D-branes

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Outline

1. String boundary conditions and a first glance to D-branes
   - Demotivation
   - Motivation
   - Interesting configurations:
     - Single Dp-brane
     - Parallel Dp-branes
     - Yang-Mills theories
2. T-duality for open strings
   - Recap T-duality for closed strings
   - T-duality in the presence of open strings
3. Gauge theories confined to D-branes
   - String endpoints coupling to D-branes
   - Dual description of a Dp-brane with E field
   - Dual description of a Dp-brane with B field
4. The Dirac-Born-Infeld action
   - Non-linear electrodynamics
   - Dirac-Born-Infeld Lagrangian

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Outline

Starting point and background

- Theory is specified in terms of an action functional. The Nambu-Goto action:

$$S = \frac{1}{2\pi\alpha'}\int_{\tau_i}^{\tau_f} d\tau \int_0^{\sigma_1} d\sigma \sqrt{-\eta_{\mu\nu}\frac{\partial X^\mu}{\partial\tau}\frac{\partial X^\nu}{\partial\sigma}} = \frac{1}{2\pi\alpha'}\int_{\tau_i}^{\tau_f} d\tau \int_0^{\sigma_1} d\sigma \sqrt{(\dot{X}\cdot X')^2 - \dot{X}^2 X'^2}$$

- Dynamical information is extracted by varying the action:

$$\delta S = \int_{\tau_i}^{\tau_f} d\tau \int_0^{\sigma_1} d\sigma \partial_\tau \left(\frac{\partial\mathcal{L}}{\partial\dot{X}^\mu}\delta X^\mu\right) + \partial_\sigma\left(\frac{\partial\mathcal{L}}{\partial X'^\mu}\delta X^\mu\right) - (\partial_\tau \mathcal{P}^\tau_\mu + \partial_\sigma \mathcal{P}^\sigma_\mu)\delta X^\mu$$

- Equations of motion

$$\partial_\tau \mathcal{P}^{\mu\tau} + \partial_\sigma \mathcal{P}^{\mu\sigma} = 0$$

Boundary Conditions

- Have to get rid of $2(d+1)$ boundary terms:

$$\int_{\tau_i}^{\tau_f} d\tau \int_0^{\sigma_1} d\sigma \, \partial_\sigma \left(\mathcal{P}^\sigma_\mu \delta X^\mu \right) = \int_{\tau_i}^{\tau_f} d\tau \, \left[\mathcal{P}^\sigma_\mu \delta X^\mu \right]_{\sigma=0}^{\sigma=\sigma_1}$$

- We need some boundary conditions. Two possibilities:
  - Neumann: $\mathcal{P}^\sigma_\mu(\tau, \sigma_*) = 0 = \frac{\partial X^\mu}{\partial \sigma}(\tau, \sigma_*); \quad \sigma_* = 0, \sigma_1$
  - Dirichlet: $\delta X^\mu(\tau, \sigma_*) = 0 = \frac{\partial X^\mu}{\partial \tau}(\tau, \sigma_*); \quad \sigma_* = 0, \sigma_1$

- For $\mu = 0$ Dirichlet boundary conditions are not allowed.

Is momentum conserved?

- The Nambu-Goto action is invariant under translations $\delta X^\mu = \epsilon^\mu$. Conserved current: $\partial_\tau \mathcal{P}^{\tau\mu} + \partial_\sigma \mathcal{P}^{\sigma\mu} = 0$

$$p_\mu(\tau) \equiv \int_0^{\sigma_1} \mathcal{P}^\tau_\mu(\tau, \sigma) d\sigma$$

$$\frac{dp_\mu}{d\tau} = \int_0^{\sigma_1} \frac{\partial \mathcal{P}^\tau_\mu}{\partial \tau} d\sigma = -\int_0^{\sigma_1} \frac{\partial \mathcal{P}^\sigma_\mu}{\partial \sigma} d\sigma = -\mathcal{P}^\sigma_\mu|_0^{\sigma_1}$$

- This is not necesarily vanishing for Dirichlet boundary conditions
- Furthermore what interpretation should we give to these boundary conditions? Where are the string endpoints attached?

There is more than just strings!

