Phenomenology of Particle Physics II

Exercise Sheet 5

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`www.itp.phys.ethz.ch/education/lectures_fs12/PPPII`

In this exercise sheet, we will compute the total decay rate of the muon, which occurs via the process,

$$\mu^-(p_1) \to \nu_\mu(p_2) + e^-(k_1) + \bar{\nu}_e(k_2).$$

Exercise 6 [*Muon decay : kinematics*] We use the notation $p_i = (E_i, \vec{p}_i), k_i = (\omega_i, \vec{k}_i)$.

(i) We consider the muon as massive ($m_\mu = m$) while the electron and the neutrinos are considered massless ($m_e = m_{\nu_\mu} = m_{\bar{\nu}_e} = 0$). We work in the rest frame of the muon, and the decay product will be lying in the xz-plane. For definiteness, the muon neutrino is chosen to go along the positive z-direction, and the electron antineutrino 3-momentum makes an angle ϑ with the latter. Write down the 4-vectors involved in the process eliminating the quantities associated to the electron.

(ii) Phase space: The 3-particle massless phase space element is,

$$d\Phi_3 = \frac{d^4p_2}{(2\pi)^4}\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}(2\pi)\delta^{(+)}(p_2^2)(2\pi)\delta^{(+)}(k_1^2)(2\pi)\delta^{(+)}(k_2^2)(2\pi)^4\delta^{(4)}(p_1-p_2-k_1-k_2).$$

Show that it can be written as,

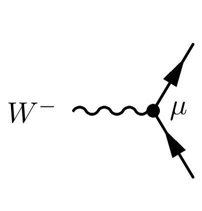
$$\begin{aligned} d\Phi_3 &= \frac{1}{2(2\pi)^3}E_2 dE_2 \omega_2 d\omega_2 d(\cos\vartheta)\delta^{(+)}\left(m^2 - 2m(E_2+\omega_2) + 2E_2\omega_2(1-\cos\vartheta)\right) \qquad (1)\\ &= \frac{1}{4(2\pi)^3} dE_2 d\omega_2. \end{aligned}$$

(iii) Flux factor: in the case of a decay it is just $F = 2E_1$.

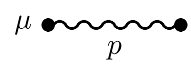
– please turn over –

Exercise 7 [*Muon decay : matrix element*]

(i) Express the matrix element $\mathcal{M}$ using the Feynman rules of the electroweak theory:

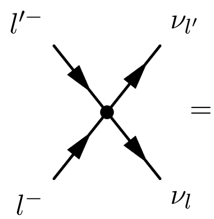


$= -i\frac{g}{\sqrt{2}}\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)$



$= i\frac{-g^{\mu\nu}+p^\mu p^\nu/m_W^2}{p^2-m_W^2}.$

(ii) Explain why the Fermi theory which describe the decay using a current-current vertex shown below is a very good approximation in the case of the muon decay.



$= i\frac{4G_F}{\sqrt{2}}$

Show that the momentum squared of the W-boson is always much smaller than m_W^2.

(iii) By comparing the matrix element obtained using Fermi and electroweak theory, derive the relation linking G_F, g and m_W.

(iv) Square the amplitude, sum over the final state spin configurations and average over the initial state spin configurations to obtain the unpolarized matrix elements squared.

(v) Compute the traces (for example using the computer algebra system FORM [1, 2]) and express the result with the the variables E_2 and ω_2 in which the phase space is expressed. *Hint*: use the δ-function in (1) to get rid of the $\cos\vartheta$.

(vi) Perform the integration of $\overline{|\mathcal{M}|^2}$ over the allowed phase space and multiply with the flux factor to get the total decay width of the muon:

$$\Gamma_\mu = \frac{1}{F}\int d\Phi_3\overline{|\mathcal{M}|^2} = \frac{G_F^2 m^5}{192\pi^3}.$$

– please turn over –

Exercise 8 [*Beta and tau decay*]

(i) Extract the value of the Fermi constant G_F^{μ} from the measured mass and lifetime of the muon (see [3]).

(ii) Find an explanation for the difference with the value extracted from neutron beta decay: $G_F^{\beta} = (1.136 \pm 0.003) \cdot 10^{-5}\,\mathrm{GeV}^{-2}$.

(iii) Consider now the same process with a tau (neutrino) instead of muon (neutrino). What is the predicted decay width? Explain the difference to the actual total decay width by assuming that the branching ratio of each possible decay mode is the same.

References

[1] http://www.nikhef.nl/~form/maindir/binaries/binaries.html

[2] An example FORM file is available on the page of the course. It can be run by calling `./form example.frm` in the folder were the executable and `example.frm` are.

[3] http://pdg.lbl.gov

Informations relative to the exercises

Testat condition : 60% of the exercise sheets worked out and solve one exercise at the blackboard.

Exercises may be solved in groups of up to 3 people.

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