FLAVOUR

Historically the first element of the CKM matrix is the CABIBBO ANGLE ϑ_C

It was introduced to explain the suppression of $\Delta S = 1^*$ decays compared with $\Delta S = 0$

$$\begin{pmatrix} u \\ d\cos\vartheta_C + s\sin\vartheta_C \end{pmatrix}$$

↑ mixing in down quarks

CABIBBO (1963)

$n \to p + e^- + \bar\nu_e$	$G^2\cos^2\vartheta_C$
$\pi^- \to \pi^0 e^- \bar\nu_e$	$G^2\cos^2\vartheta_C$
$K^- \to \pi^0 e^- \bar\nu_e$	$G^2\sin^2\vartheta_C$ ← the suppression is a factor 20 ($\vartheta_C \sim 0.23$)

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* Transitions that violate STRANGENESS

Strange particles ⇒ so called because of their long lifetime (in contrast to their copious production)

Let us investigate the consequences of the introduction of such mixing for neutral current interactions

$$\underbrace{u\bar u + d\bar d\cos^2\vartheta_C + s\bar s\sin^2\vartheta_C}_{\Delta S=0} + \underbrace{(d\bar s + s\bar d)}_{\Delta S=1}\cos\vartheta_C\sin\vartheta_C$$

⇒ Implies $\Delta S=1$ neutral current transitions that are strongly suppressed experimentally

$$\frac{K^+ \to \pi^+ \nu\bar\nu}{K^+ \to \pi^0 \mu^+ \nu_\mu} < 10^{-5}$$

Glashow - Iliopoulos - Maiani (1970) GIM mechanism

$$\begin{pmatrix} u \\ d\cos\vartheta_C + s\sin\vartheta_C \end{pmatrix} \qquad \begin{pmatrix} c \\ s\cos\vartheta_C - d\sin\vartheta_C \end{pmatrix}$$

↗ new quark

$$u\bar u + c\bar c + \underbrace{(d\bar d\cos^2\vartheta_C + s\bar s\sin^2\vartheta_C + s\bar s\cos^2\vartheta_C + d\bar d\sin^2\vartheta_C)}_{\Delta S=0}$$

$$+ \underbrace{\cancel{(s\bar d + d\bar s - s\bar d - \bar s d)}}_{\Delta S=1}\cos\vartheta_C\sin\vartheta_C$$

Great triumph for the theory when $c\bar c$ bound state was discovered ⇒ J/ψ (sharp resonance at 3.1 GeV)

Richter (BNL)
Ting (SLAC)
in 1974
⇒ NOBEL PRIZE

GIM AND CKM MATRIX

$$U = \begin{pmatrix} u \\ c \\ t \end{pmatrix} \qquad D = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$U' = V_U U \qquad D' = V_D D$$

$\uparrow$ gauge eigenstate $\quad \uparrow$ mass eigenstate $\quad V_D, V_U$ unitary matrices

The neutral current interaction for down quarks can be written as

$$\bar{D}' \Gamma D'$$

Γ made of γ matrices, Q and T_3

$$= \bar{D} V_D^+ \Gamma V_D D$$

$\Rightarrow$ COMMUTES WITH V_D (because every down quark has the same isospin and charge)

$$= \bar{D} \Gamma D$$

$\Rightarrow$ NO FLAVOUR CHANGING NEUTRAL CURRENTS PROCESSES AT TREE LEVEL

(generalization of what we have already seen for two doublets)

Let us now see what are the consequences for the charged current interaction

$$\bar{U}' \gamma_\mu (1-\gamma_5) D' = \bar{U} V_U^+ \gamma_\mu (1-\gamma_5) V_D D = \bar{U} \gamma_\mu (1-\gamma_5) V D$$

$V = V_U^+ V_D$ CKM MATRIX IT ROTATES ONLY DOWN TYPE QUARKS (BY CONVENTION)

The CKM matrix is a unitary matrix: let us study how many parameters it describes as a function of the number of families

$$2N^2 - N^2 = N^2 \text{ independent parameters}$$

$\uparrow$ complex matrix $\quad \uparrow$ unitary condition

there are $2N$ quarks and their fields can be rotated independently by a phase transformation

(but one phase rotation does not affect the V matrix)

$\Rightarrow$ we have to subtract $2N-1$ parameters

$$\Rightarrow \quad N^2 - 2N + 1 = (N-1)^2 \text{ is}$$

the number of independent parameters of the CKM matrix

A unitary REAL matrix is ortogonal $\Rightarrow$ if real V will have

$$N^2 - \Big(\underbrace{N}_{\text{Trace}} + \underbrace{\frac{N^2-N}{2}}_{\text{off diagonal terms } (VV^T \text{ is symmetric!})} \Big) = \frac{N^2-N}{2}$$

$\Rightarrow$ In general V has $(N-1)^2$ real parameters

if REAL V has $\frac{N^2-N}{2}$ parameters

$N=2 \Rightarrow$ one angle is sufficient $\Rightarrow$ CABIBBO ANGLE

$$(N-1)^2 = 1$$
$$\frac{N^2-N}{2} = 1$$
ok

$N=3$

$$(N-1)^2 = 4$$
$$\frac{N^2-N}{2} = 3$$

$\Rightarrow$ for three families the CKM matrix cannot be REAL

$\Rightarrow$ 3 ANGLES + 1 PHASE $\Rightarrow$ CP VIOLATION

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$\lambda \approx 0.22 \to \theta_C$

