The Higgs Mechanism in a Supersymmetric Context

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Contents

1 Introduction

A couple of points need to be mentioned before starting with the actual topic:

- It seems that one of the main sources of confusion in books on supersymmetry arises from the different labelling conventions used. In this paper, there are two main sections, one for the concepts in the standard model (SM) and one for the concepts in the minimal supersymmetric standard model (MSSM). There is therefore a short overview on conventions and labelling anteceding those sections.
- As additional help, the reader should know that these conventions are mainly taken from "A Supersymmetry Primer" by S. P. Martin [1] and from the book "Theory and Phenomenology of Sparticles" by M. Drees, R. M. Godbole and P. Roy [2].
- It should be noted that due to the vast amount of subtopics included in "the Higgs mechanism", it make sense to choose a phenomenological approach. All material should be viewed in this context.
- Many mechanisms, methods and tools used in the MSSM are very similar to the ones employed in the SM, where they are much easier to understand. It thus makes sense to spend time with the SM. Furthermore, the sections on the SM and MSSM should be complementary to each other.

2 The Higgs Mechanism in the Standard Model

2.1 Overview on Particles and Labelling

For this paper, the following conventions are used [2]:

- $l_{iL} = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L$, with $\nu_i = \{\nu_e, \nu_\mu, \nu_\tau\}$ and $e_i = \{e^-, \mu^-, \tau^-\}$. $Y = -1$.
- $e_{iR} = \{e^-_R, \mu^-_R, \tau^-_R\}$ with $Y = -2$.
- $q_{iL} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L$ with $u_i = \{u, c, t\}$ and $d_i = \{d, s, b\}$. $Y = \frac{1}{3}$.
- $u_{iR} = \{u_R, c_R, t_R\}$ with $Y = \frac{4}{3}$ and $d_{iR} = \{d_R, s_R, t_R\}$ with $Y = \frac{-2}{3}$.
- In a context where it is clear, the index i might be dropped in order to avoid messing up the space.

- g-type couplings are *gauge couplings*, i.e.
    1. g_s for strong coupling with gluons G_μ^a, $a = 1 \ldots 8$.
    2. g for electroweak couplings with gauge bosons $W_{\mu\nu}^a$ with $a = 1, 2, 3$ for the W-triplet.
    3. g' for electroweak coupling with the gauge boson B_μ, which is a singlet.
- λ_f are *Yukawa couplings* including fermions of type f.
- $v = 246$ GeV.

2.2 The Higgs Field Components in the Standard Model

1. The current or gauge eigenstate of the Higgs field in the standard model is given by a Higgs doublet with complex entries [1]. It can be written in a convenient way, in which the charge operator Q is diagonal:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} ,$$

   with 1 *neutral* and 1 *charged complex* component. In total 4 degrees of freedom.

2. There is one mass eigenstate, given in the *unitary gauge* [2] by:

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \eta(x) \end{pmatrix} ,$$

   where v is real valued and given by the vacuum expectation value of the Higgs boson: $v/\sqrt{2} = |\langle \Phi(x) \rangle|$. $\eta(x)$ is a real-valued field with $\langle \eta(x) \rangle = 0$ and can be considered a fluctuation from the vacuum expectation value. We see that there is only 1 degree of freedom left. 3 have been absorbed by the gauge bosons $W^\pm$ and Z and this way give them masses.

The general Lagrangian of the Higgs field in $|\phi|^4$-theory is:

$$\mathcal{L}_{SM,\,higgs} = |D_\mu \phi_i|^2 + \mu^2 \phi_i^* \phi_i - \frac{\lambda}{4} (\phi_i^* \phi_i)^2 , \tag{1}$$

[1] Capital Φ are used for *Higgs doublets*, small ϕ_i are used to indicate the *i-component* of a doublet.