- A Dp-brane is an extended object with p spatial dimensions
- The endpoints of open strings are attached to D-branes
- D-branes have a personality of their own: energy density, momentum, charge...
- The overall momentum in the string and the D-brane is conserved
- In the presence of a Dp-brane the original Lorentz symmety is broken:

$$SO(1,d) \rightarrow SO(1,p) \times SO(d-p)$$

Conventions and notation

- Open strings have parameterization range $[0, \pi]$
- Closed strings have parameterization range $[0, 2\pi]$
- Different kind of indices:
  - μ, ν... are spacetime indices that run from 0 to d
  - m, n... are world-brane indices that run from 0 to p
  - i, j... are spatial indices on the brane that run from 1 to p
  - a, b... are spatial indices normal to the brane that run from $(p + 1)$ to d
  - I is a generic index than stands for all the transverse coordinates $I \to (i, a)$

Gauge coordinates and gauge fixing

- We work in light-cone coordinates: $X^+, X^-, \{X^I\}_{I=2,\dots,d}$
- We work in light cone-gauge:

$$n \cdot X(\tau,\sigma) = \beta\alpha'(n \cdot p)\tau \qquad n \cdot p = \frac{2\pi}{\beta} n \cdot \mathcal{P}^\tau$$

$$n_\mu = \frac{1}{\sqrt{2}}(1, 1, 0, \dots, 0)$$

- Equations of motion are now easy wave equations:

$$\ddot{X}^\mu - X^{\mu''} = 0$$

- Two constraints:
  $(\dot{X} \pm X')^2 = 0 = -2(\dot{X}^+ \pm X^{+'})(\dot{X}^- \pm X^{-'}) + (\dot{X}^I \pm X^{I'})$
- The full evolution of the string is determined by:

$$X^I(\tau,\sigma), \qquad p^+, \qquad x_0^-$$

Solution of the wave equation for NN coordinates

- Coordinates satisfy the wave equation so:

$$X^i(\tau,\sigma) = \frac{1}{2}\Big(f^i(\tau+\sigma) + g^i(\tau-\sigma)\Big)$$

- Boundary conditions at $\sigma = 0$ implies:

$$X^i(\tau,\sigma) = \frac{1}{2}\Big(f^i(\tau+\sigma) + f^i(\tau-\sigma)\Big)$$

- Boundary condition at $\sigma = \pi$ implies that $f^{i'}$ is 2π periodic

Mode expansion for NN coordinates and quantization

- The mode expansion can be written as:

$$X^i(\tau,\sigma) = x_0^i + \sqrt{2\alpha'}\alpha_0^i\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^i e^{-in\tau}cos(n\sigma)$$

- Note that there is a term linear in τ so the net momentum is not vanishing. In fact $p^i = \frac{\alpha_0^i}{\sqrt{2\alpha'}}$
- To quantize we impose the communtation relations:

$$\left[X^i(\tau,\sigma), \mathcal{P}^{\tau j}(\tau,\sigma')\right] = i\eta^{ij}\delta(\sigma-\sigma')$$

$$[x_0^-, p^+] = -i$$

- In terms of oscillators the first commutator translates into:

$$\left[\alpha_m^i, \alpha_n^j\right] = m\eta^{ij}\delta_{m+n,0} \qquad \left[x_0^i, p^j\right] = i\eta^{ij}$$

States and mass operator for space-filling brane

- A general state is of the form:

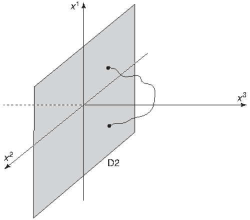
$$\prod_{n=1}^{\infty}\prod_{i=2}^{25}\left(a_n^{i\dagger}\right)^{\lambda_{n,i}}|p^+,\vec{p}\rangle$$

- The mass operator is then:

$$M^2=-p^2=2p^+p^- - p^i p^i =$$

$$=\frac{1}{\alpha'}\left(-1+\sum_{n=1}^{\infty}\sum_{i=2}^{25} n a_n^{i\dagger} a_n^i\right)=\frac{1}{\alpha'}(N^\perp-1)$$

A single Dp-brane



- $1 \leq p \leq (d-1) \longrightarrow$ *NN* and *DD* coordinates
- Remember the notation:
  - Dp tangential coordinates:

$$x^0, x^1, ..., x^p \rightarrow NN \longrightarrow X'^m(\tau,\sigma)|_{\sigma=0} = X'^m(\tau,\sigma)|_{\sigma=\pi} = 0$$

