Metal - screening

Thomas-Fermi (static) screening $\omega = 0$ $\vec{q} \ll 2k_F$

$$\chi_0(\vec{q},0) \approx -\frac{3n_0}{2\epsilon_F} \rightarrow \varepsilon(\vec{q},0) = 1 - \frac{4\pi e^2}{q^2}\chi_0(\vec{q},0) \approx 1 + \frac{k_{TF}^2}{q^2}$$

potential of point charge

$$V_a(\vec{r}) = \frac{e^2}{r} \xrightarrow{\textit{Fourier}} V_a(\vec{q}) = \frac{4\pi e^2}{q^2}$$

Coulomb

renormalize

$$V(\vec{q}) = \frac{V_a(\vec{q})}{\varepsilon(\vec{q},0)} = \frac{4\pi e^2}{q^2 + k_{TF}^2}$$

Fourier

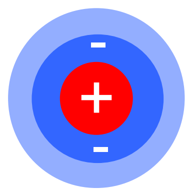
Yukawa

$$V(\vec{r}) = \frac{e^2}{r} e^{-k_{TF} r}$$

$$\vec{\nabla}^2 V = 4\pi\rho$$

$$\Downarrow$$

$$\vec{\nabla}^2 V - k_{TF}^2 V = 4\pi\rho$$



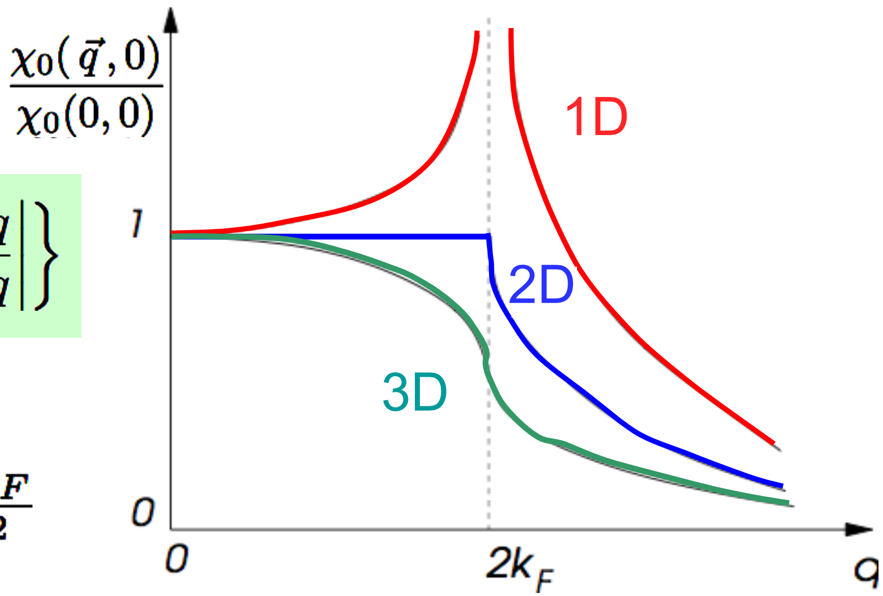
Metal - screening

static dielectric function

$$\chi_0(q,0) = \begin{cases} -\dfrac{1}{2\pi q}\ln\left|\dfrac{s+2}{s-2}\right|, & \text{1D} \\ -\dfrac{1}{2\pi}\left\{1-\left(1-\dfrac{4}{s^2}\right)\theta(s-2)\right\}, & \text{2D} \\ -\dfrac{k_F}{2\pi^2}\left\{1-\dfrac{s}{4}\left(1-\dfrac{4}{s^2}\right)\ln\left|\dfrac{s+2}{s-2}\right|\right\}, & \text{3D} \end{cases}$$

$$s = \frac{q}{2k_F}$$

singularity at $2k_F$



dielectric function in 3D

$$\varepsilon(\vec{q},0) = 1 + \frac{4e^2 m k_F}{\pi q^2}\left\{\frac{1}{2} + \frac{4k_F^2 - q^2}{8k_F q}\ln\left|\frac{2k_F+q}{2k_F-q}\right|\right\}$$