[2] The unitary gauge minimizes the number of scalar degrees of freedom by gauging the degrees of freedom of the Goldstone bosons into longitudinal degrees of freedom of the gauge bosons $W^\pm$ and Z^0. For more on the Goldstone bosons, see section 2.3.

with $iD_\mu = i\partial_\mu - gW_\mu^a \frac{\sigma^a}{2} - g' B_\mu \frac{Y}{2}$, μ a mass parameter and λ a positive
[3] perturbation parameter. There is an implicit summation over i.

2.3 The Higgs Mechanism in the SM

In the literature the Higgs mechanism generally refers to *spontaneous symmetry breaking* of the electroweak $SU(2)_L \times U(1)_Y$ symmetry.

We split the given standard model Lagrangian for the Higgs field from equation 1 into a kinetic and a potential term:

$$\mathcal{L}_{SM,\,higgs} = \mathcal{L}_{kin,\,higgs} - V_{higgs}(\phi_i) = \underbrace{|D_\mu \phi_i|^2}_{\mathcal{L}_{kin,\,higgs}} + \underbrace{\mu^2 \phi_i^* \phi_i - \frac{\lambda}{4}(\phi_i^* \phi_i)^2}_{-V_{higgs}(\phi_i)} .$$

The Lagrangian is invariant under $SU(2)_L$ gauge transformations:

$$\Phi \rightarrow \exp\left(i\theta^a \frac{\sigma^a}{2}\right)\Phi ,$$

where σ^a are the Pauli matrices and θ^a is a gauge parameter, i.e. a rotation angle.

Let us look for extrema in the potential $V(\Phi)$ now:

$$0 \overset{!}{=} \frac{\partial V}{\partial \phi_i} = -\mu^2\left(\phi_i^* + \frac{\partial \phi_i^*}{\partial \phi_i} \cdot \phi_i\right) + \frac{\lambda}{2}(\phi_i^* \phi_i)\left(\phi_i^* + \frac{\phi_i^*}{\partial \phi_i} \cdot \phi_i\right)$$

This yields two solutions:

1. $|\phi_i^*| = |\phi_i| = 0$.
2. $|\phi_i|^2 = \phi_i^* \phi_i = \frac{2\mu^2}{\lambda}$ or $|\phi_i| = \sqrt{\frac{2}{\lambda}}\mu$.

Looking at the second derivation from the potential $V(\Phi)$ with respect to ϕ_i and bearing in mind that we are looking for a minimum, i.e. $\frac{\partial^2 V(\Phi)}{\partial \phi_i^2} > 0$, we find:

$$\frac{\partial^2 V(\Phi)}{\partial \phi_i^2} = -\mu^2 + \frac{\lambda}{2}(\phi_i^* \phi_i) > 0$$

[3] λ needs to be positive so that the potential is closed, i.e. that it goes to infinity at large values of $|\phi_i|$, bearing in mind that $-V(\phi_i) = \mu^2 \phi_i^* \phi_i - \frac{\lambda}{4}(\phi_i^* \phi)^2$.

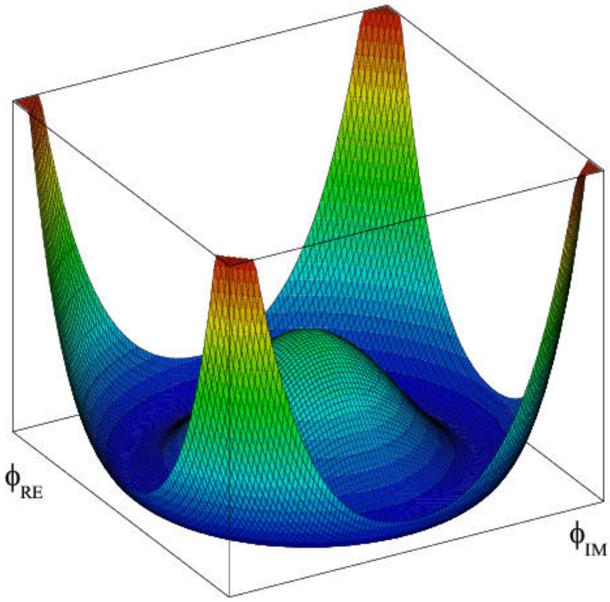


Figure 1: The Higgs potential $V(\Phi)$ in dependence of the real and imaginary part of Φ for a parameter $\mu^2 > 0$. Source: [3].

