6 Muon decay : kinematics (H&M 12.5)

i) $p_1^2 = m^2 \qquad p_2^2 = k_1^2 = k_2^2 = 0$

$p_1 = p_2 + k_1 + k_2$

Muon rest frame $\Rightarrow p_1 = \begin{pmatrix} m \\ \vec{0} \end{pmatrix}$

$p_{2,y} = k_{1,y} = k_{2,y} \stackrel{!}{=} 0$: xz-plane

$p_2 \stackrel{!}{=} \begin{pmatrix} E_2 \\ 0 \\ 0 \\ E_2 \end{pmatrix}$: muon helicity : positive z-axis

$\vec{k}_2 \cdot \vec{p}_2 \stackrel{!}{=} |\vec{k}_2||\vec{p}_2| \cos\vartheta = \omega_2 E_2 \cos\vartheta$

$\Downarrow$

$k_2 = \begin{pmatrix} \omega_2 \\ \omega_2 \sin\vartheta \\ 0 \\ \omega_2 \cos\vartheta \end{pmatrix} \Rightarrow k_1 = \begin{pmatrix} m - E_2 - \omega_2 \\ -\omega_2 \sin\vartheta \\ 0 \\ E_2 - \omega_2 \cos\vartheta \end{pmatrix}$

ii) $d\Phi_3 = (2\pi)^{-5} \underbrace{\frac{d^3 p_2}{2E_2}} \underbrace{\frac{d^3 k_2}{2\omega_2}} \delta^{(4)}\left(m^2 - 2m(E_2+\omega_2) + 2E_2\omega_2(1-\cos\vartheta)\right)$

$= (2\pi)^{-5} \frac{E_2 dE_2}{2} \frac{\omega_2 d\omega_2}{2} \delta^{(4)}(\dots) \underbrace{d\Omega_2}_{2\pi\, d(\cos\vartheta)} \cdot (4\pi)$

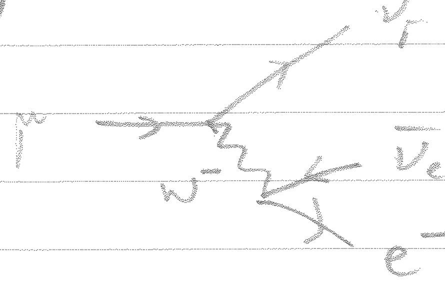
$= \frac{1}{4(2\pi)^3} dE_2 \, d\omega_2$

$\uparrow$ solving δ-function ⚠ Derivative

iii) $F = 2E_1 = 2m.$

7 Muon decay : matrix element

i)



$$\mathcal{M} = \bar{u}_{s_2}(p_2)\frac{(-i)g}{\sqrt{2}}\gamma_\mu P_L u_{s_1}(p_1)$$

$$i\,\frac{-g^{\mu\nu}+k_{12}^\mu k_{12}^\nu}{k_{12}^2-m_W^2}$$

$$\cdot\,\bar{u}_{r_1}(k_1)\,\frac{(-i)g}{\sqrt{2}}\gamma_\nu P_L v_{r_2}(k_2)$$

$$P_L = \frac{1}{2}(\mathbb{1}-\gamma_5)$$

$k_{12} = k_1 + k_2$

Note: $\underbrace{\bar{u}_{r_1}(k_1)\,\slashed{k}_1}_{=0} + \underbrace{\slashed{k}_2 P_L v_{r_2}(k_2)}_{=0\,:\ \slashed{k}_2 P_L = P_R \slashed{k}_2}$

Dirac: $=0$

$\Rightarrow k_{12}^\mu k_{12}^\nu$ can be dropped!

ii) $k_{12}^2 = (p_1-p_2)^2 = (m-E_2)^2 - E_2^2 = m^2 - mE_2$

$\leq m^2 \ll m_W^2$

$\Rightarrow k_{12}^2 - m_W^2 \approx -m_W^2$

iii) $\Rightarrow \mathcal{M}_F = i\frac{4G_F}{\sqrt{2}}\left[\bar{u}_{s_2}(p_2)\gamma^\mu P_L u_{s_1}(p_1)\right]$

typo $\left[\bar{u}_{r_1}(k_1)\gamma_\mu P_L v_{r_2}(k_2)\right]$

$$\mathcal{M} \to \frac{g^2}{2m_W^2}[\ldots][\ldots] \Rightarrow \frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

iv) $\overline{|\mathcal{M}|^2} = \frac{1}{2}\frac{g^4}{16 m_W^4} \mathrm{Tr}\left((\not{p}_1 + m)(1+\gamma_5)\gamma^\mu \not{p}_2 \gamma^{\mu'}(1-\gamma_5)\right)$

$m \ll m_W$

$\mathrm{Tr}\left(\not{k}_2 (1+\gamma_5)\gamma_\mu \not{k}_1 \gamma_{\mu'}(1-\gamma_5)\right)$

v) FORM (see solution online)

$$\overline{|\mathcal{M}|^2} = \frac{2g^4}{m_W^4}\cdot (p_2\cdot k_1)(p_1\cdot k_2)$$

$$= 2\frac{g^4}{m_W^4} E_2\left[m - \omega_2(1-\cos\vartheta)\right](m\,\omega_2)$$

From the δ function in 6 ii):

$$\omega_2(1-\cos\vartheta) = \frac{m}{E_2}(E_2+\omega_2 - m).$$

$$\overline{|\mathcal{M}|^2} = 2\frac{g^4}{m_W^4}\left[E_2 m - m(E_2+\omega_2) + m^2\right]$$

vi) $\Gamma = \frac{1}{F}\int d\Phi_2 \overline{|\mathcal{M}|^2} = \frac{1}{2m}\int_0^{m/2} dE_2 \int_{m/2-E_2}^{m/2} d\omega_2\, \overline{|\mathcal{M}|^2} = \frac{G_F^2 m^5}{192\pi^3}$

8 Beta & tau decay

i) trivial

ii) The weak eigenstates are not the mass eigenstates → CKM matrix

$$\rightarrow G_F^{\beta} = G_F^{\mu} \cdot \cos\vartheta_c < G_F^{\mu}$$

iii) $$\Gamma_{\tau \to \nu_\tau e \bar{\nu}_e} = \frac{m_\tau^5}{m_\mu^5} \Gamma_\mu = (1.5 \cdot 10^{-12}\, s)^{-1}$$

$$\Gamma_\tau = (2.9 \cdot 10^{-13}\, s)^{-1} \sim 5\, \Gamma_\mu$$

Because 5 decay channels

$\underbrace{\tau \to \nu_\tau e \bar{\nu}_e, \; \nu_\tau \mu \bar{\nu}_\mu, \; \nu_\tau d \bar{u}}$ × 3 colors → 5

⇝ "shows" that $N_c = 3$.