Electroweak interactions

• Weak interactions

Called in this way because cross section and decay rates are typically much smaller than e.m. or strong interactions

eg $\quad n \to p + e^- + \bar{\nu}_e \qquad \tau \sim 10^{3}\,s \qquad$ WEAK

$\quad \Sigma^0 \to \Lambda \pi^0 \qquad \tau \sim 10^{-23}\,s \qquad$ STRONG

↳ excited uds state in the baryon octet

$\quad \pi^0 \to 2\gamma \qquad \tau \sim 10^{-16}\,s \qquad$ E.M.

Weak processes are classified according to the leptonic content of their final state

- $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ — leptonic
- $\pi^+ \to \pi^0 e^+ \nu_e$ — semileptonic
- $\Lambda \to p \pi^-$ — hadronic

↳ Take place when much faster strong or em are forbidden by conservation laws

β decay

The neutrino hypothesis was put forward by Pauli in 1930 to explain the continuous spectrum of the electron observed in β decay

$n \to p + e^-$ ⟹ since $m_e \ll m_p, m_n$ the recoil can be neglected and we can write

$$m_n = m_p + E_e$$

$m_n = 939.6$ MeV
$m_p = 938.3$ MeV

⟹ 2body kinematics would suggest a monoenergetic spectrum of electrons

On December 4 1930 Pauli wrote a famous letter+ to the Physics Institute of ETH in which he proposed the neutrino existence (ACTUALLY HE CALLED THIS PARTICLE NEUTRON) Chadwick discovered the NEUTRON*, a much more massive nuclear particle in 1932 and thus there were two particles with the same name ⟹ Fermi in 1934 invented the word NEUTRINO

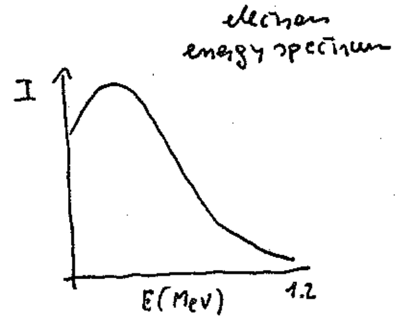
proposed mass ~ m_e spin $\frac{1}{2}$

+ Liebe Radioaktive Damen und Herren,
. . .

→ to the people who had gathered in Tubingen (where he was not able to go)

* In 1931 Bothe and Becker found that if very energetic α particles fell on light elements like beryllium, boron or lithium, a very penetrating radiation was produced

Chadwick in 1932 after a series of experiments, showed that this was a new radiation of electrically neutral particles of mass of the order of the proton mass, that were called NEUTRONS



Parity violation

Weak interactions do not conserve parity. Historically the first example of parity non conservation lead to the so called $\pi-\vartheta$ puzzle.

In 1947 Cecil Powell had identified the π-meson, that had been postulated 12 years before by Yukawa to mediate the nuclear force. Two years later Powell identified a cosmic ray particle that was decaying into three pions. He called this particle the τ MESON. Another particle called the ϑ-MESON was also discovered, that was decaying into two pions. More detailed studies showed that masses and lifetimes of the two particles were identical, within experimental uncertainties.

$\vartheta \to \pi^+\pi^0$ $\qquad\qquad$ $P\,|\pi\pi\rangle = (-1)(-1)(-1)^{\ell} = 1$

spinless $\qquad\qquad$ intrinsic parity*

$\tau \to \pi^+\pi^+\pi^-$ $\qquad\qquad$ $P\,|\pi\pi\pi\rangle = (-1)^3(-1)^{\ell_{\pi_1\pi_2}}(-1)^{\ell_{\pi_3}}$

$\ell_{\pi_1\pi_2} = \ell_{\pi_3}$ (the τ was spinless)

$\Rightarrow P\,|\pi\pi\pi\rangle = -1$

Lee and Yang famous paper "Question of parity conservation in weak interactions"

$\Rightarrow$ conclude that parity is not conserved in weak interactions $\Rightarrow$ ϑ and τ are the same particle now called K^+.

