Metals - electron-hole excitations

linear response function:

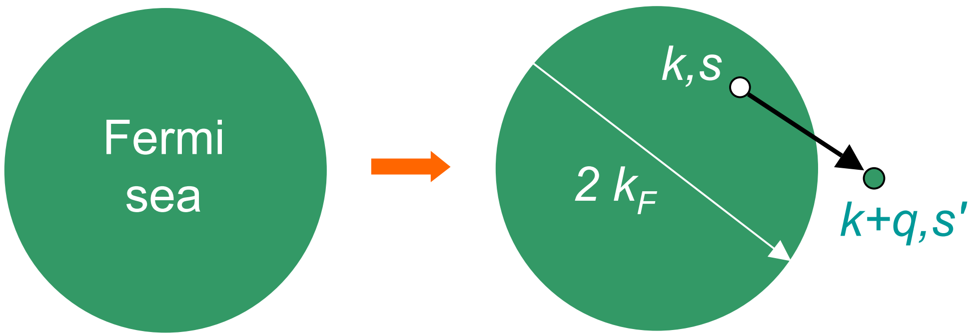
$$\chi_0(\vec{q},\omega) = \frac{1}{\Omega}\sum_{\vec{k},s} \frac{n_{0\,\vec{k}+\vec{q},s} - n_{0\,\vec{k},s}}{\epsilon_{\vec{k}+\vec{q}} - \epsilon_{\vec{k}} - \hbar\omega - i\hbar\eta}$$

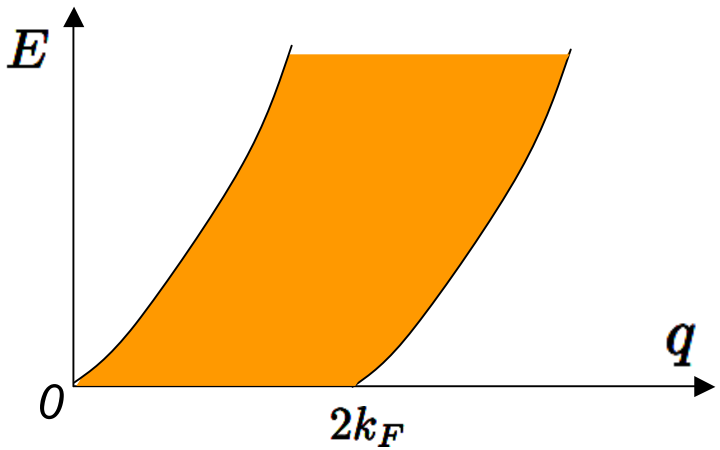
Lindhard function

$$\lim_{\eta\to 0_+} \frac{1}{z - i\eta} = \mathcal{P}\left(\tfrac{1}{z}\right) + i\pi\delta(z)$$

$$\Rightarrow \begin{cases} \chi_{01}(\vec{q},\omega) = \dfrac{1}{\Omega}\displaystyle\sum_{\vec{k},s} \mathcal{P}\left(\dfrac{n_{0,\vec{k}+\vec{q}} - n_{0,\vec{k}}}{\epsilon_{\vec{k}+\vec{q}} - \epsilon_{\vec{k}} - \hbar\omega}\right) \\ \chi_{02}(\vec{q},\omega) = \dfrac{1}{\Omega}\displaystyle\sum_{\vec{k},s} (n_{0,\vec{k}+\vec{q}} - n_{0,\vec{k}})\delta(\epsilon_{\vec{k}+\vec{q}} - \epsilon_{\vec{k}} - \hbar\omega) \end{cases}$$

$$E = \epsilon_{\vec{k}+\vec{q}} - \epsilon_{\vec{k}}$$





Metals - collective excitations

linear response function:

$$\chi_0(\vec{q},\omega) = \frac{1}{\Omega}\sum_{\vec{k},s} \frac{n_{0\,\vec{k}+\vec{q},s} - n_{0\,\vec{k},s}}{\epsilon_{\vec{k}+\vec{q}} - \epsilon_{\vec{k}} - \hbar\omega - i\hbar\eta}$$

