• Exercise 10 •

The static Vortex configuration, discussed during the Lecture, has energy

$$E = \int d^2x \left[\frac{1}{4} F_{ij} F_{ij} + (D_i \phi)^+ (D_i \phi) + \right.$$
$$\left. + \frac{\lambda}{2} \left(\phi^+ \phi - a^2 \right)^2 \right]$$

with $D_i = \partial_i + ie A_i$. Take the ansatz discussed in the Lecture,

$$A_i = - \varepsilon_{ij} \hat{x}_j \frac{A(r)}{r} \qquad \begin{cases} r = \sqrt{x_1^2 + x_2^2} \\ \vartheta = \mathrm{Arctan}\,[x_2/x_1] \\ \hat{x}_i \equiv \dfrac{x_i}{r} \end{cases}$$
$$\phi = e^{in\vartheta} f(r)$$

and plug it in the energy, <u>show</u> <u>that it</u> gives

$$E/_{2\pi} = \int dr \left\{ \frac{1}{2r} (A')^2 + r (f')^2 + \right.$$
$$\left. + \left(A - \frac{n}{e} \right)^2 \frac{e^2 f^2}{r} + \frac{\lambda}{2} r \left(f^2 - a^2 \right)^2 \right\}$$

from the above formula, <u>derive</u> <u>the</u> <u>conditions</u>

under which the energy is finite, show that they coincides with the ones found during the lecture. As last step, use E to compute the equations of motion for the functions f and A.

Hint: Use the formulas

$$\partial_i r = \hat{x}_i \ ; \quad \partial_i \hat{x}_j = \frac{1}{r}\left(\delta_{ij} - \hat{x}_i \hat{x}_j\right)$$

$$\partial_i \theta = -\varepsilon_{ij} \frac{\hat{x}_j}{r}$$