To test parity non conservation an experiment was carried out by Wu et al. (1957)

A sample of ^{60}Co at 0.01 K inside a solenoid was employed (at this temperature a high portion of ^{60}Co nuclei are aligned).

$$\boxed{e^- \longleftarrow \longrightarrow \underline{J}\,(^{60}Co) \qquad \xrightarrow{\underline{B}}}$$

$^{60}Co \to {}^{60}Ni^*$ $\qquad$ the relative electron intensities along and against the field direction were measured

$J=5 \qquad J=4$

$+\, e^- \; \bar{\nu}_e$

$J=1$

$$I(\theta) = 1 + \alpha\,\frac{\underline{\sigma}\cdot\underline{p}}{E} \qquad \alpha=-1$$

$\underline{\sigma}$ = unit vector in the direction of $\underline{J}$

pseudoscalar

$\Rightarrow$ ASYMMETRY OF THE INTENSITY

$\Rightarrow$ PARITY NON CONSERVATION

* In 1954 Chinowsky and Steinberger prove that the pion has intrinsic parity -1

They studied the decay of an "atom" $d\pi^-$ in s wave in two neutrons

$\Rightarrow$ $d\pi^-$ must have $J=1$

The two neutron final state must be antisymmetric $(-1)^{L+S+1}$ $\Rightarrow$ $L+S=$ even

$\Rightarrow$ only possibility is $L=S=1$ $\Rightarrow$ negative parity $\Rightarrow$ also the initial state must have negative parity

Since d has positive parity $\Rightarrow$ π^- must have negative parity $\Rightarrow$ PSEUDOSCALAR

Other check $\Rightarrow$ look at angular distribution of $\pi^0 \to 2\gamma$

To give J=1 electron and neutrino must have opposite helicities

Since $\alpha = -1$ the result implies that electrons are preferentially emitted opposite to the magnetic field

$$\Gamma\left({}^{60}Co \to {}^{60}Ni^{*} + e^-_L + \bar{\nu}_{eR}\right) > \Gamma\left({}^{60}Co \to {}^{60}Ni^{*} + e^-_R + \bar{\nu}_{eL}\right)$$

This observation lead to the V-A theory of weak interactions

γ_5 matrix

Recall that $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ $\quad \gamma_5^2 = 1 \quad \{\gamma_5, \gamma^\mu\} = 0 \quad \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$

$\Rightarrow$ eigenvalues of γ_5 can be only ± 1 (CHIRALITY) $\quad \gamma_5\psi_+ = \psi_+$, $\gamma_5\psi_- = -\psi_-$ eigenvectors

$$W_\mu = -\frac{1}{4}\varepsilon_{\mu\nu\rho\sigma}k^\nu\sigma^{\rho\sigma}$$

spin 4-vector $\quad$ one can show that $\varepsilon_{\mu\nu\rho\sigma}\sigma^{\rho\sigma} = \frac{2}{i}\gamma_5\sigma_{\mu\nu}$

In the rest frame $\quad W_\mu = -\frac{1}{4}k^\nu\frac{2}{i}\gamma_5\sigma_{\mu\nu} = \frac{i}{2}k^\nu\gamma_5\sigma_{\mu\nu}$

$$W_i = \frac{i}{2}k_0\gamma_5\sigma_{i0} = -\frac{1}{4}m\gamma_5(\gamma_i\gamma_0 - \gamma_0\gamma_i) = \frac{1}{2}m\gamma_5\gamma_0\gamma_i = \frac{1}{2}m\underline{\Sigma}$$

$\underline{\Sigma} = \gamma^5\gamma^0\underline{\gamma}$

$\underline{\Sigma}\cdot\underline{p}\,\psi = \gamma^5 p^0\psi$ massless particles

(take $\not{p}\psi = 0$

$\gamma^5\gamma^0(\gamma_0 p_0 - \gamma^i p_i)\psi = 0$

$\Rightarrow \underline{\Sigma}\cdot\underline{p} = \gamma^5 p^0\psi$)

