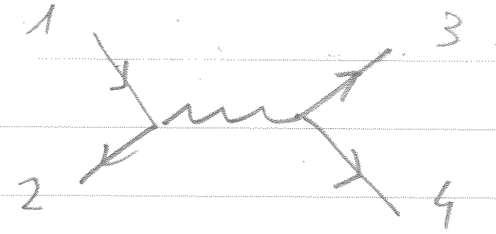
9 Forward-backward asymmetry in $e^+e^- \to \mu^+\mu^-$

(H&M 13.6

i)



$\bar{u}_1 = \bar{e} \quad u_2 = e$

$\bar{u}_3 = \bar{\mu} \quad u_4 = \mu$

$$\mathcal{M}_Z = \left(\bar{e}(-i)\frac{g}{\cos\vartheta_W}\gamma_\mu (c_V^e - c_A^e\gamma_5) e\right)$$

$$i\,\frac{-g^{\mu\nu} + p_{12}^\mu p_{12}^\nu / m_Z^2}{s - m_Z^2 + i m_Z \Gamma_Z}$$

$p_{12} = p_1 + p_2$

$$\left(\bar{\mu}\frac{g}{\cos\vartheta_W}\gamma_\nu (c_V^\mu - c_A^\mu\gamma_5)\mu\right)$$

$$\mathcal{M}_\gamma = (\bar{e}\, ie\, \gamma_\mu\, e)\, i\frac{-g}{s}(\bar{\mu}\, ie\, \gamma_\nu\, \mu)$$

ii) Dirac equation $\bar{e}\not{p}_2 = \not{p}_1 e = 0$

$$\Rightarrow \underbrace{\bar{e}(\not{p}_1 + \not{p}_2)}_{=0}(c_V^e - c_A^e\gamma_5)e$$

$$= c_V \bar{e}\underbrace{\not{p}_1 e}_{=0} - c_A^e \bar{e}\not{p}_1\gamma_5 e$$

$$\overset{\{\gamma^\mu,\gamma_5\}=0}{=} + c_A \bar{e}\gamma_5\underbrace{\not{p}_1 e}_{=0} = 0$$

iii) $P_L + P_R = \mathbb{1}$: trivial

$$P_L^2 = \frac{1}{4}(\mathbb{1} - \gamma_5 - \gamma_5 + \underbrace{\gamma_5^2}_{=\mathbb{1}} = \frac{1}{2}(\mathbb{1} - \gamma_5 = P_L.$$

$$P_L P_R = \frac{1}{4}\left(\cancel{1\!\!1} + \gamma_5 - \gamma_5 - \underbrace{\gamma_5^2}_{=1\!\!1}\right) = 0.$$

iv) $c_R = c_V - c_A \qquad c_L = c_V + c_A.$

$$\gamma_\mu = \gamma_\mu P_R + \gamma_\mu P_L.$$

v) Replace γ_μ & $(c_V^f - c_A^f\gamma_5)$ by

$$\gamma_\mu P_R,\ \gamma_\mu P_L \quad \& \quad c_L^f P_L + c_R^f P_R.$$

Use $e = g \sin\vartheta_W$.

$$r = \frac{g^2}{4\cos^2\vartheta_W}\ \frac{1}{s - m_Z^2 + i m_Z \Gamma_Z}\ \frac{s}{e^2}$$

vi) $$|A_{Li}|^2 = (\bar{\mu}_L \gamma^\mu \mu_L)(\bar{\mu}_L \gamma^\nu \mu_L)(\ldots)$$
$$= \mathrm{Tr}(\not{p}_3 P_R \gamma^\mu P_L \not{p}_4 P_R \gamma^\nu P_L)\cdot(\ldots)$$

$$\gamma_\mu P_L = P_R \gamma_\mu \Rightarrow \text{OK}.$$

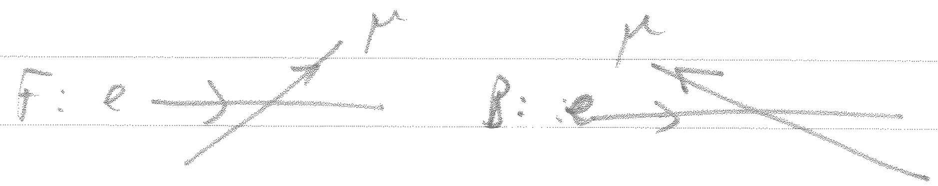
$$A_{Ri}^* A_{Lj} = (\bar{\mu}_R \gamma^\mu \mu_R)(\bar{\mu}_L \gamma^\nu \mu_L)(\ldots)$$
$$= \mathrm{Tr}(\not{p}_3 P_L \gamma^\mu \underbrace{P_R \underbrace{\not{p}_4 P_R}_{=P_L\not{p}_4}}_{=0:\ \text{iii)}} \gamma^\nu P_L)(\ldots)$$

vii) – ix) : See Mathematica file. etc. // Effect already visible for $\sqrt{s} < m_Z$.

x) ϑ: angle of $e^-\mu^-$: $\vec{p}_1 \cdot \vec{p}_3 = E_1 E_3 \cos\vartheta$

$\cos\vartheta \in [0,1] \Rightarrow \vec{p}_1 \cdot \vec{p}_3 > 0 \Rightarrow$ "forward"

$\cos\vartheta \in [-1,0] \Rightarrow \vec{p}_1 \cdot \vec{p}_3 < 0 \Rightarrow$ "backward"



10 Forward-backward asymmetry

i) Same: - diagrams

Different: - charges ($e \to e_q$), colors ($\times 3$).
- L, R (V, A) couplings.

ii)-iii) See Mathematica file.