$A, \rho, \eta \sim 1$

↑ Wolfenstein parametrization

hierarchical structure : the more you go off diagonal and the smaller are the matrix elements

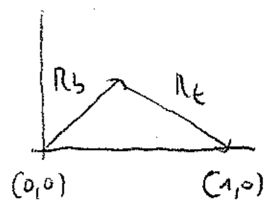
$(V^\dagger V) = I \Rightarrow$ 6 triangular relations

$$(V^\dagger V)_{31} = V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

normalise to $V_{cb}^* V_{cd} \Rightarrow$ unitarity triangle

$$R_b = \left| \frac{V_{ud} V_{ub}^*}{V_{cb}^* V_{cd}} \right| \qquad R_t = \left| \frac{V_{td} V_{tb}^*}{V_{cb}^* V_{cd}} \right|$$

COMPLEX PLANE



AREA DIFFERENT FROM ZERO $\Rightarrow$ CP VIOLATION

Why a phase in the CKM matrix implies CP violation?

$$\bar{U}_L \gamma_\mu V_{CKM} D_L W^\mu \quad + \quad \bar{D}_L \gamma_\mu V^\dagger_{CKM} U_L W^{+\mu}$$

h.c.

What are the transformation properties under C, P, T?

$$P \psi_L P^\dagger = \psi_R$$

$$C \psi_L C^\dagger = \bar{\psi}_R^T$$

T is special: it is antiunitary and antilinear

$$T c \psi T^\dagger = c^* T \psi T^\dagger$$

$$\bar{u}_L \gamma_\mu V_{CKM} D_L W^\mu \xrightarrow{P} \bar{U}_R \gamma_\mu V_{CKM} D_R W^\mu$$

$$\bar{u}_R \gamma_\mu V_{CKM} D_R W^\mu = \bar{u}_{R\,i} \gamma_\mu V_{CKM\,ij} D_{R\,j} W^\mu \xrightarrow{C} U_{L\,i} \gamma_\mu V_{CKM\,ij} \bar{D}_{L\,j} W^{\mu+}$$

$$= \bar{D}_L \gamma_\mu V^T_{CKM} U_L W^{\mu+}$$

$\Rightarrow$ If V_{CKM} is real $\rightarrow$ $V^T_{CKM} = V^\dagger_{CKM}$ and CP exchanges $\bar{u}_L \gamma_\mu V_{CKM} D_L W^\mu$ with its h.c.

Notice that

$$TCP \left(\bar{u}_L \gamma_\mu V_{CKM} D_L W^\mu \right) (TCP)^{-1} = \bar{D}_L \gamma_\mu V^\dagger_{CKM} U_L W^{+\mu}$$

CPT theorem

$$\mathcal{L} = \lambda A B^\dagger C^\dagger + h.c. = \lambda A B^\dagger C^\dagger + \lambda^* A^\dagger B C \quad (\text{normal ordering})$$

$$CP \mathcal{L} CP^{-1} = \lambda A^\dagger B C + \lambda^* A B^\dagger C^\dagger = \mathcal{L} \quad \underline{\text{only if } \lambda \text{ is real!}}$$

$$CPT \mathcal{L} (CPT)^{-1} = \lambda^* A^\dagger B C + \lambda A B^\dagger C^\dagger = \mathcal{L} \quad \Rightarrow$$

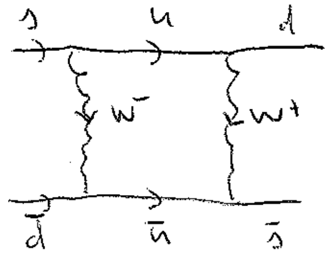
EVERY LORENTZ INVARIANT LOCAL AND HERMITIAN LAGRANGIAN IS CPT INVARIANT

Flavour changing neutral currents effects are strongly suppressed also at loop level

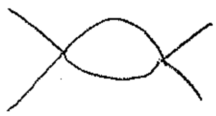
Let us consider the $K_0 \bar{K}_0$ system

$$K_1 = \frac{K_0 + \bar{K}_0}{\sqrt{2}} \qquad K_2 = \frac{K_0 - \bar{K}_0}{\sqrt{2}} \qquad K_0 \sim d\bar{s} \quad \bar{K}_0 \sim s\bar{d} \qquad M_{K_0} \simeq 498\ MeV$$

Mass differences are due to $\Delta S = 2$ transitions $\qquad \Delta m = 3.48 \cdot 10^{-12}\ MeV$



$\longrightarrow G_2 =$



$G_F^2 m_W^2$

↳ normal cutoff of loop integration

The result is too large to match the extremely small experimental value for Δm $\Rightarrow$ HOW TO EXPLAIN IT?

Let us keep into account all quarks (neglect t and b for their very small mixing)

u, u	$\sin^2\theta_c \cos^2\theta_c$
c, c	$\sin^2\theta_c \cos^2\theta_c$
u, c	$-\sin^2\theta_c \cos^2\theta_c$
c, u	$-\sin^2\theta_c \cos^2\theta_c$

$\Rightarrow$ if all quarks have the same masses the net result is zero!

The contribution from momenta in the loop larger than quark masses is zero
The only contributing region is the one of small momenta in the loop

$\Rightarrow$ The dominant contribution is

$$G_F^2 (m_c^2 - m_u^2)$$

↳ perfectly in agreement with the observed value of Δm