$\Rightarrow$ $\boxed{\text{CHIRALITY} \Leftrightarrow \text{HELICITY}}$

left handed massless fermion $\quad \overset{p}{\longleftarrow}\, e^- \,\Rightarrow$ spin

$u_L \equiv \frac{1-\gamma_5}{2}u$

NOTE THAT IF $P_L = \frac{1-\gamma_5}{2}$ $\quad P_L^2 = P_L$

IDEMPOTENCE

$\psi_L = \frac{1-\gamma_5}{2}\psi$ $\quad \bar{\psi}_L = \psi_L^\dagger\gamma^0 = \psi^\dagger\frac{1-\gamma_5}{2}\gamma^0 = \bar{\psi}\frac{1+\gamma_5}{2}$

Fermi theory

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}}\bar{\nu}_\mu\gamma^\mu(1-\gamma_5)\mu\;\bar{e}\gamma_\mu(1-\gamma_5)\nu_e$$

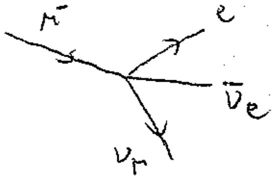
$G_F = 1.166\cdot 10^{-5}\ \text{GeV}^{-2}$ dimensionful constant

the structure is that of a current-current interaction

$$\mathcal{L} = -\frac{4G_F}{\sqrt{2}}J^i_\mu J^{+}_{\mu j}$$

$$J_i^\mu = \frac{1}{2}\,\bar{\psi}_i\,\gamma^\mu(1-\gamma_5)\,\psi_{\nu_i}$$

$$J_i^{\mu +} = \frac{1}{2}\,\bar{\psi}_{\nu_i}\,\gamma^\mu(1-\gamma_5)\,\psi_i$$



CHARGED CURRENT INTERACTION

$\Downarrow$

FLAVOR VIOLATION

Nice phenomenological description of muon and β decay but

- Dimensionful constant $\Rightarrow$ non renormalizable

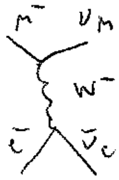
$\Downarrow$ violates unitarity

$$[\sigma] = \frac{1}{E^2} \quad \Rightarrow \quad \sigma \sim G_F^2 E^2 \quad \text{by dimensional analysis}$$

$\Rightarrow$ cross sections increase arbitrarily with the energy

<u>IVB theory</u>

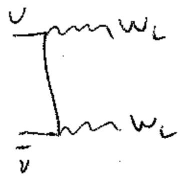
We can look at the four fermion interactions as the low energy limit of



$$\frac{g_W^2}{q^2 - m_W^2} \rightarrow \frac{g_W^2}{m_W^2} \equiv \frac{G_F}{\sqrt{2}}$$

But the vector boson propagator is $\dfrac{-g_{\mu\nu} + \frac{k^\mu k^\nu}{m_W^2}}{k^2 - m_W^2}$ $\rightarrow$ bad behavior at large k (UV region) to ensure renormalizability of such theory we NEED GAUGE INVARIANCE

Also



has a bad high energy behavior

$\Rightarrow$ SOLUTION IS <u>GAUGE THEORY</u>

Glashow (1961) $\rightarrow$ NO SSB

Weinberg-Salam (1967) $\rightarrow$ with Higgs mechanism

GAUGE THEORY :

brief recap

QED $\quad \mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi$

$\partial_\mu \rightarrow D_\mu = \partial_\mu - ieQA_\mu$

$\downarrow$

U(1) generator in the representation of ψ

global phase invariance (U(1) symmetry)

$\Rightarrow$ <u>violates causality</u> (an observer in a point X does a phase transformation $\Rightarrow$ also an observer in a point Y with $(x-y)^2 < 0$ must do the same transformation)

$J_\mu = J_\mu^+ = \bar{u}_\nu \gamma_\mu u_{eL}$

$J_\mu^\dagger = J_\mu^- = \bar{u}_e \gamma_\mu u_{L\nu}$

these currents correspond to transitions between fermions with charges differing by ONE UNIT

In analogy to the case of isospin, when proton and neutron are considered as the two isospin eigenstates of the NUCLEON $\begin{pmatrix} p \\ n \end{pmatrix}$, we can postulate a WEAK ISOSPIN STRUCTURE

$$\chi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$\tau_\pm = \frac{1}{2}(\tau_1 \pm i\tau_2) \qquad \tau_1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \qquad \tau_2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}$$

$$\tau_+ = \begin{pmatrix} 0 & 1 \\ & \end{pmatrix} \qquad \tau_- = \begin{pmatrix} & \\ 1 & 0 \end{pmatrix}$$

$J_\mu^+ = \bar{\chi}_L \gamma_\mu \tau_+ \chi_L$

$J_\mu^- = \bar{\chi}_L \gamma_\mu \tau_- \chi_L$

we can then POSTULATE an $SU(2)$ symmetry of this current

$\Rightarrow$ a neutral current should exist $\bar{\chi}_L \gamma_\mu \frac{\tau_3}{2} \chi_L = J_\mu^3$

Can we use it TO UNIFY weak and EM. interactions?

(DIFFICULTY IN UNIFICATION
$r_W = \frac{\hbar}{m_W c} \approx 2.5 \cdot 10^{-16}$ cm
$r_\gamma = \infty$!

The answer is no because the sum of charges is not zero in the doublet! We need another $U(1)$

$\Rightarrow$ WE SHOULD CHOOSE $SU(2) \otimes U(1)$ as GAUGE GROUP

The electromagnetic current $J_\mu^{em} = \bar{e}\gamma_\mu Q e = \bar{e}_R \gamma_\mu Q e_R + \bar{e}_L \gamma_\mu Q e_L$ is NOT invariant under $SU(2)$

$\Rightarrow$ Let us introduce a $SU(2)$ invariant current $J_\mu^Y = \bar{e}_R \gamma_\mu Y_R e_R + \bar{\chi}_L \gamma_\mu Y_L \chi_L$

quantum numbers associated to the additional $U(1)$ group (<u>hypercharge</u>)

$$\bar{e}_L \gamma_\mu Q e_L + \bar{e}_R \gamma_\mu Q e_R$$

$$= J_\mu^3 + \frac{1}{2} J_\mu^Y$$

we try to construct J_μ^{em} as a linear combination of J_μ^3 and J_μ^Y

$$= \frac{1}{2}\bar{\nu}_L \gamma_\mu \nu_L - \frac{1}{2}\bar{e}_L \gamma_\mu e_L + \frac{1}{2}\bar{e}_R \gamma_\mu Y_R e_R + \frac{1}{2}\bar{\chi}_L \gamma_\mu Y_L \chi_L$$

from which we need $Y_R = 2Q$ $\qquad Q = -\frac{1}{2} + Y_L$

THIS DEFINITION HAS THE PROPERTY THAT EACH COMPONENT OF A $SU(2)$ DOUBLET HAS THE SAME HYPERCHARGE

Introducing $T_3 = \pm\frac{1}{2}$ $\qquad T_3(e_R) = 0$

we can write

$$\boxed{Q = T_3 + \frac{Y}{2}} \quad (*)$$

IN THIS WAY THE EM CURRENT IS A SUPERPOSITION OF THE THIRD COMPONENT OF THE ISOSPIN AND OF THE HYPERCHARGE

(a) Note that ν_R are completely decoupled (they are singlet of $SU(2) \otimes U(1)$)

	T	T_3	Q	Y
ν_L	$1/2$	$+\frac{1}{2}$	0	-1
e^-_L	$1/2$	$-\frac{1}{2}$	-1	-1
ν_R	0	0	0	0
e^-_R	0	0	-1	-2

	T	T_3	Q	Y
u_L	$1/2$	$1/2$	$2/3$	$1/3$
d_L	$1/2$	$-1/2$	$-1/3$	$1/3$
u_R	0	0	$2/3$	$4/3$
d_R	0	0	$-\frac{1}{3}$	$-\frac{2}{3}$

Construction of $SU(2) \otimes U(1)$ Lagrangian

$$\mathcal{L} = \bar\Psi_L i \not{D} \Psi_L + \bar\Psi_R i \not{D} \Psi_R$$

$$= \bar\Psi_L i \gamma_\mu \left(\partial^\mu + ig\frac{\tau_i}{2} W_i^\mu + ig'\frac{Y_L}{2} B_\mu\right)\Psi_L \quad + \bar\Psi_R \gamma^\mu i \left(\partial_\mu + ig'\frac{Y_R}{2} B_\mu\right)\Psi_R$$

g, g' gauge couplings of $SU(2)_L$ and $U(1)_Y$

charged current interaction

$$\bar\Psi_L ig\frac{1}{2}\left(\tau_1 W_1 + \tau_2 W_2\right)\Psi_L \qquad W = \frac{1}{\sqrt2}(W_1 - iW_2) \qquad \tau_1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$$

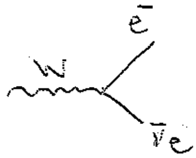
$$W^+ = \frac{1}{\sqrt2}(W_1 + iW_2) \qquad \tau_2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}$$

$$\rightarrow \bar\Psi_L ig\frac{1}{2}\left(\frac{W+W^+}{\sqrt2}\tau_1 + \frac{W^+-W}{\sqrt2\, i}\tau_2\right)\Psi_L \qquad \tau_1 + i\tau_2 = \begin{pmatrix} 0 & 2 \\ & \end{pmatrix}$$

$$\tau_1 - i\tau_2 = \begin{pmatrix} & \\ 2 & 0 \end{pmatrix}$$

$$= \bar\Psi_L ig\frac{1}{2}\left(W\frac{\tau_1+i\tau_2}{\sqrt2} + W^+\frac{\tau_1-i\tau_2}{\sqrt2}\right)\Psi_L$$

$$= ig/\sqrt2\left(J^+_{\mu L} W^\mu + J_{\mu L} W^{+\mu}\right) \qquad J_{\mu L} = \bar e \gamma_\mu \nu_L$$



neutral current interaction

$W_3 = \cos\theta\, Z + \sin\theta\, A$ $\qquad$ θ mixing angle

$B = \cos\theta\, A - \sin\theta\, Z$

$$\bar\Psi_L \gamma_\mu \left(ig\frac{\tau_3}{2} W_3^\mu + ig'\frac{Y_L}{2} B^\mu\right)\Psi_L + \bar\Psi_R \gamma_\mu ig'\frac{Y_R}{2} B^\mu \Psi_R$$

$$\rightarrow \bar\Psi_L \gamma_\mu \left[ig\frac{\tau_3}{2}(\cos\theta\, Z^\mu + \sin\theta\, A^\mu) + ig'\frac{Y_L}{2}(\cos\theta\, A^\mu - \sin\theta\, Z^\mu)\right]\Psi_L$$

$$+ \bar\Psi_R \gamma_\mu ig'\frac{Y_R}{2}(\cos\theta\, A^\mu - \sin\theta\, Z^\mu)\Psi_R$$

$$= \left\{\bar\Psi_L \gamma_\mu \left(ig\frac{\tau_3}{2}\sin\theta + ig'\frac{Y_L}{2}\cos\theta\right)\Psi_L + \bar\Psi_R\gamma_\mu ig'\frac{Y_R}{2}\cos\theta\, \Psi_R\right\} A^\mu$$

$$+ \left\{\Psi_L \gamma_\mu \left(ig\frac{\tau_3}{2}\cos\theta - ig'\frac{Y_L}{2}\sin\theta\right)\Psi_L - \bar\Psi_R\gamma_\mu ig'\frac{Y_R}{2}\sin\theta\, \Psi_R\right\} Z^\mu$$

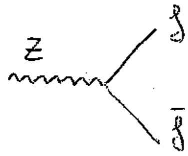
$\Rightarrow$ we must identify $\quad g\sin\theta = g'\cos\theta = e$

$$\rightarrow \quad \bar{\Psi} i \gamma_\mu Q \Psi A^\mu$$

$$+ \left\{ \bar{\Psi}_L \gamma_\mu \left(i g \cos\theta \frac{\tau_3}{2} - i g' (Q - T_3) \sin\theta \right) \Psi_L - \bar{\Psi}_R \gamma^\mu i g' Q \sin\theta \, \Psi_R \right\} Z^\mu$$

$$= \frac{ig}{\cos\theta} \bar{\Psi} \gamma^\mu \left(T_3^L - \sin^2\theta \, Q \right) \Psi Z_\mu$$

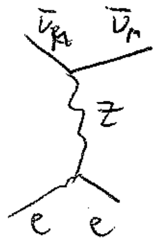
$$\hookrightarrow \frac{(1-\gamma_5)}{2} T_3$$



COMMENTS

1) SU(2) ⊗ U(1) THEORY PREDICTS THE EXISTENCE OF NEUTRAL CURRENT INTERACTIONS (mediated by the Z boson) THEY WILL BE DISCOVERED IN 1973 IN THE GARGAMELLE BUBBLE CHAMBER

$$\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$$



2) Charged currents interactions are the only known interactions able to change flavour : $e^- \rightarrow \nu_e$ $\quad u \rightarrow d$ $\quad$ etc.

ON THE CONTRARY : NO FLAVOUR CHANGING NEUTRAL CURRENTS AT TREE LEVEL

(also at one loop we have strong cancellations → GIM MECHANISM)

3) SU(2) ⊗ U(1) IS A CHIRAL THEORY ⇒ LEFT HANDED AND RIGHT HANDED FIELDS COUPLE DIFFERENTLY TO THE GAUGE GROUP

⇒ MASS TERMS FOR FERMIONS ARE FORBIDDEN BY GAUGE INVARIANCE

QED, QCD $\qquad m \bar{\Psi} \Psi$ is OK

SU(2) ⊗ U(1) $\qquad \bar{\Psi}\Psi = \bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L \rightarrow$ NOT INVARIANT !

4) Weak interactions violate both P and C : CP is (approximately) conserved

$$\Gamma(\pi^+ \to \mu^+_R + \nu_L) \neq \Gamma(\pi^+ \to \mu^+_L + \nu_R) \qquad \text{P violation}$$

$$\Gamma(\pi^+ \to \mu^+_R + \nu_L) \neq \Gamma(\pi^- \to \mu^-_R + \bar{\nu}_L) \qquad \text{C violation}$$

$$\Gamma(\pi^+ \to \mu^+_R + \nu_L) = \Gamma(\pi^- \to \mu^-_L + \bar{\nu}_R) \qquad \text{CP conservation}$$

5)

$$g \sin\theta = g' \cos\theta = e$$

$\Rightarrow$ e and g should be of the same order of magnitude

$\Rightarrow$ WHY IS THE WEAK INTERACTION SO MUCH WEAKER THAN ELECTROMAGNETIC INTERACTION ?

The answer is in the massive propagator of the gauge field.

$$\frac{-g^{\mu\nu} + k^\mu k^\nu / m_W^2}{k^2 - m_W^2}$$

unless k is very large, the propagator becomes suppressed by the heavy W mass

GAUGE SECTOR

$$\mathcal{L}_{gauge} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} \sum_{a=1}^{3} F^a_{\mu\nu} F^{\mu\nu}_a \qquad F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\,\varepsilon^{abc} W^b_\mu W^c_\nu$$

$$W_3 = \cos\theta\, Z + \sin\theta\, A$$

$$B = \cos\theta\, A - \sin\theta\, Z$$

$$\Gamma^{\mu\nu\lambda}_{W^+W^-V} = i\, g_{W^+W^-V} \left[g^{\mu\nu}(q-p)^\lambda + g^{\mu\lambda}(p-r)^\nu + g^{\nu\lambda}(r-q)^\mu \right]$$

$$g_{W^+W^-\gamma} = e$$

$$g_{W^+W^-Z} = g\cos\theta$$

Quadrilinear couplings :