1 & 1 \\ \varphi & \sqrt{\varphi} \\ 1 & \sqrt{\varphi} & -\frac{1}{\varphi} \end{pmatrix} \]

where \( \varphi \) is the golden ratio \( \varphi = \frac{1 + \sqrt{5}}{2} \)

From hexagon equation and Yang-baxter relation:

\[ R^{\tau 1 \tau} = R^{1 \tau \tau} = 1 \]

\[ R^{\tau \tau} = \begin{pmatrix} e^{i4\pi/5} & 0 \\ 0 & -e^{i2\pi/5} \end{pmatrix} \]
Braiding matrices obtained from $F$ and $R$:

$$\begin{pmatrix}
|0\rangle \\
|1\rangle \\
|N\rangle
\end{pmatrix} \rightarrow
\begin{pmatrix}
e^{-\frac{4\pi i}{5}} & 0 & 0 \\
0 & -e^{-\frac{2\pi i}{5}} & 0 \\
0 & 0 & -e^{-\frac{2\pi i}{5}}
\end{pmatrix}
\begin{pmatrix}
|0\rangle \\
|1\rangle \\
|N\rangle
\end{pmatrix} = \rho(\sigma_1)
$$

$$\begin{pmatrix}
|0\rangle \\
|1\rangle \\
|N\rangle
\end{pmatrix} \rightarrow
\begin{pmatrix}
eg^{-\frac{\pi i}{5}}/\phi & -ie^{-\frac{i\pi}{10}}/\sqrt{\phi} & 0 \\
egie^{-\frac{i\pi}{10}}/\sqrt{\phi} & -1/\phi & 0 \\
0 & 0 & -e^{-\frac{2\pi i}{5}}
\end{pmatrix}
\begin{pmatrix}
|0\rangle \\
|1\rangle \\
|N\rangle
\end{pmatrix} = \rho(\sigma_2)
$$
Fibonacci anyonic model

Dimension of Hilbert space for Fibonacci model with the constraint that no two consecutive 1’s can appear:

Bratelli diagram:

Dimension of the space is $\propto \Phi^n$
Fibonacci anyonic model

Universality

- Solovay and Kitaev (version of brute force search algorithm): Combine short braids $\rightarrow$ can obtain a long braid that with arbitrary accuracy $\epsilon$ will simulate a desired single qubit unitary operation

- Bonesteel, Hormozi: 2-qubit entangling gate CNOT

1st observation:
Fibonacci anyonic model

2nd observation:

Case 1: upper qubit is $|0\rangle = \begin{array}{c} \bullet \bullet 1 \end{array}$

Case 2: upper qubit is $|1\rangle = \begin{array}{c} \bullet \bullet \end{array}$
\[ \rho(\sigma_1) \& \rho(\sigma_2) \] acting on the logical states can perform any unitary evolution in SU(N)

\[ \rightarrow \text{universal computations} \]

Conclusion:
Fibonacci anyonic model:

- Can achieve universal computing
- well-controlled accuracy
- Requires 4n physical anyons for encoding n logical qubits (i.e. polynomial scaling)
Fibonacci anyons and $SU(2)_3$

- Spin-1 particles: $1 \otimes 1 = 0 \oplus 1 \oplus 2$
- Similarity to: $\tau \otimes \tau = 0 \oplus \tau$ if spin-2 is cut off

$SU(2)_k$: "quantized" version of $SU(2)$ obtained by truncating the possible values of the angular momentum to

$$j = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots, \frac{k}{2}$$

E.g. $SU(2)_3 = \{0, \frac{1}{2}, 1, \frac{3}{2}\}$

- Consider only 0 & 1 particles of $SU(2)_3$
  - Subgroup ("even" part) of $SU(2)_3 \cong$ superselection sector of Fibonacci model

- A model described by such symmetry is the $SU(2)_3$ WZW model coupled to a U(1) gauge field
Non-abelian Chern-Simons Theory

\[
S_{CS}(A) = \frac{k}{4\pi} \int_M d^3x \varepsilon^{\mu\nu\rho} \text{tr} \left( A_\mu \partial_\nu A_\rho + i \frac{2}{3} A_\mu A_\nu A_\rho \right) = \frac{1}{4\pi} \int_M d^3x L_{CS}(A)
\]

where \( k \): coupling constant , \( M = \Sigma \times \mathbb{R} \) (2+1D) , \( A \) is a gauge field

