

Electroweak interactions

Weak interactions

Called in this way because cross section and decay rates are typically much smaller than e.m. or strong interactions

eg	$M \rightarrow P + e^- + \bar{\nu}_e$	$\tau \sim 10^3 s$	WEAK
	$\Sigma^0 \rightarrow \Lambda + \pi^0$	$\tau \sim 10^{-23} s$	STRONG
	↳ excited udh state in the baryon octet		
	$\pi^0 \rightarrow 2\gamma$	$\tau \sim 10^{-16} s$	E.M.

Weak processes are classified according to the leptonic content of their final state

- $M^+ \rightarrow e^+ + \nu_e + \bar{U}_M$ leptonic
- $\pi^+ \rightarrow \pi^0 e^+ \nu_e$ semileptonic
- $\Lambda \rightarrow p \pi^-$ hadronic

↳ Take place when much faster strong or em are forbidden by conservation laws

β decay

The neutrino hypothesis was put forward by Pauli in 1930 to explain the continuous spectrum of the electron observed in β decay

$n \rightarrow p + e^-$ \Rightarrow same $m_e \ll m_p, m_n$ the recoil can be neglected and we can write $m_n = m_p + E_e$

$m_n = 939.6$ MeV
 $m_p = 938.3$ MeV

\Rightarrow 2body kinematics would suggest a monoenergetic spectrum of electrons

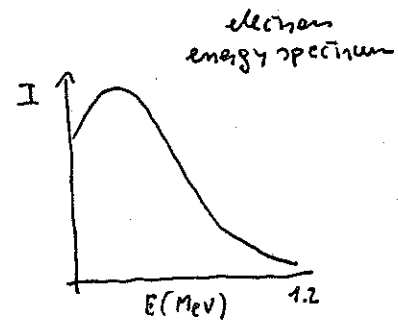
On December 6 1930 Pauli wrote a famous letter[†] at the Physics Institute of ETH in which he proposed the neutrino existence (ACTUALLY HE CALLED THIS PARTICLE NEUTRON) Chadwick discovered the NEUTRON^{*}, a much more massive nuclear particle in 1932 and thus there were two particles with the same name \Rightarrow Fermi in 1934 invented the word NEUTRINO

proposed may $\sim m_e$ so in $\frac{1}{2}$

[†] Liebe Radioaktive Dement und Hemen,

\rightarrow to the people who had gathered in Tubingen (where he was NOT able to go)

* In 1931 Bothe and Becker found that if very energetic α particles fell on light elements like beryllium, boron or lithium, a very penetrating radiation was produced. Chadwick in 1932 after series of experiments, showed that this was a new sort of electrically neutral particles of mass of the order of the proton mass, that were called NEUTRONS



Parity violation

Weak interactions do not conserve parity. Historically the first example of parity non conservation led to the so called $\pi-\theta$ puzzle.

In 1947 Cecil Powell had identified the π -meson, that had been postulated 12 years before by Yukawa to mediate the nuclear force. Two years later Powell identified a cosmic ray particle that was decaying into three pions. He called this particle the τ meson. Another particle called the θ -meson was also discovered, that was decaying into two pions. More detailed studies showed that masses and lifetimes of the two particles were identical, within experimental uncertainties.

$$\theta \rightarrow \pi^+ \pi^0$$

↓
spinless

$$P(\pi\pi) = (-1)(-1)(-1)^L = 1$$

↑
intrinsic parity*

$$\tau \rightarrow \pi^+ \pi^+ \pi^-$$

$$P(\pi\pi\pi) = (-1)^3 (-1)^{L_{\pi_1\pi_2}} (-1)^{L_{\pi_3}}$$

$L_{\pi_1\pi_2} = L_{\pi_3}$ (the τ was spinless)

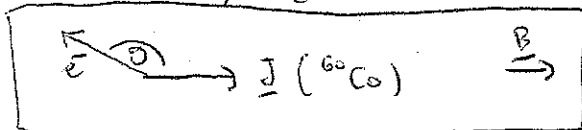
$$\Rightarrow P(\pi\pi\pi) = -1$$

Lee and Yang famous paper "Question of parity conservation in weak interactions"

\Rightarrow conclude that parity is not conserved in weak interactions $\Rightarrow \theta$ and τ are the same particle now called K^+ .

