18 Rapidity & pseudorapidity

i) $m \to 0 \Rightarrow E \to |\vec{p}| \Rightarrow p_L \to E\cos\vartheta$

$$y = \frac{1}{2}\ln\frac{1+\cos\vartheta}{1-\cos\vartheta} = \frac{1}{2}\ln\frac{(1+\cos\vartheta)^2}{1-\cos^2\vartheta}$$

$$\overset{0\leq\vartheta\leq\pi/2}{=} \ln\frac{1+\cos\vartheta}{\sin\vartheta} = \ln\frac{\cos\vartheta/2}{\sin\vartheta/2} = -\ln\tan\vartheta/2$$

$$1+\cos\vartheta = 1+\cos^2\vartheta/2 - \sin^2\vartheta/2 = 2\cos^2\vartheta/2$$
$$\sin\vartheta = 2\sin\vartheta/2\,\cos\vartheta/2$$

ii) For $p_L > 0$ $\quad E+p_L > E-p_L$

$$\Rightarrow y \geq 0. \quad , y \text{ monotonic in } p_L$$

Max y for: $p_L = \sqrt{E^2-m^2} = p_L^{max}$

$$y_{max} \leq \frac{1}{2}\ln\frac{E+\sqrt{E^2-m^2}}{E-\sqrt{E^2-m^2}} = \frac{1}{2}\ln\frac{\left(E+\sqrt{E^2-m^2}\right)^2}{\cancel{E^2}-\cancel{E^2}+m^2}$$

$$= \ln\frac{E+\sqrt{E^2-m^2}}{m} \leq \ln\frac{2E}{m}$$

because $\sqrt{E^2-m^2} < E$. And similarly for $y<0$

Heavy: $E = \frac{\sqrt{s}}{2}(x_1+x_2)$ $\quad p_L = \frac{\sqrt{s}}{2}(x_1-x_2)$ $\quad (x_1 \geq x_2)$

$\Downarrow$ central

$$\Rightarrow y = \frac{1}{2}\ln\frac{x_1}{x_2} = 0 \text{ for } x_1 \to x_2$$

iii) $p_i = \begin{pmatrix} E_i \\ p_{L i} \end{pmatrix}$ $\Lambda_L = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix}$ $p_i' = \Lambda_L p_i$

$$y_1' - y_2' = \frac{1}{2} \ln \frac{E_1' + p_{L1}'}{E_1' - p_{L1}'} - (1 \to 2)$$

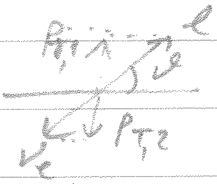
$$= \frac{1}{2} \ln \frac{\gamma E_1 + \beta\gamma p_{L1} + \beta\gamma E_1 + \gamma p_{L1}}{\gamma E_1 + \beta\gamma p_{L1} - \beta\gamma E_1 - \gamma p_{L1}} - (1 \to 2)$$

$$= \frac{1}{2} \ln \frac{(1+\beta)(E_1 + p_{L1})}{(1-\beta)(E_1 - p_{L1})} - \frac{1}{2} \ln \frac{(1+\beta)}{(1-\beta)} \frac{(E_2 + p_{L2})}{(E_2 - p_{L2})}$$

$$= \frac{1}{2} \ln \frac{E_1 + p_{L1}}{E_1 - p_{L1}} - \frac{1}{2} \ln \frac{E_2 + p_{L2}}{E_2 - p_{L2}} = y_1 - y_2$$

19 Transverse mass.

Leading order $q\bar{q}' \to W' \to l \nu_e$



$p_{T,1} = p_{T,2}$ & $\varphi_{12} = \pi$ (since there is no initial p_T)

$$M_T^2 = 4 p_T^2 = 4 \left(\frac{m_W}{2}\right)^2 \sin^2\vartheta = m_{W'}^2 \sin^2\vartheta$$

$$\cos\vartheta = \sqrt{1 - \frac{M_T^2}{m_{W'}^2}}$$

$$\frac{1}{\sigma} \frac{d\sigma}{d(M_T^2)} = \frac{1}{\sigma} \frac{d\sigma}{d(\cos\vartheta)} \underbrace{\frac{d\cos\vartheta}{d(M_T^2)}}_{= \left(\frac{d(M_T^2)}{d\cos\vartheta}\right)^{-1}}$$

$$\frac{d(M_T^2)}{d\cos\vartheta} = m_{W'}^2 (-2\cos\vartheta) = -2 m_{W'}^2 \sqrt{1 - \frac{M_T^2}{m_{W'}^2}}$$

$\to$ very big for $M_T \to m_{W'}$

$\to$ Jacobian peak.