Lindhard function

small-q-limit: $$\epsilon_{\vec{k}+\vec{q}} \approx \epsilon_{\vec{k}} + \vec{q}\cdot\vec{\nabla}_{\vec{k}}\,\epsilon_{\vec{k}} = \epsilon_{\vec{k}} + \vec{q}\cdot\hbar\vec{v}_{\vec{k}}$$

$$n_{0,\vec{k}+\vec{q}} \approx n_{0,\vec{k}} + \frac{\partial n_0}{\partial\epsilon}\vec{q}\cdot\vec{\nabla}_{\vec{k}}\,\epsilon_{\vec{k}} = n_{0,\vec{k}} - \delta(\epsilon_{\vec{k}} - \epsilon_F)\,\vec{q}\cdot\hbar\vec{v}_{\vec{k}}$$

$$\chi_0(\vec{q},\omega) \approx -2\int \frac{d^3k}{(2\pi)^3}\,\frac{\vec{q}\cdot\vec{v}_F\,\delta(\epsilon_{\vec{k}} - \epsilon_F)}{\vec{q}\cdot\vec{v}_F - \omega - i\eta}$$

$$\approx \frac{n_0 q^2}{m(\omega+i\eta)^2}\left(1 + \frac{3}{5}\frac{v_F^2 q^2}{(\omega+i\eta)^2}\right)$$

Metals - collective excitations

$$\chi_0(\vec{q},\omega) \approx \frac{n_0 q^2}{m(\omega+i\eta)^2}\left(1+\frac{3}{5}\frac{v_F^2 q^2}{(\omega+i\eta)^2}\right) = \frac{n_0 q^2}{m(\omega+i\eta)^2}R(q,\omega)^2$$

$$\varepsilon(\vec{q},\omega) = 1 - \frac{4\pi e^2}{q^2}\chi_0(\vec{q},\omega) \approx 1 - \frac{\omega_p^2}{\omega^2}$$

plasma frequency

$$\omega_p^2 = \frac{4\pi e^2 n_0}{m}$$

renormalized: $$\chi(\vec{q},\omega) = \frac{\chi_0(\vec{q},\omega)}{1-\frac{4\pi e^2}{q^2}\chi_0(\vec{q},\omega)}$$

$$\chi(\vec{q},\omega) \approx \frac{n_0 q^2 R(q,\omega)^2}{(\omega+i\eta)^2 - \frac{4\pi e^2 n_0^2}{m}R(q,\omega)^2}$$

$$= \frac{n_0 q^2 R(q,\omega)}{\omega_p}\left\{\frac{1}{\omega+i\eta-\omega_p R(q,\omega)} - \frac{1}{\omega+i\eta+\omega_p R(q,\omega)}\right\}$$