  - Dp normal coordinates:

$$x^{p+1}, x^{p+2}, ..., x^d \rightarrow DD \longrightarrow X^a(\tau,\sigma)|_{\sigma=0} = X^a(\tau,\sigma)|_{\sigma=\pi} = \bar{x}^a$$

Solution of the wave equation for DD coordinates

- Solution of the wave equation for the DD coordinates is slightly different than for NN coordinates
- Coordinates satisfy the wave equation so:

$$X^a(\tau,\sigma) = \frac{1}{2}\left(f^a(\tau+\sigma) + g^a(\tau-\sigma)\right)$$

- Boundary conditions at $\sigma = 0$ implies:

$$X^a(\tau,\sigma) = \bar{x}^a + \frac{1}{2}\left(f^a(\tau+\sigma) - f^a(\tau-\sigma)\right)$$

- Boundary condition at $\sigma = \pi$ implies that the function f is 2π periodic

Mode expansion for DD coordinates and quantization

- The mode expansion can be written as:

$$X^a(\tau,\sigma) = \bar{x}^a + \sqrt{2\alpha'} \sum_{n\neq 0} \frac{1}{n} \alpha_n^a e^{-in\tau} sin(n\sigma)$$

- Note that the term linear in τ present for *NN* coordinates is now missing $\longrightarrow$ No net time-averaged momentum, no zero mode
- To quantize we impose the communtation relations:

$$\left[X^a(\tau,\sigma), \mathcal{P}^{\tau b}(\tau,\sigma')\right] = i\delta^{ab}\delta(\sigma-\sigma')$$

- In terms of oscillators this means:

$$\left[\alpha_m^a, \alpha_n^b\right] = m\delta^{ab}\delta_{m+n,0} = 0, \qquad m, n \neq 0$$

States and mass operator for a Dp-brane

- A general state is of the form:

$$\left[\prod_{n=1}^{\infty}\prod_{i=2}^{p}\left(a_n^{i\dagger}\right)^{\lambda_{n,i}}\right]\left[\prod_{m=1}^{\infty}\prod_{a=p+1}^{d}\left(a_m^{a\dagger}\right)^{\lambda_{m,a}}\right]|p^+,\vec{p}\rangle$$

- The mass operator is then:

$$M^2=-p^2=2p^+p^- - p^ip^i=$$

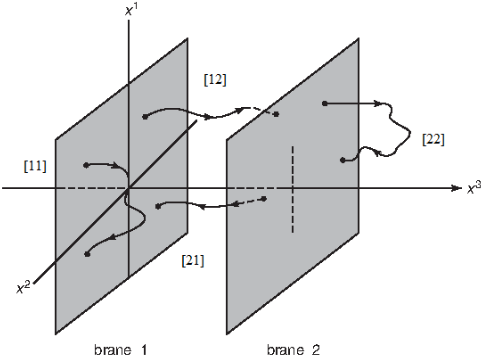
$$=\frac{1}{\alpha'}\left(-1+\sum_{n=1}^{\infty}\sum_{i=2}^{p}na_n^{i\dagger}a_n^i+\sum_{m=1}^{\infty}\sum_{a=p+1}^{d}ma_m^{a\dagger}a_m^a\right)=\frac{1}{\alpha'}(N^{\perp}-1)$$

Ground states and first excited states

- The ground states $|p^+, \vec{p}\rangle$ have a mass $M^2 = \frac{-1}{\alpha'}$
    - Tachyons
    - Lorentz scalars
- First excited states can be either tangent or normal to the brane:
    - Tangent: $a_1^{i\dagger} |p^+, \vec{p}\rangle \qquad i = 2, ..., p$
        - p-1 massless states
        - Lorentz vector
        - Photons$\rightarrow$ a Dp-brane has a Maxwell field living on its world volume
    - Normal : $a_1^{a\dagger} |p^+, \vec{p}\rangle \qquad a = p + 1, ..., d$
        - d-p massless states
        - scalars $\rightarrow$ a Dp-brane has a massless scalar field for each normal direction

Parallel Dp-branes

- Different sectors, depending on which brane the string begins/ends. Chan-Paton indices [ij]



