Phenomenology of Particle Physics II

Exercise Sheet 2

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Exercise 2 [*Callan-Gross Relation*]

Consider the lepton-proton Deep Inelastic Scattering (DIS) in the laboratory frame

$$l(k) + P(p) \longrightarrow l(k') + X(W),$$

where $X(W)$ represents an unspecified hadronic final state with total four momentum W^μ, originating from the breaking up of the proton. Defining the variables

$$\nu = \frac{p \cdot q}{M}, \qquad q = k - k',$$

where M is the proton mass, the differential cross section for this process can be parametrized in function of two form factors $W_{1,2}(\nu, q^2)$ as:

$$\left(\frac{d\sigma}{dE'\,d\Omega}\right)_{lab} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left[W_2(\nu, q^2)\,\cos^2\frac{\theta}{2} + 2\,W_1(\nu, q^2)\,\sin^2\frac{\theta}{2}\right]. \tag{1}$$

In the parton model the proton is made up of point-like free partons which carry a fraction x of the total momentum of the proton, and which interact with the lepton through the exchange of a photon.

Define the parton-momentum distribution for the parton i:

$$f_i(x) = \frac{dP_i}{dx} \qquad \text{with} \qquad \sum_i \int dx\, x\, f_i(x) = 1.$$

Work in the massless approximation, i.e.

$$s \simeq 2k \cdot p, \qquad t = q^2 = -2k \cdot k', \qquad u \simeq -2p \cdot k',$$

and neglect any parton momentum transverse to the proton momentum, i.e. the parton's momentum is $\hat{p} = xp$.

– please turn over –

(i) Notice that with these conventions the cross section can be written as

$$\left(\frac{d\sigma}{dt\,du}\right)_{lp\to lX} = \sum_i \int dx\, f_i(x)\,\left(\frac{d\sigma}{dt\,du}\right)_{lq_i\to lq_i}. \tag{2}$$

Can you motivate this form physically?

(ii) Show that the invariants for the partonic subprocess are:

$$\hat{s} = xs, \qquad \hat{u} = xu, \qquad \hat{t} = t.$$

(iii) Starting from the cross section for $e\,\mu \to e\,\mu$ show that the cross section for the partonic subprocess becomes:

$$\left(\frac{d\sigma}{dt\,du}\right)_{lq_i\to lq_i} = x\,\left(\frac{d\sigma}{d\hat{t}\,d\hat{u}}\right) = x\,\frac{2\pi\alpha^2 e_i^2}{t^2}\left(\frac{s^2+u^2}{s^2}\right)\delta(t+x(s+u)),$$

where e_i is the charge of the parton i.

(iv) Consider (1) and (2). Define the new form factors $F_2 = \nu W_2, \quad F_1 = M W_1$ and show that the left-hand side of (2) can be written as:

$$\left(\frac{d\sigma}{dt\,du}\right)_{lp\to lX} = \frac{4\pi\alpha^2}{t^2 s^2}\,\frac{1}{s+u}\Big[(s+u)^2 x\,F_1 - u\,s\,F_2\Big].$$

(v) Putting everything together you should obtain the **Callan-Gross relation** among the form factors, valid for spin 1/2 particles:

$$2\,x\,F_1(x) = F_2(x) = \sum_i e_i^2\,x\,f_i(x).$$

Informations relative to the exercises

Testat condition : 60% of the exercise sheets worked out and solve one exercise at the blackboard.

Exercises may be solved in groups of up to 3 people.

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