To test parity non conservation an experiment was carried out by Wu et al. (1957)

A sample of ^{60}Co at 0.01 K inside a solenoid was employed (at this temperature a high portion of ^{60}Co nuclei are aligned).



$^{60}\text{Co} \rightarrow ^{60}\text{Ni}^{\gamma} + e^- + \bar{\nu}_e$
 $J=5 \quad \quad \quad J=4$
 $J=1$

the relative electron intensities along and against the field direction were measured

$$I(\theta) = 1 + d \frac{\sigma \cdot \underline{p}}{E} \quad d = -1$$

$\sigma =$ unit vector in the direction of \underline{J}

\Rightarrow ASYMMETRY OF THE INTENSITY

\Rightarrow PARITY NON CONSERVATION

↓
polarization

* In 1956 Chinnowsky and Steinberg prove that the pion has intrinsic parity -1

They studied the decay of a "eta" $d\pi^-$ in s wave in two neutrons

$\Rightarrow d\pi^-$ must have $J=1$

The two neutron final state must be antisymmetric $(-1)^{L+S+1} \Rightarrow L+S = \text{even}$

\Rightarrow only possibility is $L=S=1 \Rightarrow$ negative parity \Rightarrow also the initial state must have negative parity

Since d has positive parity $\Rightarrow \underline{\pi^-}$ must have negative parity \Rightarrow PSEUDOSCALAR

Other check \Rightarrow look at angular distribution of $\pi^0 \rightarrow 2\gamma$

To give $J=1$ electron and neutrinos must have opposite helicities
 Since $d=-1$ the result implies that electrons are preferentially emitted opposite to the magnetic field

$$\Gamma(^{60}\text{Co} \rightarrow ^{60}\text{Ni}^* + e^-_L + \bar{\nu}_{eR}) > \Gamma(^{60}\text{Co} \rightarrow ^{60}\text{Ni}^* + e^-_R + \bar{\nu}_{eL})$$

This observation led to the V-A theory of weak interactions

γ_5 matrix

Recall that $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ $\gamma_5^2 = 1$ $\{\gamma_5, \gamma^\mu\} = 0$ $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$

\Rightarrow eigenvalues of γ_5 can be only ± 1 (CHIRALITY) $\gamma_5 \psi_+ = \psi_+$ $\gamma_5 \psi_- = -\psi_-$ eigenvectors

$W_\mu = -\frac{1}{4} \epsilon_{\mu\nu\rho\sigma} k^\nu \sigma^{\rho\sigma}$ spin 4-vector one can show that $\epsilon_{\mu\nu\rho\sigma} \sigma^{\rho\sigma} = \frac{2}{i} \gamma_5 \sigma_{\mu\nu}$

In the rest frame $W_\mu = -\frac{1}{4} k^\nu \frac{2}{i} \gamma_5 \sigma_{\mu\nu} = \frac{i}{2} k^\nu \gamma_5 \sigma_{\mu\nu}$

$W_i = \frac{i}{2} k_0 \gamma_5 \sigma_{i0} = -\frac{1}{4} m \gamma_5 (\gamma_i \gamma_0 - \gamma_0 \gamma_i) = \frac{1}{2} m \gamma_5 \gamma_0 \gamma_i = \frac{1}{2} m \underline{\Sigma}$

$\underline{\Sigma} = \gamma_5 \gamma^0 \underline{\gamma}$

$\underline{\Sigma} \cdot \underline{P} \psi = \gamma_5 \not{P} \psi$ massless particles

take $\not{P} \psi = 0$
 $\gamma_5 \not{P} (\gamma_0 p_0 - \gamma^i p_i) \psi = 0$