singular at $2k_F$, unlike

small-q approximation $\varepsilon(\vec{q},0) \approx 1 + \dfrac{k_{TF}^2}{q^2}$

Metal - screening

Friedel oscillation $\quad \omega = 0$

point charge

$$e n_a(\vec{r}) = e\delta(\vec{r}) \xrightarrow[\text{redistribution}]{\text{charge}} \boxed{\delta n(\vec{q})} = \chi_0(\vec{q}) V(\vec{q}) = \chi_0(\vec{q}) \frac{V_a(\vec{q})}{\varepsilon(\vec{q})}$$

$$= \frac{\chi_0(\vec{q})}{\varepsilon(\vec{q})} \frac{4\pi e^2}{q^2} n_a(\vec{q}) = \boxed{\frac{1-\varepsilon(\vec{q})}{\varepsilon(\vec{q})}}$$

$$\boxed{\delta n(\vec{r})} = \int \frac{d^3 q}{(2\pi)^3} \delta n(\vec{q}) e^{i\vec{q}\cdot\vec{r}} = \int_0^\infty \frac{dq}{(2\pi)^2} q^2 \int_{-1}^{+1} d\cos\theta\, e^{iqr\cos\theta} \delta n(\vec{q})$$

$$= \int_0^\infty \frac{dq\, q}{2\pi^2 r} \frac{1-\varepsilon(q)}{\varepsilon(q)} \sin(qr) = \frac{1}{r}\int_0^\infty dq g(q)\sin(qr) = \boxed{-\frac{1}{r^3}\int_0^\infty dq g''(q)\sin(qr)}$$

2x integration by parts

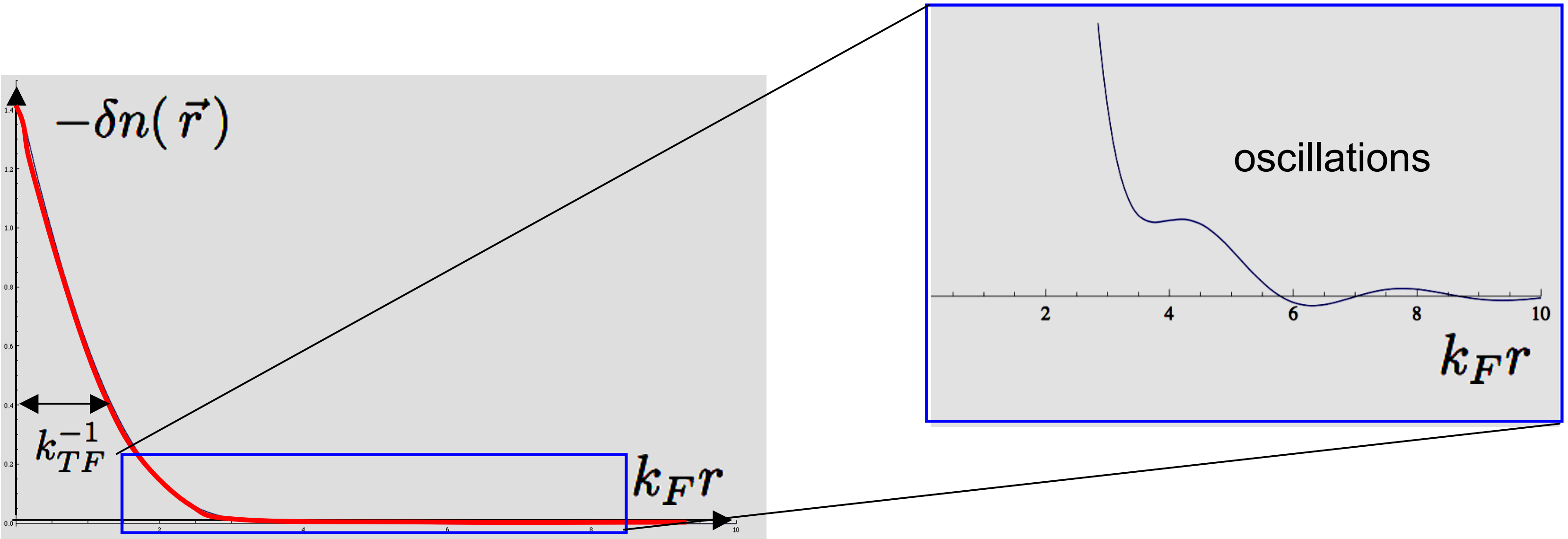
$$\varepsilon(\vec{q},0) = 1 + \frac{4e^2 m k_F}{\pi q^2}\left\{\frac{1}{2} + \frac{4k_F^2 - q^2}{8k_F q}\ln\left|\frac{2k_F + q}{2k_F - q}\right|\right\} \xrightarrow[\text{part}]{\text{most singular}} \boxed{g''(q) \approx \frac{A}{q - 2k_F}}$$