1. $\mu^2 < 0$: We see that this choice of μ^2 is consistent with the first solution $|\phi_i| = 0$, the trivial solution. This means that for this μ^2, there is a minimum at the origin and thus no symmetry breaking.
2. $\mu^2 > 0$: With this choice of μ^2, the inequality cannot be fulfilled by the trivial solution. We resort to the second solution given by $|\phi_i| = \sqrt{\frac{2}{\lambda}}\mu$, which has degenerate minima.

Hence, for a positive μ^2 we get a vacuum expectation value of $v = \frac{2}{\sqrt{\lambda}}\mu$, where v is a measure of the distance from the origin of the potential to the degenerate ground states. The potential $V(\Phi)$ with $\mu^2 > 0$ is shown in figure 1.

We consider an ansatz for the 4 degrees of freedom of the Higgs doublet given by:

$$\Phi = \begin{pmatrix} r_1 e^{i\varphi_1} \\ r_2 e^{i\varphi_2} \end{pmatrix},$$

where r_i are normalization constants. The second solution connects r_1 with r_2 by $r_1^2 + r_2^2 = \frac{2\mu^2}{\lambda}$ and we thus obtain:

$$\Phi = \begin{pmatrix} r_1 e^{i\varphi_1} \\ \sqrt{\frac{2\mu^2}{\lambda} - r_1^2} e^{i\varphi_2} \end{pmatrix} .$$

The minima determine 1 degree of freedom, which is the distance. This means that 3 degrees of freedom can be chosen freely, which we do by keeping it simple: $r_1 = \varphi_1 = \varphi_2 = 0$. This *breaks the symmetry* and we obtain a vacuum expectation value given by:

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

With this choice of r_1, φ_1 and φ_2, a zero is obtained in the top entry of the vacuum expectation value. As we will see, the unitary gauge is the logical choice as gauge. Since the top entry is generally chosen to be the charged one, the $U(1)$ symmetry will be preserved.

Taking the obtained result for the vacuum expectation value as motivation, we now parametrize the Higgs components for small deviations from the ground state as: $\phi_1 = \frac{1}{\sqrt{2}} (\alpha + i\beta)$ and $\phi_2 = \frac{1}{\sqrt{2}} (v + \eta + i\chi)$ with $\alpha(x)$, $\beta(x)$, $\eta(x)$ and $\chi(x)$ real fields. We can then write the Lagrangian as follows:

$$
\begin{aligned}
\mathcal{L}_{SM,\,higgs} &= \tfrac{1}{2}\left(D^\mu \begin{pmatrix} \alpha + i\beta \\ v+\eta+i\chi \end{pmatrix}\right)^\dagger \left(D_\mu \begin{pmatrix} \alpha + i\beta \\ v+\eta+i\chi \end{pmatrix}\right) \\
&\quad + \tfrac{\mu^2}{2}\left(\alpha^2+\beta^2+(v+\eta)^2+\chi^2\right) - \tfrac{\lambda}{16}\left(\alpha^2+\beta^2+(v+\eta)^2+\chi^2\right)^2 \\
&= \tfrac{1}{2}\begin{pmatrix} D^\mu\alpha + iD^\mu\beta \\ D_\mu\eta + iD_\mu\chi \end{pmatrix}^\dagger \begin{pmatrix} D_\mu\alpha + iD_\mu\beta \\ D_\mu\eta + iD_\mu\chi \end{pmatrix} + \tfrac{1}{2}\left|D_\mu v\right|^2 \\
&\quad + \tfrac{\mu^2}{2}\left(\alpha^2+\beta^2+(v+\eta)^2+\chi^2\right) - \tfrac{\lambda}{16}\left(\alpha^2+\beta^2+(v+\eta)^2+\chi^2\right)^2 \\
&= \tfrac{1}{2}\left|D_\mu\alpha\right|^2 + \tfrac{1}{2}\left|D_\mu\beta\right|^2 + \tfrac{1}{2}\left|D_\mu\eta\right|^2 + \tfrac{1}{2}\left|D_\mu\chi\right|^2 + \tfrac{1}{2}\left|D_\mu v\right|^2 \\
&\quad + \tfrac{1}{2}\mu^2\alpha^2 - \tfrac{\lambda}{8}v^2\alpha^2 - \tfrac{\lambda}{16}\alpha^4 \\
&\quad + \tfrac{1}{2}\mu^2\beta^2 - \tfrac{\lambda}{8}v^2\beta^2 - \tfrac{\lambda}{16}\beta^4 \\
&\quad + \mu^2 v\eta - \tfrac{\lambda}{4}v^3\eta + \tfrac{1}{2}\mu^2\eta^2 - \tfrac{\lambda}{4}v^2\eta^2 - \tfrac{\lambda}{8}v^2\eta^2 - \tfrac{\lambda}{4}v\eta^3 - \tfrac{\lambda}{16}\eta^4 \\
&\quad + \tfrac{1}{2}\mu^2\chi^2 - \tfrac{\lambda}{8}v^2\chi^2 - \tfrac{\lambda}{16}\chi^4 \\
&\quad + \text{mixed terms} + \text{const.} \\
&\overset{v=\frac{2\mu}{\sqrt{\lambda}}}{=} \tfrac{1}{2}\left|D_\mu\alpha\right|^2 + \tfrac{1}{2}\left|D_\mu\beta\right|^2 + \tfrac{1}{2}\left|D_\mu\eta\right|^2 + \tfrac{1}{2}\left|D_\mu\chi\right|^2 + \tfrac{1}{2}\left|D_\mu v\right|^2 \\
&\quad + 0\cdot\alpha^2 + 0\cdot\beta^2 + 0\cdot\eta - \mu^2\cdot\eta^2 + 0\cdot\chi^2 \\
&\quad + \text{higher order terms} + \text{mixed terms} + \text{const.}
\end{aligned}
$$

We thus see kinetic and mass terms of four real fields:

1. α and β are two real scalar fields and since they are massless, they can be identified with two Goldstone bosons: $G^\pm$.

2. η is a real scalar field with mass $m_\eta = \sqrt{2}\mu$. η can be identified with the standard modell Higgs boson. The linear term is gone due to fixing the vacuum expectation value.

3. χ can be viewed as another massless Goldstone boson: G^0.

The *higher order terms* of η, i.e. η^3 and η^4 terms, represent Higgs self couplings. As the other fields are absorbed by the gauge bosons $W^\pm$ and

Z^0, their higher order terms contribute to WW and ZZ scattering.

The *mixed terms* can be gauged away by choosing an appropriate gauge, the *unitary gauge*. We know that $\mathcal{L}_{SM,\,higgs}$ is invariant under the transformation:

$$\Phi \to e^{i\theta^a \frac{\sigma^a}{2}} \Phi$$

We can then parametrize the 4 degrees of freedom of Φ as:

$$\Phi = \frac{1}{\sqrt{2}} e^{i\frac{\zeta^a}{v}\frac{\sigma^a}{2}} \begin{pmatrix} 0 \\ v+\eta \end{pmatrix} .$$

Choosing the parameter $\theta^a = -\frac{\zeta^a}{v}$, we see that the Goldstone bosons have been gauged away and we obtain the result for Φ suggested in the previous section:

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+\eta \end{pmatrix} .$$

The 3 degrees of freedom of the Goldstone bosons have been gauged into 3 longitudinal degrees of freedom for the 3 gauge bosons $W^{\pm}$ and Z^0. As we will see in section 2.4, this implies that the gauge bosons get mass terms.