$\Rightarrow \underline{\Sigma} \cdot \underline{P} = \gamma_5 \not{P} \psi$

\Rightarrow CHIRALITY \Leftrightarrow HELICITY

Left handed massless fermion $\leftarrow \frac{P}{e^-} \Rightarrow$ spin

$u_L \equiv \frac{1-\gamma_5}{2} u$

NOTE THAT IF $P_L = \frac{1-\gamma_5}{2}$ $P_L^2 = P_L$

IDEMPOTENCE

Fermi theory

$\psi_L = \frac{1-\gamma_5}{2} \psi$

$\bar{\psi}_L = \psi_L^\dagger \gamma^0 = \psi^\dagger \frac{1-\gamma_5}{2} \gamma^0 = \bar{\psi} \frac{1+\gamma_5}{2}$

$\mathcal{L} = -\frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma^\mu (1-\gamma_5) \mu e^- \gamma_\mu (1-\gamma_5) \nu_e$

$G_F = 1.166 \cdot 10^{-5} \text{ GeV}^{-2}$

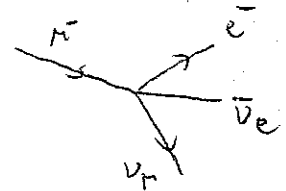
dimensionful constant

the structure is that of a current-current interaction

$\mathcal{L} = -\frac{G_F}{\sqrt{2}} J_\mu^+ J_\mu^-$

$$J_c^\mu = \frac{1}{2} \bar{\psi}_i \gamma^\mu (1 - \gamma_5) \psi_{\nu_i}$$

$$J_c^{\mu\dagger} = \frac{1}{2} \bar{\psi}_{\nu_i} \gamma^\mu (1 - \gamma_5) \psi_i$$



CHANGES
CURRENT
INTERACTION
↓
FLOREN
VIOLATION

Nice phenomenological description of muon and beta decay but

- Dimension full constant \Rightarrow non renormalizable

\Downarrow violates unitarity

$$[\sigma] = \frac{1}{E^2} \Rightarrow \sigma \sim G_F^2 E^2 \text{ by dimensional analysis}$$

\Rightarrow cross sections increase arbitrarily with the energy

UVB theory

We can look at the four fermion interactions as the low energy limit of

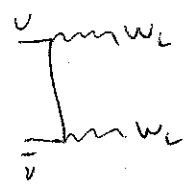


$$\frac{g_w^2}{q^2 - m_w^2} \rightarrow \frac{g_w^2}{m_w^2} = \frac{G_F}{\sqrt{2}}$$

But the vector boson propagator is

$$\frac{-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_w^2}}{k^2 - m_w^2}$$

\Rightarrow bad behavior at large k (UV region) to ensure renormalizability of such theory we NEED GAUGE INVARIANCE

Also  has a bad high energy behavior

\Rightarrow SOLUTION IS GAUGE THEORY

Glashow (1961) \rightarrow NOSSB

Weinberg-Salam (1967) \rightarrow with Higgs mechanism

GAUGE THEORY :
brief recap

$$L = \bar{\psi} (i \not{\partial} - m) \psi$$

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ieQ A_\mu$$



U(1) generators in the representation of ψ

global phase invariance (U(1) symmetry)

