Ex sheet 2 PPP II (4)

i) $\left(\frac{d\sigma}{dt\,du}\right)_{\ell p \to \ell X} = \sum_i' \int dx\, f_i(x) \left(\frac{d\sigma}{dt\,du}\right)_{\ell q_i \to \ell q_i}$

Parton model prediction, no QCD corrections, just scattering from free particles with probability $f_i(x)$

ii) $\hat{s} = (k+\hat{p})^2 = 2k\cdot\hat{p} = x\, 2k\cdot p = xS$

$\hat{t} = (k-k')^2 = k$

$\hat{u} = (k'-\hat{p})^2 = -2k'\cdot\hat{p} = -x\, 2k'\cdot p = xu$

iii) it's obvious that $\left(\frac{d\sigma}{dt\,du}\right)_{\ell q_i \to \ell q_i} = x\left(\frac{d\sigma}{d\hat{t}\,d\hat{u}}\right)_{\ell q_i \to \ell q_i}$

in proper variables $\left(\frac{d\sigma}{d\hat{t}\,d\hat{u}}\right)_{\ell q_i \to \ell q_i} \equiv \left(\frac{d\sigma}{dt\,du}\right)_{e\mu \to e\mu} \cdot \left(\frac{e_i}{e}\right)$

charge factor

Now we know that

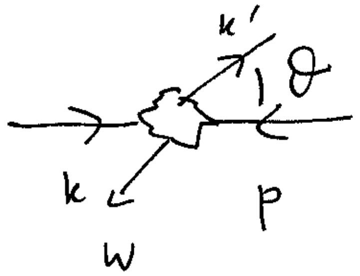
$$|M|^2_{e\mu \to e\mu} = 2e^4 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$$

notice this depends on invariants, so it doesn't matter in which frame you compute it → CoM easier so $\left(\frac{d\sigma}{d\hat{t}\,d\hat{u}}\right)_{e\mu\to e\mu} = \frac{|M|^2}{F}\,\frac{d\phi_2}{d\hat{t}\,d\hat{u}}$

F = flux factor $= 2\hat{s}$ for massless particles

$d\phi_2$ = phase space , let's work in CoM frame

$k+p \to k'+w$



since massless particles $\Rightarrow \left\| \; E = |\vec{k}| = p^0 = |\vec{k}'| = E' = \frac{\sqrt{\hat{s}}}{2} \; \right\|$

$$d\phi_2 = \frac{d^4k'}{(2\pi)^4}\,\frac{d^4w}{(2\pi)^4}\,(2\pi)^2\,\delta^{(+)}(k'^2)\,\delta^{(+)}(w^2)\,(2\pi)^4\,\delta(k+p-k'-w)$$

$$= \frac{d^4k'}{(2\pi)^2}\,\delta^{(+)}(k_0'^2-|\vec{k}'|^2)\,\delta^{(+)}((k+p-k')^2)$$

$$= \frac{d^3\vec{k}'}{(2\pi)^2}\,\frac{1}{2|\vec{k}'|}\,\delta^{(+)}(\hat{s}+\hat{t}+\hat{u})$$

↑ momentum conservation

notice that

$(k+p-k')^2 =$

$= \underbrace{k^2}_{=0}+\underbrace{p^2}_{=0}+\underbrace{k'^2}_{\downarrow}+\hat{s}+\hat{u}+\hat{t}$

put to zero by the $\delta^{(+)}(k_0^2-|\vec{k}|^2)$

neglect $\delta^{(+)}$ from now on $\to \delta$

$$= \frac{\cancel{(2\pi)}\,d(\cos\theta)\,|\vec{k}'|^{\cancel{2}}\,d|\vec{k}'|}{(2\pi)^{\cancel{2}}\;2\cancel{|\vec{k}'|}}\;\delta(\hat{s}+\hat{t}+\hat{u})$$

Use $\underline{|\vec{k}'| = E'}$

$$= \frac{d(\cos\theta)\,E'\,dE'}{4\pi}\;\delta(\hat{s}+\hat{t}+\hat{u})$$

Change of variables $E', \vartheta \to \hat{t}, \hat{u}$

$$\left.\begin{aligned}\hat{t} &= -2k\cdot k' = -2EE'(1-\cos\vartheta)\\ \hat{u} &= -2k'\cdot p = -2EE'(1+\cos\vartheta)\end{aligned}\right\}$$ always in C.M frame

$$J = \begin{pmatrix} -2E(1-\cos\vartheta) & 2EE' \\ -2E(1+\cos\vartheta) & -2EE' \end{pmatrix} \Rightarrow |\det J| = 8E^2E' = 2\hat{S}E'$$

since $\boxed{E^2 = \frac{\hat{S}}{4}}$

this gives $E'dE'\,d(\cos\vartheta) = \frac{d\hat{t}\,d\hat{u}}{2\hat{S}}$ and so

$$d\phi_2 = \frac{d\hat{t}\,d\hat{u}}{8\pi\hat{S}}\,\delta(\hat{s}+\hat{u}+\hat{t})$$ Putting everything together

$$\left(\frac{d\sigma}{d\hat{t}\,d\hat{u}}\right)_{e\mu\to e\mu} = \frac{1}{\cancel{2}\hat{S}}\,\cancel{2}e^4\,\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\,\frac{\delta(\hat{s}+\hat{u}+\hat{t})}{8\pi\hat{S}}$$ $e^2 = 4\pi\alpha$

$$= \frac{2\cancel{16}\pi^{\cancel{2}}\alpha^2}{\hat{t}^2}\,\frac{\hat{s}^2+\hat{u}^2}{\hat{s}^2}\,\delta(\hat{s}+\hat{u}+\hat{t})\,\frac{1}{\cancel{8\pi}}$$

$$= \frac{2\pi\alpha^2}{\hat{t}^2}\,\frac{\hat{s}^2+\hat{u}^2}{\hat{s}^2}\,\delta(\hat{s}+\hat{u}+\hat{t})$$

now convert to $eq_i \to eq_i$, 1 of the two particles charge $e^2 \to e^2\cdot \underline{e_i}$