2.4 Introducing the Mass of Gauge Bosons

In the Weinberg-Salam model for standard model particles, the full Lagrangian $\mathcal{L}$ writes as [4]

$$\begin{aligned}
\mathcal{L}_{SM} \quad = \quad & -\tfrac{1}{4} W^a_{\mu\nu} \cdot W^{a\mu\nu} - \tfrac{1}{4} B_{\mu\nu} B^{\mu\nu} \\
& + \bar{f}_L \gamma^\mu \left(i\partial_\mu - g\tfrac{\sigma_a}{2} \cdot W^a_\mu - g'\tfrac{Y}{2} B_\mu \right) f_L + \bar{f}_R \gamma^\mu \left(i\partial_\mu - g'\tfrac{Y}{2} B_\mu \right) f_R \\
& + \left| \left(i\partial_\mu - g\tfrac{\sigma_a}{2} \cdot W^a_\mu - g'\tfrac{Y}{2} B_\mu \right) \Phi(x) \right|^2 - V(\Phi) \\
& - \left(\lambda_f \bar{f}_L \Phi(x) f_R + \lambda_f \bar{f}_L \Phi^c(x) f_R + h.c. \right) ,
\end{aligned}$$

with the following quantities:

$W^a_{\mu\nu}$: The gauge eigenstates of the $SU(2)$ group, forming a triplet and thus $a = 1, 2, 3$ is summed over.

$B_{\mu\nu}$: The gauge eigenstate of the $U(1)$ group, forming a singlet.

$f_{L,R}$: Left chiral and right chiral fermions. f_L is a *doublet* under $SU(2)_L$, f_R a *singlet*.

σ_a: The Pauli matrices with $a = 1, 2, 3$. They are the generators of the $SU(2)$ group.

Y: The weak hypercharge defined through $Q = T_3 + \frac{Y}{2}$, where Q is the *electric charge*, T_3 the *third component of the weak isospin* and the factor 1/2 conventional. The weak hypercharge is the generator of the $U(1)$ group.

Φ: The Higgs field in its mass eigenstate.

$V(\Phi)$: The Higgs potential.

λ_f: Yukawa couplings to the corresponding fermion f.

Furthermore, we can interprete the lines of $\mathcal{L}_{SM}$ as:

1st line: kinetic terms and self-interaction for gauge bosons $W^\pm$, Z, γ.

2nd line: kinetic terms for l and q and their interactions with $W^\pm$, Z, γ.

3rd line: kinetic terms of the Higgs field: the v term yields Gauge boson mass terms, the η term allows Higgs-gauge boson couplings. The Higgs potential $V(\Phi)$ introduces Higgs self couplings.

4th line: Yukawa terms $\Phi \bar{f} f$: the v term yields fermion masses, the η term gives Higgs-anitfermion-fermion couplings.

The mass terms of the gauge bosons come from the *kinetic terms of the v component of the Higgs field* in the 3rd line. Since we are interested in the mass of gauge bosons, we only consider terms *bilinear* in the gauge fields. Furthermore, we only consider the v component in the following:

$$
\begin{aligned}
|D_\mu\langle\Phi\rangle|^2 &= \left|\left(ig\tfrac{\sigma_a}{2}\cdot W^a_\mu + ig'\tfrac{Y}{2}B_\mu\right)\langle\Phi\rangle\right|^2 \\
&= \tfrac{1}{8}\left|\begin{pmatrix} gW^3_\mu + g'B_\mu & g\left(W^1_\mu - iW^2_\mu\right) \\ g\left(W^1_\mu + iW^2_\mu\right) & -gW^3_\mu + g'B_\mu \end{pmatrix}\begin{pmatrix}0\\ v\end{pmatrix}\right|^2 \\
&= \tfrac{v^2}{8}\left(g^2\left|W^1_\mu - iW^2_\mu\right|^2 + \left(-gW^3_\mu + g'B_\mu\right)^2\right) \\
&= \tfrac{v^2}{8}\left(\underbrace{g^2\left(\left(W^1_\mu\right)^2 + \left(W^2_\mu\right)^2\right)}_{(A)} + \underbrace{\left(-gW^3_\mu + g'\right)\left(-gW^3_\mu + g'B_\mu\right)}_{(B)}\right) \\
&= \tfrac{v^2}{8}\left(W^{1*}_\mu, \quad W^{2*}_\mu, \quad W^{3*}_\mu, \quad B^*_\mu\right)\underbrace{\begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^2 \end{pmatrix}}_{(C)}\begin{pmatrix} W^1_\mu \\ W^2_\mu \\ W^3_\mu \\ B_\mu \end{pmatrix}
\end{aligned}
$$

The basic idea on the following is to have mass matrices, which we want to modify by diagonalizing, without changing the value of the Lagrangian:

$$
\mathcal{L}_{SM,\, mass\, only} = \frac{1}{2}G^\dagger M^2 G = \frac{1}{2}G^\dagger \mathbb{I}_{4\times4} M^2 \mathbb{I}_{4\times4} G = \frac{1}{2}\underbrace{G^\dagger R^\dagger}_{(D)}\underbrace{RM^2R^\dagger}_{(E)}\underbrace{RG}_{(D)},
$$

where G is any gauge field [4] and R a unitary matrix. The terms (D) are the gauge eigenstates changed to mass eigenstates, while the term (E) is the diagonalized mass matrix.

Bearing this in mind, we study the two parts (A) and (B), which are responsible for the masses of the charged gauge bosons $W^\pm$, as well as for the Z and γ respectively.

(A): The mass matrix is already diagonlized and we can write term (A) as

$$
\frac{1}{8}(vg)^2\left(W^1_\mu - iW^2_\mu\right)\left(W^1_\mu + iW^2_\mu\right) = \left(\frac{1}{2}vg\right)^2 W^+_\mu W^-_\mu ,
$$

with $W^\pm_\mu \equiv \frac{1}{\sqrt{2}}\left(W^1_\mu \mp iW^2_\mu\right)$.

We can identify the prefactor to $W^+_\mu W^-_\mu$ as the mass:

[4]Note: It works also for scalar frields ϕ or fermion fields f. For fermion fields there will be left chiral and right chiral transformations that generally differ.

$$m_{W^\pm} = \frac{1}{2} v g \approx 80.4 \text{ GeV}\,.$$

(B): We are looking for the physical masses of the gauge boson, which is the masses of Z and γ, which is represented by the photon field A_μ. In order to obtain those, we have to rotate W_μ^3 and B_μ by an angle that will be called the *Weinberg angle* θ_W later:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}\,. \tag{2}$$

If we thus square the 2×2 matrix in the lower part of (C) and call it $\mathbb{M}_{2\times 2}$, we obtain:

$$\begin{aligned} \mathbb{M}^2_{2\times 2} &= \tfrac{v^2}{4} \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix}^T \\ &= \tfrac{v^2}{4} \begin{pmatrix} g^2 c_W^2 + 2gg' s_W c_W + g'^2 s_W^2 & \left(g^2 - g'^2\right) s_W c_W + gg' \left(s_W^2 - c_W^2\right) \\ \left(g^2 - g'^2\right) s_W c_W + gg' \left(s_W^2 - c_W^2\right) & g^2 s_W^2 - 2gg' s_W c_W + g'^2 c_W^2 \end{pmatrix}\,. \end{aligned}$$

where we have set $\cos\theta_W = c_W$ and $\sin\theta_W = s_W$.

