Phenomenology of Particle Physics II

Exercise Sheet 7

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Exercise 11 [*Pion Decay*]

The meson π^-, which consists of a quark-antiquark bound state:

$$|\pi^-\rangle = |d\,\bar{u}\,\rangle,$$

can decay through charged weak interaction as:

$$\pi^-(q) \longrightarrow \mu^-(p) + \bar{\nu}_\mu(k). \tag{1}$$

Compute the lifetime of the π^- following the steps listed below:

(i) Write down the matrix element for the process. Notice that the W^* couples to bound quarks in the π, so that the simple form for the quark current coupled to the W:

$$\bar{u}_d\,\gamma^\mu(1-\gamma_5)\,u_{\bar{u}}$$

cannot be used anymore. The amplitude will be instead of the form:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\,C^\mu_{d\bar{u}W}(q)\,\big[\,\bar{u}(p)\gamma_\mu(1-\gamma_5)\,v(k)\,\big]. \tag{2}$$

One can prove on invariance considerations that $C^\mu_{d\bar{u}W}(q)$ can only have the general structure:

$$C^\mu_{d\bar{u}W}(q) = f_\pi\,q^\mu,$$

where f_π is a constant. Can you justify this expression?

(ii) Working in the π^- rest frame, and using the familiar expression for the 2-particle phase space, prove that:

$$\Gamma = \frac{1}{\tau} = \frac{G_F^2}{8\pi}\,f_\pi^2\,m_\pi^2\,m_\mu^2\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2.$$

Notice that the decay rate depends quadratically on the muon mass ! What happens if we assume a massless lepton? Comment on that.

– please turn over –

(iii) Following the steps above repeat the calculation for $\pi^-(q) \to e^-(p) + \bar{\nu}_e(k)$ and show that the ratio:

$$\frac{\Gamma(\pi^- \to e^-\bar{\nu}_e)}{\Gamma(\pi^- \to \mu^-\bar{\nu}_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 = 1.2 \times 10^{-4},$$

which means that the decay of the pion to the more massive state (μ) is 10^4 times more likely! What would you expect from phase-space considerations? Can you justify physically this apparently counterintuitive result?

Exercise 12 [*Higgs production at LEP*]

LEP (Large Electron-Positron collider) was a lepton collider operating in the LHC tunnel at CERN from 1989 until 2000. It was able to collide electron and positron beams with a maximum centre of mass energy of 209 GeV reached in year 2000.

The main production channel for the Higgs boson at LEP was the so-called Higgs-strahlung:

$$e^-(p_1) + e^+(p_2) \longrightarrow Z(p_3) + H(p_4). \tag{3}$$

There is only one diagram contributing to the process above, see Figure 1.

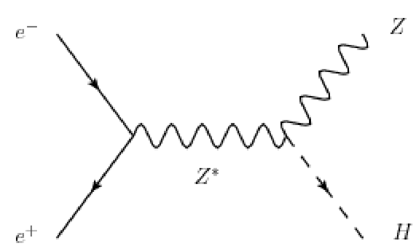

Figure 1: H Z production

We assume massless leptons, and massive final state particles, i.e. the kinematics of the process becomes:

$$p_1 + p_2 \to p_3 + p_4 \qquad \text{with} \qquad p_1^2 = p_2^2 = 0, \quad p_3^2 = m_Z^2, \quad p_4^2 = m_H^2,$$

and

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2, \quad u = (p_2 - p_3)^2, \qquad s + t + u = m_Z^2 + m_H^2.$$

– please turn over –

(i) We start analyzing the phase-space for the process. Show that in the C.o.M. frame the 2-particle phase-space reads:

$$\int d\Phi_2 = \frac{1}{8(2\pi)^2}\sqrt{\lambda(s, m_Z^2, m_H^2)}\int d\Omega_3 \tag{4}$$

Where

$$\int d\Omega_3 = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\vartheta\, d\vartheta$$

is the three-dimensional angular integration, and

$$\lambda(s, m_Z^2, m_H^2) = \lambda = \left(1 - \frac{m_Z^2}{s} - \frac{m_H^2}{s}\right)^2 - \frac{4m_Z^2 m_H^2}{s^2},$$

with m_Z being the Z mass, and m_H being the Higgs mass.
Show in addition that the flux factor simply becomes $F = 2\,s$.

(ii) Using the electroweak Feynman rules write down the amplitude squared, averaged over the leptons' spins, and summed over external polarizations. Defining $s_w = \sin\theta_w$ and $c_w = \cos\theta_w$ the Feynman rules read:

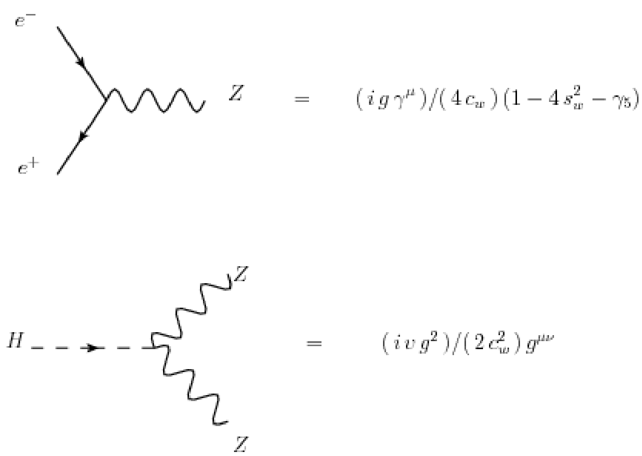

Figure 2: Electroweak Feynman rules

Prove that:

$$\begin{aligned}\overline{|\mathcal{M}|^2} &= \frac{1}{4}\sum_{spins}|\mathcal{M}|^2\\ &= \frac{1}{128}\left(\frac{g^3 v}{c_w^3}\right)^2 \frac{(1+c^2)}{(s-m_Z^2)^2}\left[\frac{m_H^2}{m_Z^2}(t - m_Z^2) + (t + 2\,s) - \frac{t\,(t+s)}{m_Z^2}\right],\end{aligned} \tag{5}$$

where we defined

$$c = 1 - 4s_w^2.$$

– please turn over –

(iii) Putting everything together, prove that the cross section for the process can be written as:

$$\sigma = \frac{1}{F} \int d\Phi_2 \, \overline{|\mathcal{M}|^2} = \frac{G_F^2 \, m_Z^4}{96 \, \pi \, s} \, (\, 1 + c^2 \,) \, \lambda^{1/2} \, \frac{\left(\lambda + 12 \, m_Z^2/s\right)}{\left(\, 1 - m_Z^2/s\,\right)^2}. \tag{6}$$

Hints: Notice that the sum over polarizations for the massive vector boson Z reads

$$\sum_{pol} \epsilon^{*\mu}(p) \epsilon^{\nu}(p) = -g^{\mu\nu} + \frac{p^\mu p^\nu}{m_Z^2}.$$

Moreover, in writing down the final formula, recall the important relations:

$$c_w = \frac{m_W}{m_Z}, \qquad m_W^2 = \frac{\sqrt{2}}{8 \, G_F} \, g^2, \qquad v = \frac{1}{(\sqrt{2} G_F)^{1/2}},$$

where m_W is the W mass, G_F is the Fermi constant, and v is the Higgs vacuum expectation value (VEV).

Informations relative to the exercises

Testat condition : 60% of the exercise sheets worked out and solve one exercise at the blackboard.

Exercises may be solved in groups of up to 3 people.

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