$e_i \to q_i$ charge in units of e

sending $e^2 \to e_i^2 e^2$ gives $\alpha^2 \to \alpha^2 e_i^2$

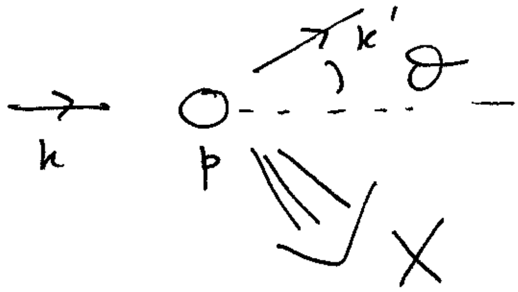
and so finally

$$\left(\frac{d\sigma}{d\hat{t}\,d\hat{u}}\right)_{eq_i \to eq_i} = \frac{2\pi\alpha^2 e_i^2}{\hat{t}^2} \; \frac{\hat{s}^2 + \hat{u}^2}{\hat{s}^2} \; \delta(\hat{s} + \hat{u} + \hat{t})$$

$$\delta(\hat{s} + \hat{u} + \hat{t}) = \delta(t + x(s+u)) \to x = -\frac{t}{s+u}$$

iv) now need to rewrite also formula (1) in terms of (s, t, u) for $ep \to eX$ scattering

now we work in __LAB__ frame



use approximate kinematics

notice ϑ now is different then ϑ __before__

$$\begin{cases} s \simeq 2k \cdot p = 2EM + O(M^2) \\ t = -2k \cdot k' = -2EE'(1-\cos\vartheta) \\ u \simeq -2k' \cdot p = -2E'M + O(M^2) \end{cases}$$

Again change of variable $(E', \vartheta) \to (t, u)$

$$\frac{\partial t}{\partial E'} = -2E(1-\cos\vartheta) \qquad \frac{\partial u}{\partial E'} = -2M$$

$$\frac{\partial t}{\partial \cos\vartheta} = 2EE' \qquad \frac{\partial u}{\partial \cos\vartheta} = 0$$

$$|\det J| = \left| \left(\quad \right) \right| = \underline{\underline{4EE'M}}$$

this gives

$$dt\,du = 4EM\,E'\,dE'\,d(\cos\vartheta) = \frac{4EM}{2\pi}\,E'dE'\,d\Omega$$

take now (1), reexpress everything in terms of (s,t,u)

you should find

$$\left(\frac{d\sigma}{dE'd\Omega}\right)_{lab} = \frac{\alpha^2}{4E^2\sin^4\frac{\vartheta}{2}}\left[W_2\cos^2\frac{\vartheta}{2} + 2W_1\sin^2\frac{\vartheta}{2}\right]$$

$$= \frac{4\pi\alpha^2}{t^2s^2}\,\frac{1}{s+u}\left[(s+u)^2\,x F_1 - us\,F_2 + O(M^2)\right]$$

neglected

where
$$\begin{cases} F_2 = \nu W_2 \\ F_1 = M W_1 \end{cases}$$

notice $\quad EM = \dfrac{s}{2}\,; \qquad t = -4EE'\sin^2\dfrac{\vartheta}{2} \Rightarrow EE'\sin^2\dfrac{\vartheta}{2} = -\dfrac{t}{4}$

$E'M = -\dfrac{u}{2}\,; \quad \Rightarrow \quad E = \dfrac{s}{2M} \qquad E' = -\dfrac{u}{2M}$

$$\sin^2\frac{\vartheta}{2} = -\frac{t}{4EE'} = \frac{t\,\cancel{4}M^2}{\cancel{4}\,su}$$

$$= M^2\,\frac{t}{su} \qquad \underline{\underline{\text{etc}}}\ \ldots$$

with $\left(x = -\dfrac{t}{s+u}\right)$ usual definition

Using now point i)

$$\frac{4\pi\alpha^2}{t^2 s^2}\frac{1}{(s+u)}\left[(s+u)^2 x F_1 - us F_2\right] = \sum_i{}' \int dx\, f_i(x) \cdot$$

$$\otimes\; x\,\frac{2\pi\alpha^2 e_i^2}{t^2}\left(\frac{s^2+u^2}{s^2}\right)\delta(t+x(s+u))$$

(notice $\boxed{\frac{\hat{s}^2+\hat{u}^2}{\hat{s}^2} \equiv \frac{s^2+u^2}{s^2}}$)

you read $\boxed{2x F_1(x) = F_2(x) = \sum_i{}' e_i^2 x f_i(x)}$