

Ub 3.3

1

The Laplace-de Rham-Hodge operator Δ on $\Omega^*(M)$ is defined as

$$\Delta = d\delta + \delta d$$

It follows directly that

$$d\Delta = d^2\delta + d\delta d = d\delta d = d\delta d + \delta d^2 = \Delta d$$

and

$$\delta\Delta = \Delta\delta \text{ since } d^2 = \delta^2 = 0.$$

Let be $\Delta^{-1}\alpha = \beta$ with $\alpha = \Delta\beta$.

We have

$$d\Delta^{-1}\alpha = d\Delta^{-1}\Delta\beta = d\beta$$

and

$$\begin{aligned} \Delta^{-1}d\alpha &= \Delta^{-1}d\Delta\beta = \Delta^{-1}\Delta d\beta \\ &= d\beta. \end{aligned}$$

Q.E.D.