Mode expansion and Mass of stretched strings

- Mode expansion for the normal coordinates picks and extra term:

$$X^a(\tau,\sigma) = \bar{x}_1^a + (\bar{x}_2^a - \bar{x}_1^a)\frac{\sigma}{\pi} + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^a e^{-in\tau} sin(n\sigma)$$

- Mass operator picks up an extra term accordingly:

$$M^2 = \left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi\alpha'}\right)^2 + \frac{1}{\alpha'}(N^{\perp} - 1)$$

Sectors and states

- The vaccum state of sector [ij] is $|p^+, \vec{p}; [ij]\rangle$. It can be excited as usual:

$$\left[\prod_{n=1}^{\infty}\prod_{i=2}^{p}\left(a_n^{i\dagger}\right)^{\lambda_{n,i}}\right]\left[\prod_{m=1}^{\infty}\prod_{a=p+1}^{d}\left(a_m^{a\dagger}\right)^{\lambda_{m,a}}\right]|p^+, \vec{p}; [ij]\rangle$$

- Excitations do not mix different sectors, so there is no need to label the operators

Ground state of the [12] sector

- Where do the fields in a mixed sector live?
- The ground state $|p^+, \vec{p}; [ij]\rangle$ is now massive:

$$M^2 = \left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi\alpha'}\right)^2 - \frac{1}{\alpha'}$$

- Depending on the separation of the branes we can have a massive, massless or tachyonic scalar field

First excited states in the [12] sector

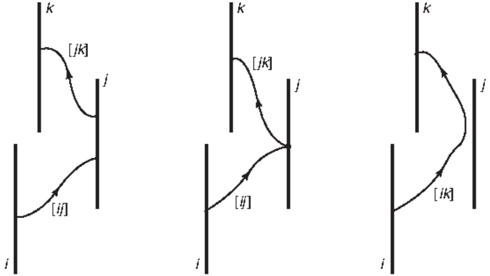
- First excited states can be either normal or tangent to the branes
  - Normal: $a_1^{a\dagger} |p^+, \vec{p}; [12]\rangle \qquad a = p+1, ..., d$
    - have mass $M^2 = \left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi\alpha'}\right)^2$
    - d-p massive states
  - Tangent: $a_1^{i\dagger} |p^+, \vec{p}; [12]\rangle \qquad i = 2, ..., p$
    - have mass $M^2 = \left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi\alpha'}\right)^2$
    - p-1 massive states $\rightarrow$ massive gauge field? Almost...
- These (p-1) states are not enough degrees of freedom to represent a massive gauge field
- One of the scalar states joins the (p-1) set and this new set transform as a massive gauge field

$$\sum_a (\bar{x}_2^a - \bar{x}_1^a) a_1^{a\dagger} |p^+, \vec{p}; [12]\rangle$$

Yang-Mill theories

- If the two D-branes overlap the mass of all the first excited states vanishes and we are left with the same field content as in the case of a single Dp-brane: a massless gauge field and (d-p) massless scalar.
- Considering all sectors we have:
  - $4(d-p)$ massless scalars
  - Four massless gauge fields$\Longrightarrow U(2)$ Yang-Mills theory

- In general N coincident D-branes carry $U(N)$ massles gauge fields $\longrightarrow$ AdS/CFT (Next talk)
- Can visualize interactions:



$$[ij] * [jk] = [ik]$$

Recap T-duality for closed strings

- When a spatial dimension is compactified TWO interesting things happen:
  - The momentum gets quantized: $p = \frac{n}{R}$
  - A new form of momentum (which is also quantized) appears: the winding $\omega = \frac{mR}{\alpha'}$
- Mode expansion for a compactified coordinate:

$$X(\tau,\sigma) = x_0 + \alpha' p\tau + \alpha'\omega\sigma + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{e^{-in\tau}}{n}\left(\bar{\alpha}_n e^{-in\sigma} + \alpha_n e^{in\sigma}\right)$$

- There is an extra term and so the quantum theory will have an extra operator, the winding: ω.
- This operator will also contribute to the mass square

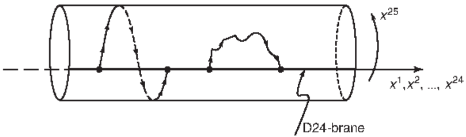
$$M^2 = \frac{n^2}{R^2} + \frac{m^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}(N^\perp + \bar{N}^\perp - 2)$$