\Rightarrow violates causality (an observer in a point X does a phase transformation \Rightarrow also an observer in a point Y with $(X-Y)^T < 0$ must do the same transformation)

$$J_M^+ = J_M^+ = \bar{u}_D \gamma_M u_{eL}$$

these currents corresponds to transitions

$$J_M^- = J_M^- = \bar{e} \gamma_M u_{eL}$$

between fermions with charges differing by ONE UNIT

In analogy to the case of isospin, when proton and neutron are considered as the two isospin eigenstates of the NUCLEON $\begin{pmatrix} p \\ n \end{pmatrix}$, we can postulate a WEAK ISOSPIN structure

$$\chi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$T_{\pm} = \frac{1}{2} (T_1 \pm iT_2) \quad T_1 = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

$$T_3 = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \quad T_- = \begin{pmatrix} & \\ & 0 \end{pmatrix} \quad T_2 = \begin{pmatrix} & \\ i & -i \end{pmatrix}$$

$$J_M^+ = \bar{\chi}_L \gamma_M T_+ \chi_L$$

we can then postulate an SU(2) symmetry of this current

$$J_M^- = \bar{\chi}_L \gamma_M T_- \chi_L$$

$$\Rightarrow \text{a neutral current should exist } \bar{\chi}_L \gamma_M T_3 \chi_L = J_M^3$$

Can we use it to UNIFY weak and EM. interactions?

DIFFICULTY IN UNIFICATION

$$Z_W = \frac{\hbar}{m_{Wc}} \approx 2.5 \cdot 10^{-16} \text{ cm}$$

$$Z_\gamma = \infty !$$

The answer is no because the sum of charges is not zero in the doublet! We need another U(1)

\Rightarrow WE SHOULD CHOOSE SU(2) \otimes U(1) as GAUGE GROUP

The electromagnetic current $J_M^{\text{em}} = \bar{e} \gamma_M Q e = \bar{e}_R \gamma_M Q e_R + \bar{e}_L \gamma_M Q e_L$ is NOT invariant under SU(2)

$$\Rightarrow \text{Let us introduce a SU(2) invariant current } J_M^Y = \bar{e}_R \gamma_M Y e_R + \bar{\chi}_L \gamma_M Y \chi_L$$

$$\bar{e}_L \gamma_M Q e_L + \bar{e}_R \gamma_M Q e_R$$

$$= J_M^3 + \frac{1}{2} J_M^Y$$

we try to construct J_M^{em} as a linear combination of J_M^3 and J_M^Y

Quantum numbers associated to the additional U(1) group (hypercharge)

$$= \frac{1}{2} \bar{\nu}_L \gamma_M \nu_L - \frac{1}{2} \bar{e}_L \gamma_M e_L + \frac{1}{2} \bar{e}_R \gamma_M Y e_R + \frac{1}{2} \bar{\chi}_L \gamma_M Y \chi_L$$

from which we need $Y_n = 2Q$

$$Q = -\frac{1}{2} + Y_L$$

Introducing $T_3 = \pm \frac{1}{2}$ $T_3(e_R) = 0$

$$Q = T_3 + \frac{Y}{2} \quad (*)$$

we can write

THIS DEFINITION HAS THE PROPERTY THAT EACH COMPONENT OF A SU(2) DOUBLET HAS THE SAME HYPERCHARGE

IN THIS WAY THE EM CURRENT IS A SUPERPOSITION OF THE THIRD COMPONENT OF THE ISOSPIN AND OF THE HYPERCHARGE

(*) Note that ν_R are completely decoupled (they are singlet of $SU(2) \otimes U(1)$)

	T	T_3	Q	Y
ν_L	$\frac{1}{2}$	$+\frac{1}{2}$	0	-1
\bar{e}_L	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-1
ν_R	0	0	0	0
\bar{e}_R	0	0	-1	-2

	T	T_3	Q	Y
u_L	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$
d_L	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$
u_R	0	0	$\frac{2}{3}$	$\frac{4}{3}$
d_R	0	0	$-\frac{1}{3}$	$-\frac{2}{3}$

