Sheet 6

Due: 19/04/11

Question 1 [*BRST invariance*]:

In the lecture, the BRST symmetry was discussed after we had introduced an auxiliary field in order to linearise the gauge fixing term. In this exercise we want to study the BRST symmetry of the action without auxiliary field.

(i) Consider the Lagrangian density

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 - \frac{1}{2\xi}(\partial^\mu A^a_\mu)^2 + \bar{\psi}(i\not{D} - m)\psi + \bar{c}^a(-\partial^\mu D^{ac}_\mu)c^c$$

and analyse the BRST transformation defined by

$$\delta A^a_\mu = \epsilon D^{ac}_\mu c^c \tag{1}$$

$$\delta\psi = ig\epsilon c^a t^a \psi \tag{2}$$

$$\delta c^a = -\frac{1}{2} g\epsilon f^{abc} c^b c^c \ . \tag{3}$$

Determine the BRST transformation of $\bar{c}^a$ by requiring that the corresponding action is invariant.

(ii) Define the BRST operator Q as in the lectures, and analyse its nilpotency. Check that the BRST operator is not nilpotent on $\bar{c}^a$, but that it acts as

$$QQ\bar{c}^a = -\frac{1}{\xi}\partial^\mu D^{ac}_\mu c^c \ .$$

Note that the right-hand side is proportional to the equation of motion derived for $\bar{c}^a$, and thus the BRST operator is still nilpotent on-shell.

Question 2 [*Arnowitt-Fickler gauge*]:

Perform the Faddeev-Popov quantisation of Yang-Mills theory in the gauge $A^{3a} = 0$, and write the Feynman rules. Show that there are no propagating ghosts, and that the gauge field is reduced to two positive-metric degrees of freedom. (Although the gauge condition violates Lorentz invariance, this symmetry is restored in the calculation of the gauge-invariant S-matrix elements.)

This gauge condition is often called “axial gauge” and it allows to compute physical S-matrix elements without introducing Faddeev-Popov ghost fields.

Hints:

(i) Write the Faddeev-Popov Lagrangian using the gauge fixing condition $n \cdot A = 0$ where $n^\mu = (0, 0, 0, 1)$ and show that ghosts do not couple to any field so they can be simply dropped.

(ii) Derive the Feynman rules for the above Lagrangian showing that the gauge boson propagator is not Lorentz invariant any longer.