- No metric \( \rightarrow \) invariant under diffeomorphisms
- Non Abelian Gauge transformations:
  \[
  A'_\mu = gA_\mu g^{-1} - ig\partial_\mu g^{-1} \quad \text{where} \quad g: M \rightarrow G
  \]

\[
\rightarrow L_{CS}(A') = L_{CS}(A) - k\varepsilon^{\mu\nu\rho} \partial_\mu \text{tr} \left( (\partial_\nu g)g^{-1}A_\rho \right) - \frac{k}{3} \varepsilon^{\mu\nu\rho} \text{tr} \left( g^{-1}(\partial_\mu g)g^{-1}(\partial_\nu g)g^{-1}(\partial_\rho g) \right)
\]

For suitable boundary conditions the \( 1^{\text{st}} \) extra term vanishes in the action.
In the case of simple compact groups e.g. G=SU(2):

\[ \omega(g) = \frac{1}{24\pi^2} \varepsilon^{\mu\nu\rho} tr(g^{-1}(\partial_{\mu}g)g^{-1}(\partial_{\nu}g)g^{-1}(\partial_{\rho}g)) \]

where \( \omega(g) \) = winding number and we realize that is proportional to the 2\textsuperscript{nd} extra term in the Lagrangian. Hence the action becomes:

\[ \rightarrow S_{CS}(A') = S_{CS}(A) + 2\pi k \omega(g) \]

- If \( \omega(g) = 0 \) (small gauge transformations, low energies)
  \[ \rightarrow \text{action is invariant} \]
- If \( \omega(g) \neq 0 \) (large gauge transformations, high energies)
  \[ \rightarrow \text{failure of gauge invariance} \]

Case 1: \( k \) integer \( \rightarrow \) OK

Case 2: \( k \) not an integer \( \rightarrow \) gapped pure gauge degrees of freedom for the high energy theory
Non-abelian Chern-Simons Theory

\[ H = \frac{k}{4\pi} tr(A_2 \partial_0 A_1 - A_1 \partial_0 A_2) - L = 0 \]

Easily seen if we choose gauge \( A_0 = 0 \) where the momenta canonically conjugated to:

\[ A_1: -\frac{k}{4\pi} A_2 \quad , \quad A_2: \frac{k}{4\pi} A_1 \]

Introduce spatial boundaries \( M = \partial \Sigma \times \mathbb{R} \)

- Locally (bulk part): Gauge invariance
- BUT globally: topological obstruction in making the gauge field zero everywhere if \( \Sigma \) topologically non-trivial

\[ \text{Chern-Simons gauge invariant up to a surface term} \]

\[ \text{physical topological degrees of freedom} \]

\[ \text{Chern-Simons: theory of ground state of a 2D topologically ordered system in } \Sigma \]
Non-abelian Chern-Simons Theory

Conclusion

• For low energies:
  → Chern Simons theory describes non abelian anyons.

• For high energies:
  → Difficult to disentangle physical topological degrees of freedom from unphysical local gauge degrees of freedom

→ hence have to consider Chern Simons as low energy effective field theory

What is the theory that describes the excitations of these quasiparticles?
Recap of WZW models

- WZW models describe Symmetry Protected topological Phases (SPT) in 2D at an open boundary with symmetry SU(2).
- Showed global and local SU(2) invariance of $J_+$ (the WZW charge carrying covariant current density) coupled with an external field action.
- Showed that integrating action with an external field leads to an effective field theory:

  \[
  \text{WZW action coupled with external field at low energies} \cong \text{Chern Simon action}
  \]

For WZW models:

- $k$ = number of anyon species in the theory → integer
- gapless WZW gauge degrees of freedom = CS pure gauge degrees of freedom

→ **Solved problem of Chern-Simons in higher energies**
Summary

1. Defined Quantum Computer and identified problem of decoherence

2. Identified topological properties as a remedy for the problem

3. Identified anyons as systems that exhibit such topological properties and hence under specific conditions can accommodate a Topological quantum computer

4. Examined Fibonacci anyons as candidate particles for performing Universal Topological quantum computing

5. Showed that such particles can be theoretically described in the context of a CFT model