Construction of $SU(2)_L \otimes U(1)$ Lagrangian

$$\mathcal{L} = \bar{\Psi}_L i \not{\partial} \Psi_L + \bar{\Psi}_R i \not{\partial} \Psi_R$$

$$= \bar{\Psi}_L i \gamma_M \left(\partial^M + i g \frac{\tau_i}{2} W_i^M + i g' \frac{Y_L}{2} B_M \right) \Psi_L + \bar{\Psi}_R \gamma^M i \left(\partial_M + i g' \frac{Y_R}{2} B_M \right) \Psi_R$$

g, g' gauge couplings of $SU(2)_L$ or $U(1)_Y$

charged current interaction

$$\bar{\Psi}_L i g \frac{1}{2} (\tau_1 W_1 + \tau_2 W_2) \Psi_L$$

$$W = \frac{1}{\sqrt{2}} (W_1 - i W_2)$$

$$W^\dagger = \frac{1}{\sqrt{2}} (W_1 + i W_2)$$

$$\tau_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\rightarrow \bar{\Psi}_L i g \frac{1}{2} \left(\frac{W+W^\dagger}{\sqrt{2}} \tau_1 + \frac{W^\dagger - W}{\sqrt{2}i} \tau_2 \right) \Psi_L$$

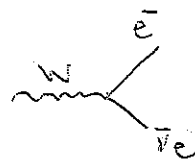
$$\tau_1 + i \tau_2 = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\tau_1 - i \tau_2 = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$$

$$= \bar{\Psi}_L i g \frac{1}{2} \left(W \frac{\tau_1 + i \tau_2}{\sqrt{2}} + W^\dagger \frac{\tau_1 - i \tau_2}{\sqrt{2}} \right) \Psi_L$$

$$= i g / \sqrt{2} \left(J_{ML}^+ W^M + J_{ML}^- W^{M\dagger} \right)$$

$$J_{ML} = \bar{\Psi} \gamma_M \Psi_L$$



neutral current interaction

$$W_3 = \cos \theta Z + \sin \theta A$$

θ mixing angle

$$B = \cos \theta A - \sin \theta Z$$

$$\bar{\Psi}_L \left(i g \frac{\tau_3}{2} W_3^M + i g' \frac{Y_L}{2} B^M \right) \Psi_L + \bar{\Psi}_R \gamma_M i g' \frac{Y_R}{2} B^M \Psi_R$$

$$\rightarrow \bar{\Psi}_L \gamma_M \left[i g \frac{\tau_3}{2} (\cos \theta Z^M + \sin \theta A^M) + i g' \frac{Y_L}{2} (\cos \theta A^M - \sin \theta Z^M) \right] \Psi_L$$

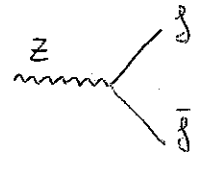
$$+ \bar{\Psi}_R \gamma^M i g' \frac{Y_R}{2} (\cos \theta A^M - \sin \theta Z^M) \Psi_R$$

$$= \left\{ \bar{\Psi}_L \gamma_M \left(i g \frac{\tau_3}{2} \sin \theta + i g' \frac{Y_L}{2} \cos \theta \right) \Psi_L + \bar{\Psi}_R \gamma^M i g' \frac{Y_R}{2} \cos \theta \Psi_R \right\} A^M$$

$$+ \left\{ \bar{\Psi}_L \gamma_M \left(i g \frac{\tau_3}{2} \cos \theta - i g' \frac{Y_L}{2} \sin \theta \right) \Psi_L - \bar{\Psi}_R \gamma^M i g' \frac{Y_R}{2} \sin \theta \Psi_R \right\} Z^M$$

\Rightarrow we must identify $g \sin \theta = g' \cos \theta = e$

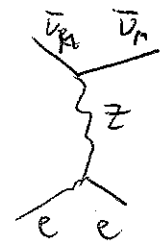
$$\begin{aligned} &\rightarrow \bar{\Psi} i \gamma_M \alpha \Psi A^M \\ &+ \alpha \left(\bar{\Psi}_L \gamma_M \left(i g \cos \theta \frac{T_3}{2} - i g' (Q - T_3) \right) \Psi_L \right. \\ &\quad \left. - \bar{\Psi}_R i g' \alpha \sin \theta \Psi_R \right) Z^M \\ &= \frac{i g}{\cos \theta} \bar{\Psi} \gamma^M \left(T_3^L - \sin^2 \theta Q \right) \Psi Z^M \\ &\quad \hookrightarrow \frac{(1 - g_s^2)}{2} T_3 \end{aligned}$$



