Phenomenology of Particle Physics II

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Exercise Sheet 1

Spring semester 2012 | Issued : 21.02.2012

Lecturers: Dr. M. Dittmar, Dr. M. Grazzini | Due : 28.02.2012

www.itp.phys.ethz.ch/education/lectures_fs12/PPPII

Exercise 1 [*e^- scattering off a static Coulomb potential*]

Consider the scattering of a high-energy electron off a distributed fixed Coulomb potential

$$A^\mu(\vec{y}) = (\phi(\vec{y}), \vec{0}) \tag{1}$$

where

$$\phi(\vec{y}) = \int d^3\vec{y}'\, \frac{\rho(\vec{y}')}{|\vec{y} - \vec{y}'|} \tag{2}$$

$$\int d^3\vec{y}'\, \rho(\vec{y}') = Ze. \tag{3}$$

Remembering that the electron's current is given by

$$j^\mu(x) = e\bar{\psi}(x)\gamma^\mu\psi(x), \tag{4}$$

compute the differential cross section $\frac{d\sigma}{d\Omega}$ for such a process.

Hints

Consider the fermion fields

$$\psi(x) = u_s(p)e^{ip\cdot x} \qquad \bar{\psi}(x) = \bar{u}_{s'}(p')e^{-ip'\cdot x}. \tag{5}$$

Compute the quantity

$$T_{fi} = -i\int d^4x'\, H_{int}(x'), \tag{6}$$

where

$$H_{int}(x) = j^\mu(x)A_\mu(x). \tag{7}$$

Perform the time integration using

$$\int dt\, e^{i(E-E')t} = (2\pi)\delta(E-E'), \tag{8}$$

and the space integration using ($\vec{q} \equiv \vec{p} - \vec{p}'$)

$$\int d^3\vec{x}\, \frac{e^{i\vec{q}\cdot\vec{x}}}{|\vec{x}|} = \frac{4\pi}{|\vec{q}|^2}, \tag{9}$$

and defining the form factor

$$F(\vec{q}) \equiv \int d^3\vec{y}' \, \frac{\rho(\vec{y}')}{Ze} e^{i\vec{q}\cdot\vec{y}'}. \tag{10}$$

Then, squaring the amplitude, summing final and averaging initial electron spins using

$$|\delta(E-E')|^2 = \lim_{T\to\infty} \left| \frac{\sin\left((E-E')\frac{T}{2}\right)}{(E-E')\pi} \right|^2 \tag{11}$$

$$\delta(E-E') = \lim_{T\to\infty} \frac{2}{\pi T} \frac{\sin^2\left((E-E')\frac{T}{2}\right)}{(E-E')^2} \tag{12}$$

$$\frac{1}{2}\sum_s \sum_{s'} |u^\dagger_{s'}(p')u_s(p)|^2 = 4E^2 \left[1 - \beta^2 \sin^2\left(\frac{\theta}{2}\right)\right], \tag{13}$$

compute the quantity

$$w = \frac{|T_{fi}|^2}{T}. \tag{14}$$

Considering now the two particles phase space, compute

$$d\sigma = \frac{d^3\vec{p}'}{(2\pi)^3 2E'} \frac{w}{\beta 2E} \tag{15}$$

where $\beta = \frac{|\vec{p}|}{E}$. Upon integration you should get

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} |F(\vec{q})|^2, \tag{16}$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{(Z\alpha)^2 E^2}{4|\vec{p}|^4 \sin^4\left(\frac{\theta}{2}\right)} \left[1 - \beta^2 \sin^2\left(\frac{\theta}{2}\right)\right]. \tag{17}$$

Informations relative to the exercises

Testat condition : 60% of the exercise sheets worked out and solve one exercise at the blackboard.

Exercises may be solved in groups of up to 3 people.

Teaching assistants:
Julián Cancino, HIT K21.4, `cancinoj@itp.phys.ethz.ch`
Lorenzo Tancredi, I36 K36, `tancredi@physik.uzh.ch`
Andrey Starodumov, HPK E27, `starodumov@phys.ethz.ch`