T-duality in the presence of open strings

- When a spatial dimension is compactified ONE interesting thing happens:
  - Momentum gets quantized: $p = \frac{n}{R}$
- T-duality is a good symmetry only if branes transform

$$(D25; R) \Longrightarrow \left(D24; \tilde{R} = \frac{\alpha'}{R}\right)$$

- The brane has changed dimesion as a consequence:
  - Nontrivial winding states appear
  - There is no momentum



Left/right movers and dual coordinate

- For an *NN* coordinate:

$$X(\tau,\sigma) = x_0 + \sqrt{2\alpha'}\alpha_0\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n cos\ n\sigma\, e^{-in\tau}$$

- Right and left movers:
  - $X_L = \frac{1}{2}(x_0 + q_0) + \sqrt{\frac{\alpha'}{2}}\alpha_0(\tau+\sigma) + \frac{i}{2}\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n e^{-in\tau}e^{-in\sigma}$
  - $X_R = \frac{1}{2}(x_0 - q_0) + \sqrt{\frac{\alpha'}{2}}\alpha_0(\tau-\sigma) + \frac{i}{2}\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n e^{-in\tau}e^{in\sigma}$
- Guided by closed string T-duality we define the dual coordinate $\tilde{X}(\tau,\sigma) \equiv X_L(\tau+\sigma) - X_R(\tau-\sigma)$

- $\tilde{X}(\tau,\sigma) = q_0 + \sqrt{2\alpha'}\alpha_0\sigma + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n e^{-in\tau} sin\, n\sigma$
- This is just the mode expansion of a *DD* coordinate since:

$$\sqrt{2\alpha'}\alpha_0 = \frac{(\bar{x}_2 - \bar{x}_1)}{\pi}$$

- $[\tilde{X}(\tau,\sigma),\tilde{\mathcal{P}}(\tau,\sigma')] = i\delta(\sigma-\sigma')$ provided $[X(\tau,\sigma),\mathcal{P}(\tau,\sigma')] = i\delta(\sigma-\sigma')$
- $\tilde{X}(\tau,\pi) - \tilde{X}(\tau,0) = \sqrt{2\alpha'}\alpha_0\pi = 2\pi\alpha' p = 2\pi\frac{\alpha'}{R}n = 2\pi\tilde{R}n$
- Note that the duality interchanges boundary conditions:
  - $\partial_\sigma X = X_L'(\tau+\sigma) - X_R'(\tau-\sigma) = \partial_\tau\tilde{X}$
  - $\partial_\tau X = X_L'(\tau+\sigma) + X_R'(\tau-\sigma) = \partial_\sigma\tilde{X}$

Interacting Lagrangian and boundary conditions

- The Maxwell field that lives on a brane couples to the string's endpoints like if they were point charges:

$$S = S_{NG} + \int d\tau A_m(X)\frac{dX^m}{d\tau}|_{\sigma=\pi} - \int d\tau A_m(X)\frac{dX^m}{d\tau}|_{\sigma=0}$$

- We consider only field configurations such that F_{mn} is constant, thus:

$$A_n(X) = \frac{1}{2}F_{mn}X^m$$

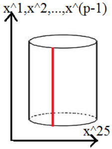
- Boundary conditions are:

$$\mathcal{P}^\sigma_m + F_{mn}\partial_\tau X^n = 0 \qquad \sigma = 0, \pi$$

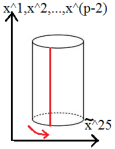
$$\partial_\sigma X_m - 2\pi\alpha' F_{mn}\partial_\tau X^n = 0 \qquad \sigma = 0, \pi$$

Dual description of a Dp-brane with E field

Scenario 1



Scenario 2



- Boundary conditions:

$$\partial_+ \begin{pmatrix} X^0 \\ X \end{pmatrix} = \begin{pmatrix} \frac{1+\mathcal{E}^2}{1-\mathcal{E}^2} & \frac{2\mathcal{E}}{1-\mathcal{E}^2} \\ \frac{2\mathcal{E}}{1-\mathcal{E}^2} & \frac{1+\mathcal{E}^2}{1-\mathcal{E}^2} \end{pmatrix} \partial_- \begin{pmatrix} X^0 \\ X \end{pmatrix}$$

$$\mathcal{E} \equiv 2\pi\alpha' E$$

- Boundary conditions:

$$\partial_+ \begin{pmatrix} X'^0 \\ \tilde{X}' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \partial_- \begin{pmatrix} X'^0 \\ \tilde{X}' \end{pmatrix}$$