COMMENTS

1) $SU(2) \otimes U(1)$ THEORY PREDICTS THE EXISTENCE OF NEUTRAL CURRENT INTERACTIONS (mediated by the Z boson) THEY WILL BE DISCOVERED IN 1973 IN THE GARGAMERE BUBBLE CHAMBER

$$\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$$



2) Charged currents interactions are the only known interactions able to change flavour: $\bar{e} \rightarrow \nu_e$ $u \rightarrow d$ etc.

ON THE CONTRARY: NO FLAVOUR CHANGING NEUTRAL CURRENTS AT TREE LEVEL (also at one loop we have strong cancellations \rightarrow GIM MECHANISM)

3) $SU(2) \otimes U(1)$ IS A CHIRAL THEORY \Rightarrow LEFT HANDED AND RIGHT HANDED FIELDS COUPLE DIFFERENTLY TO THE GAUGE GROUP

\Rightarrow MASS TERMS FOR FERMIONS ARE FORBIDDEN BY GAUGE INVARIANCE

QED, QCD $m \bar{\Psi} \Psi$ is OK

$SU(2) \otimes U(1)$ $\bar{\Psi} \Psi = \bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L \rightarrow$ NOT INVARIANT!

4) Weak interactions violate both P and C ; CP is (approximately) conserved

$$\Gamma(\pi^+ \rightarrow \mu^+ + \nu_L) \neq \Gamma(\pi^+ \rightarrow \mu^+ + \nu_R) \quad \text{P violation}$$

$$\Gamma(\pi^+ \rightarrow \mu^+ + \nu_L) \neq \Gamma(\pi^- \rightarrow \mu^- + \bar{\nu}_L) \quad \text{C violation}$$

$$\Gamma(\pi^+ \rightarrow \mu^+ + \nu_L) = \Gamma(\pi^- \rightarrow \mu^- + \bar{\nu}_R) \quad \text{CP conservation}$$

5)

$$g \sin \theta = g' \cos \theta = e$$

⇒ e and g should be of the same order of magnitude

⇒ WHY IS THE WEAK INTERACTION SO MUCH WEAKER THAN ELECTROMAGNETIC INTERACTION?

The answer is in the massive propagator of the gauge field.

$$\text{propagator } \frac{-g^{\mu\nu} + \frac{k^\mu k^\nu}{m_W^2}}{k^2 - m_W^2}$$

unless k is very large, the propagator becomes suppressed by the heavy W mass

GAUGE SECTOR

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} \sum_{a=1}^3 F_{\mu\nu}^a F^{\mu\nu a}$$

$$F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g \epsilon^{abc} W_\mu^b W_\nu^c$$

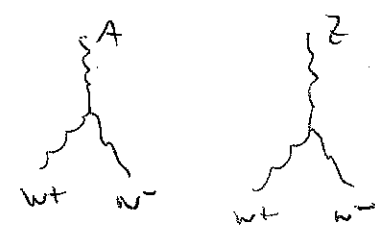
$$W_3 = \cos \theta Z + \sin \theta A$$

$$B = \cos \theta A - \sin \theta Z$$

$$\Gamma_{W^+ W^- \gamma}^{\mu\nu\lambda} = i g_{W^+ W^- \gamma} \left[g^{\mu\nu} (\gamma - P)^\lambda + g^{\nu\lambda} (P - 2)^\mu + g^{\lambda\mu} (2 - P)^\nu \right]$$

$$g_{W^+ W^- \gamma} = e$$

$$g_{W^+ W^- Z} = g \cos \theta$$



Quadrilinear couplings:

