Phenomenology of Particle Physics II

Exercise Sheet 6

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`www.itp.phys.ethz.ch/education/lectures_fs12/PPPII`

In this exercise sheet, we will investigate the forward-backward asymmetry induced by the production of Z-bosons in the s-channel of e^+e^--anhihilation.

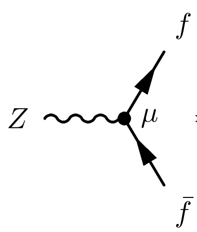
$$e^+e^- \to \gamma/Z \to \mu^+\mu^-, q\bar{q}$$

Exercise 9 [*Forward-backward asymmetry in* $e^+e^- \to \mu^+\mu^-$] Both electrons and muons are considered massless. The differential cross section in the center of mass frame can thus be written,

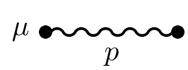
$$\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2 s},$$

where $\mathcal{M} = \mathcal{M}_\gamma + \mathcal{M}_Z$.

(i) Using the Feynman rules of the electroweak theory [1]:



$$= -i\frac{g}{\cos\theta_W}\gamma_\mu \frac{1}{2}(c_V^f - c_A^f\gamma_5)$$



$$= i\frac{-g^{\mu\nu} + p^\mu p^\nu/m_Z^2}{p^2 - m_Z^2 + i m_Z \Gamma_Z}.$$

Write down the amplitudes $\mathcal{M}_\gamma$ and $\mathcal{M}_Z$, keeping track of the flavors.

(ii) Use the Dirac equation to show that the term $p_\mu p_\nu/m_Z^2$ in the numerator of the Z-propagator vanishes leaving only $-g_{\mu\nu}$.

(iii) Show that

$$P_R = \frac{\mathbb{1} + \gamma_5}{2}, \qquad P_L = \frac{\mathbb{1} - \gamma_5}{2}.$$

are projectors, i.e. that,

$$P_L + P_R = \mathbb{1}, \qquad P_i^2 = P_i, \qquad P_L P_R = 0.$$

– please turn over –

[1] The term $i m_Z \Gamma_Z$ in the propagator becomes important for $s \approx m_Z^2$.

(iv) For both amplitudes reexpress γ_μ and $\frac{1}{2}(c_V^f - c_A^f\gamma_5)$, using P_R and P_L:

$$\frac{1}{2}(c_V^f - c_A^f\gamma_5) = c_R^f P_R + c_L^f P_L.$$

(v) Split $\mathcal{M}$ according to helicity configurations, to write it in the form,

$$\mathcal{M} = \frac{e^2}{s}\left[(1 + r c_R^\mu c_R^e)\mathcal{A}_{RR} + (1 + r c_R^\mu c_L^e)\mathcal{A}_{RL} + (1 + r c_L^\mu c_R^e)\mathcal{A}_{LR} + (1 + r c_L^\mu c_L^e)\mathcal{A}_{LL}\right],$$

with

$$\mathcal{A}_{ij} = (\bar{\mu}_i\gamma^\mu\mu_i)(\bar{e}_j\gamma_\mu e_j), \qquad i,j = L,R,$$

and find $r = r(s)$.

(vi) Argue why $|\mathcal{M}|^2$ is simply the sum of each summand squared.

(vii) Using the fact that,

$$\begin{aligned} |\mathcal{A}_{RR}|^2 = |\mathcal{A}_{LL}|^2 &= s^2(1+\cos\vartheta)^2 \\ |\mathcal{A}_{RL}|^2 = |\mathcal{A}_{LR}|^2 &= s^2(1-\cos\vartheta)^2, \end{aligned}$$

average over the initial spin configurations and find A_0 and A_1 such that,

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left[A_0(1+\cos^2\vartheta) + A_1\cos\vartheta\right]. \tag{1}$$

(viii) Plot the differential cross section in QED ($c_A^f = c_V^f = 0$) and in electroweak theory for $\sqrt{s} = 34\,\mathrm{GeV}$ (PETRA) and for $\sqrt{s} = m_Z$ (LEP I) and comment.

(ix) Defining the forward-backward asymmetry as,

$$A \equiv \frac{F-B}{F+B},$$

where,

$$F \equiv \int_0^1 d(\cos\vartheta)\frac{d\sigma}{d\Omega}, \quad B \equiv \int_{-1}^0 d(\cos\vartheta)\frac{d\sigma}{d\Omega},$$

show that,

$$A = \frac{3A_1}{8A_0}.$$

(x) Explain why it is called forward-backward asymmetry.

– please turn over –

Exercise 10 [*Forward-backward asymmetry in* $e^+e^- \to q\bar{q}$]

(i) If we want to perform the same analysis for the quark production at an e^+e^- collider, what are the differences and the similarities.

(ii) Using your previous result, repeat the points (viii)-(ix) in the case of up and down type quarks.

(iii) If only the Z-boson exchange is considered (which is a reasonable approximation when $s \approx m_Z^2$) the asymmetry in $e^+e^- \to f\bar{f}$ can be factorized as,

$$A \propto k_e k_f, \qquad k_f \equiv \frac{(c_L^f)^2 - (c_R^f)^2}{(c_L^f)^2 + (c_R^2)^2}.$$

Explain why the asymmetry discrepancy between theory and experiment observed by LEP in the b-sector (see [1], chapter 10) could be a hint for physics beyond the standard model.

Hint: Look at

$$\frac{1}{k_f}\frac{dk_f}{d(\sin^2\theta_W)},$$

at $s = m_Z^2$ for $f = \mu^+$ and $f = b$.

References

[1] http://pdg.lbl.gov

Informations relative to the exercises

Testat condition : 60% of the exercise sheets worked out and solve one exercise at the blackboard.

Exercises may be solved in groups of up to 3 people.

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