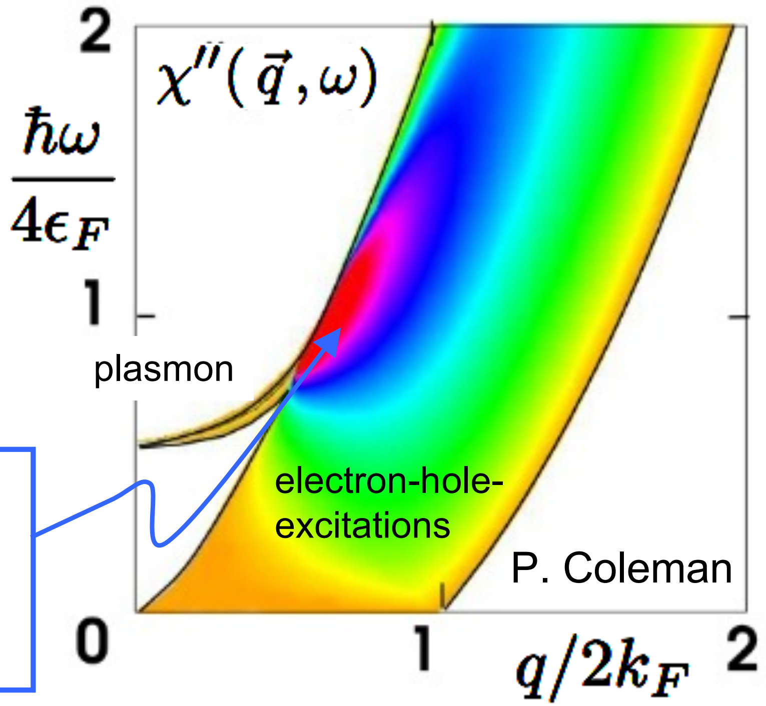
Metals - collective excitations

$$\chi(\vec{q},\omega) \approx \frac{n_0 q^2 R(q,\omega)}{\omega_p} \left\{ \frac{1}{\omega + i\eta - \omega_p R(q,\omega)} - \frac{1}{\omega + i\eta + \omega_p R(q,\omega)} \right\}$$

spectrum in imagnary part

$$\chi''(\vec{q},\omega) \approx \frac{\pi n_0 q^2 R(q,\omega_p)}{\omega_p} \left[\delta(\omega - \omega_p R(q,\omega_p)) - \delta(\omega + \omega_p R(q,\omega_p)) \right]$$

$$\begin{aligned} \omega(\vec{q}) &= \omega_p R(q,\omega_p) \\ &= \omega_p \left\{ 1 + \frac{3 v_F^2 q^2}{10 \omega_p^2} + \cdots \right\} \end{aligned}$$



Landau damping in e-h-continuum

decay of plasmon into electron-hole excitations → *finite lifetime*

Metals - collective excitations

$$\omega_p^2 = \frac{4\pi e^2 n_0}{m}$$

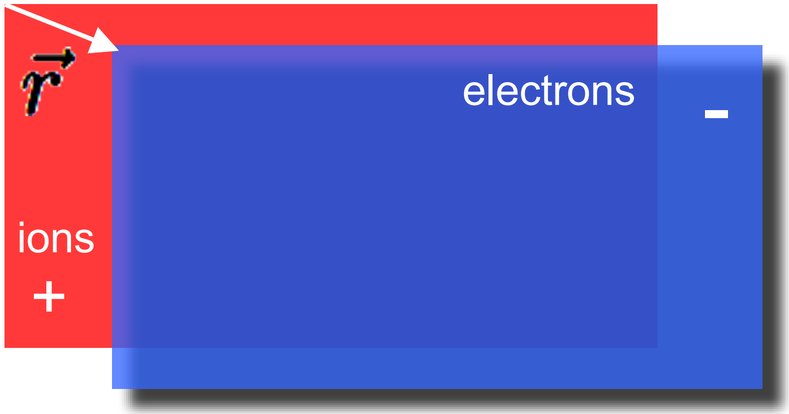
simple metals $n_0 = \frac{3}{4\pi d^3} = \frac{3}{4\pi (r_s a_B)^3}$

$r_{s,Li} = 3.22 \quad r_{s,Na} = 3.96 \quad r_{s,K} = 4.86$

metal	$\omega_p^{(exp)}$ [eV]	$\omega_p^{(theo)}$ [eV]
Li	7.1	8.5
Na	5.7	6.2
K	3.7	4.6
Mg	10.6	-
Al	15.3	-

Metals - collective excitations

classical picture



restoring force

$$\vec{P} = -e n_0 \vec{r}$$

$$\vec{E} = -4\pi \vec{P}$$

equation of motion

$$m \frac{d^2 \vec{r}}{dt^2} = -e \vec{E} = -4\pi e^2 n_0 \vec{r}$$

harmonic oscillator

oscillation frequency

$$\omega_p^2 = \frac{4\pi e^2 n_0}{m}$$

plasma frequency