Metal - screening

Friedel oscillation

$$\boxed{\delta n(r)} \approx -\frac{A}{r^3}\int_{2k_F-\Lambda}^{2k_F+\Lambda} \frac{\sin[(q-2k_F)r]\cos 2k_F r + \cos[(q-2k_F)r] sin2k_F r}{q-2k_F} dq$$

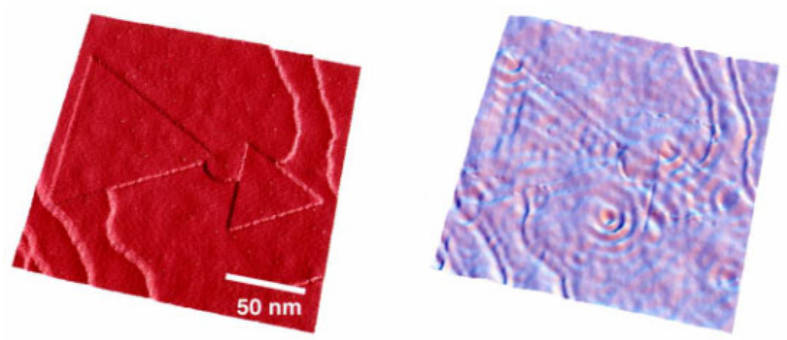
$$\longrightarrow \boxed{-\pi A\frac{\cos 2k_F r}{r^3}} \qquad k_F r \gg 1 \qquad \Lambda \to \infty$$



Metal - screening

Friedel oscillation

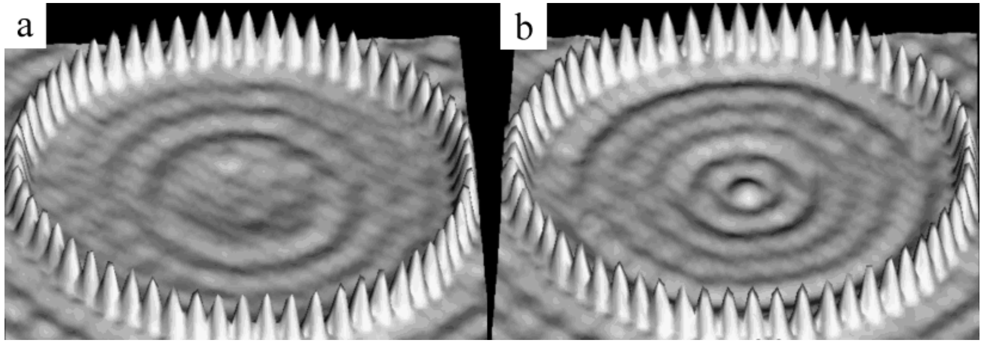
scanning tunneling microscope - pictures



Topography

Local Density of States (LDOS)
(Friedel Oscillation)

NTT: report 2000 - 2D semiconductor surface



quantum corral by Fe-atoms on Cu surface

D. Eigler, IBM Almaden

total charge cloud

$$\delta n_{tot} = \int d^3 r \, \delta n(\vec{r}) = \delta n(\vec{q}) = \lim_{\vec{q}\to 0} \frac{1-\varepsilon(\vec{q})}{\varepsilon(\vec{q})} = -1$$

$$\lim_{\vec{q}\to 0} \varepsilon(\vec{q}) = \infty$$

point charge at $\vec{r} = 0$

completely compensated