Chapter 3

Question 1. *Waves and Potentials*

How do I best approach a particle-in-a-potential-well-problem?

Answer: First thing is to split the wave function into the relevant sections: before the well, in the well, after the well. The next thing is to split the wave into incoming parts (the part with e^{ikx}) and reflected parts (with e^{-ikx}) and think about which type of solution we expect for each part. For instance, this can give us that there is no left-travelling part behind the well (so in the final section we do not have a e^{-ikx}-type part in the solution), or tells us whether to expect real or imaginary solutions for k (depending on if we expect plane waves or exponential decay), depending on the particular problem we study.

After this, we can impose the boundary conditions where the different sections share borders: here, both the wave function itself needs to match, as well as the first derivative thereof. By setting those equal at the boundaries, we obtain a set of equations that we can then solve for the respective coefficients.

One final thing to note: When solving for the coefficients, one can of course only find everything depending on the incoming part of the wave: either set in the left-most, incoming part the coefficient in front of e^{ikx} equal to 1 or find relative transmission and reflection coefficients.

Chapter 7

Question 2. *Reducible and irreducible representations*

Sometimes what we call "the representation of a group" seems to refer to the operators that perform the group action (e.g. rotation), sometimes the Hilbert space they act on. What exactly is a representation, and what exactly means reducible and irreducible here?

Answer: Let me take the simple example of SO(3). It is like this:

1. We have a group G, namely a set of items g.
2. We can represent each item of this group in different ways using linear operators. Such a function U from the group elements to a set of operators is called a representation, the resulting elements form a set of operators {U(g) for all g $\in$ G}.
3. Actually, on a side note, the group elements themselves are defined via their fundamental representation and associated with the resulting elements (this may seem circular; well, anyway).
4. Now, to each such representation there is also the space that these operators act on. For example, we can represent the rotation group elements in terms of 3x3 rotation matrices, and these act on 3-D physical space. But now the rotation operators give the space they act on structure: We can find invariant subspaces (in this case spheres of all radii), that is subspaces whose elements are mapped onto each other and not onto elements outside under action of the group. Or similarly, we can represent the group elements by rotation operators acting on Hilbert space, thus rotating states or wave functions. Then again the associated space is part of the specification of the operators and hence part of the specification of the function U, of the representation.
5. Now if we can find invariant subspaces in the space the operators act on, we say that the representation is reducible. This means that we can write the Hilbert space as $\mathcal{H} = \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus ...$ and similarly the representations as $U = U_1 \oplus U_2 \oplus ...$
6. The matrices of U can then be written in Block-diagonal form.

Chapter 10

Question 3. ***EPR paradox***

How can one resolve the EPR paradox?

Answer: Well, the EPR 'paradox' is not really a paradox, but rather captures an interesting property of quantum mechanics: Although one can predict the outcome of a distant measurement by a local measurement on an entangled state, this does not mean that faster-than-light signalling is possible or that faster-than-light causality applies. The trick is: although these correlations appear paradoxical, the outcome probability distributions for both parties stay the same regardless of what we choose to measure! It's crucial that party A cannot control its measurement outcome, and thus cannot control B's measurement outcome either!

For anyone interested in this further, read the suggested reference for the next question, or check out Bell's book: 'Speakable and Unspeakable in Quantum Mechanics'.

Question 4. ***Bell inequalities***

Which conclusion can we draw from Bell inequalities?

Answer: What follows from Bell inequalities is that certain quantum correlations cannot be reproduced by a local hidden variable model. Local here means that we can factorize the joint probabilities for the measurement outcomes in the following manner:

$$P(XY|AB) = \int d\lambda P(\lambda) * P(X|A\lambda) * P(Y|B\lambda) \tag{L.1}$$

As these correlations appear in nature (as backed up with experiments), this shows that nature cannot be local in the above sense. This sense in which we observe non-local correlations must not be confused with the possibility of signalling: indeed, Bell showed also that we cannot use these correlations to transmit signals faster than light. This is because the outcome probabilities for either party A and B do not depend on the other party's input:

$$P(X|AB) = P(X|A) \tag{L.2}$$

More on Bell inequalities can be found in the HS11 Quantum Information Theory course website, denoted as Roger Colbeck's lecture notes".