- We can make a boost to express the boundary conditions in the sytem where the D($p-1$)-brane is moving:

$$\partial_+ \begin{pmatrix} X^0 \\ \tilde{X} \end{pmatrix} = M^{-1} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} M \, \partial_- \begin{pmatrix} X^0 \\ \tilde{X} \end{pmatrix}$$

Dual description of a Dp-brane with E field

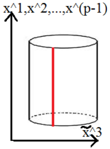
- Using the duality relations we obtain:

$$\partial_+ \begin{pmatrix} X^0 \\ X \end{pmatrix} = \begin{pmatrix} \frac{1+\beta^2}{1-\beta^2} & \frac{2\beta}{1-\beta^2} \\ \frac{2\beta}{1-\beta^2} & \frac{1+\beta^2}{1-\beta^2} \end{pmatrix} \partial_- \begin{pmatrix} X^0 \\ X \end{pmatrix}$$

- Boundary conditions match those of Scenario 1 provided we identify $\mathcal{E}$ and $\beta \implies \mathcal{E} \equiv 2\pi\alpha' E = \beta$
- Since the brane can not move faster than light the E field is bounded: $E \leq \frac{1}{2\pi\alpha'} = T_0$

Dual description of a Dp-brane with B field

Scenario 1

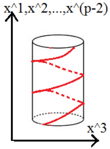


- Boundary conditions:

$$\partial_+ \begin{pmatrix} X^2 \\ \tilde{X}^3 \end{pmatrix} = \begin{pmatrix} \frac{1-\mathcal{B}^2}{1+\mathcal{B}^2} & \frac{2\mathcal{B}}{1-\mathcal{B}^2} \\ \frac{-2\mathcal{B}}{1+\mathcal{B}^2} & \frac{1-\mathcal{B}^2}{1+\mathcal{B}^2} \end{pmatrix} \partial_- \begin{pmatrix} X^2 \\ \tilde{X}^3 \end{pmatrix}$$

$$\mathcal{B} \equiv 2\pi\alpha' B$$

Scenario 2



- Boundary conditions:

$$\partial_+ \begin{pmatrix} X'^2 \\ X'^3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \partial_- \begin{pmatrix} X'^2 \\ X'^3 \end{pmatrix}$$

- We can make a rotation to express the boundary conditions in the sytem where the D$(p-1)$-brane is not tilted:

$$\partial_+ \begin{pmatrix} X^2 \\ X^3 \end{pmatrix} = R^{-1} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} R \, \partial_- \begin{pmatrix} X^2 \\ X^3 \end{pmatrix}$$

Dual description of a Dp-brane with B field

- Using the duality relations we obtain:

$$\partial_+ \begin{pmatrix} X^2 \\ \tilde{X}^3 \end{pmatrix} = \begin{pmatrix} cos2\alpha & -sin2\alpha \\ sin2\alpha & cos2\alpha \end{pmatrix} \partial_- \begin{pmatrix} X^2 \\ \tilde{X}^3 \end{pmatrix}$$

- Boundary conditions match those of Scenario 1 provided we identify $-\mathcal{B}$ and $tan\alpha \Longrightarrow \mathcal{B} \equiv 2\pi\alpha' B = -tan\,\alpha$
- A zero magnetic field produces no rotation, an infinite magnetic field is required to rotate the D-brane by ninety degrees

Non-linear electrodynamics

- Relation $(\vec{E}, \vec{B}) \sim (\vec{D}, \vec{H})$
- Maxwell theory allows arbitrary large E fields

Dirac-Born-Infeld Lagrangian

- Dirac-Born-Infeld Lagrangian:

$$\mathcal{L} = -b^2\sqrt{-det\left(\eta_{\mu\nu} + \frac{1}{b}F_{\mu\nu}\right)} + b^2$$

- Gauge and Lorentz invariant but also:
    - Reduces to Maxwell Lagrangian for small E and B
    - E is bounded when $B = 0$
    - Yields finite electrostatic self-energy:

$$U_Q = \frac{1}{4\pi}\frac{1}{3}(\Gamma(\frac{1}{4}))^2 b^{\frac{1}{2}} Q^{\frac{3}{2}}$$

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THANK YOU