From diagonalization, we find that the off-diagonal elements disappear for:

$$\tan\theta_W = g'/g\,.$$

We thus get

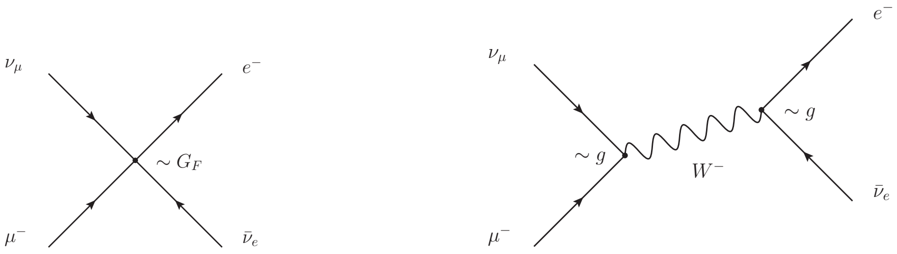
$$\mathbb{M}_{2\times 2} = \frac{v^2}{4} \begin{pmatrix} g^2 + g'^2 & 0 \\ 0 & 0 \end{pmatrix}\,,$$

and hence,

$$m_Z = \frac{v}{2}\sqrt{g^2 + g'^2} \approx 91.2 \text{ GeV} \qquad \text{and} \qquad m_\gamma = 0\,.$$

We see that the three gauge bosons $W^\pm$ and Z^0 get a mass, while γ stays massless. The latter leaves the symmetry $U(1)_{em}$ unbroken. From plugin in the obtained value for $\tan\theta_W$ into equation 2, we have the mass eigenstates:

$$Z_\mu = \frac{g W_3^3 - g' B_\mu}{\sqrt{g^2 + g'^2}} \qquad \text{and} \qquad A_\mu = \frac{g W_3^3 + g' B_\mu}{\sqrt{g^2 + g'^2}}$$



(a) (b)

Figure 2: μ decay: figure 2(a) shows the decay in the Fermi limit of low energies. Figure 2(b) shows the decay as an emission of a W^- boson.

2.5 Summary of Higgs Mechanism in SM

In short, the Higgs mechanism can be summarized by four steps:

1. Symmetry Breaking: Choosing the parameter $\mu^2 > 0$ in the Higgs potential $V(\Phi)$ and fixing the vacuum expectation value, spontaneously breaks the $SU(2)_L \times U(1)_Y$ symmetry down to a $U(1)_{em}$ symmetry.

2. Goldstone Bosons: This yields 3 massless Goldstone bosons, 1 particle remains with mass, the Higgs boson.

3. Gauge Transformation: By applying the unitary gauge, the degrees of freedom of the massless Goldstone bosons can be transformed into longitudinal degrees of freedom of the gauge bosons $W^\pm$ and Z^0.

4. Mass of Gauge Bosons: Having a longitudinal degree of freedom implies that they obtain a mass.

2.5.1 Experimental Verification of the Relation Between G_F and m_W

The μ-decay can be used to verify the connection between G_F, m_W and ultimately v^2 experimentally. We study the decay $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$, which is shown in figure 2.

The propagator of the W^- in figure 2(b) is proportional to $\sim \frac{1}{p^2 - m_W^2}$, with p the momentum of the W^-. For a limit, in which the transferred momentum p^2 is small compared to the W-mass m_W, this proportionality simplifies to $\sim \frac{1}{m_W^2}$. This limit can also be described by the Fermi approach shown in figure 2(a). We thus find the theoretical prediction:

$$\frac{8}{\sqrt{2}}G_F = g\frac{1}{m_W^2}g\,.$$

We can measure G_F experimentally. The Weinberg angle is obtained from $\cos\theta_W = \frac{m_W}{m_Z}$, which can be measured as well. This allows us to express g in terms of the Weinberg angle: $g^2 = \frac{e^2}{\sin^2\theta_W}$. The measured data confirms the theoretical prediction and justifies the usage of the W boson.

Noting the following relation, we obtain the vacuum expectation value:

$$g\frac{1}{m_W^2}g = \frac{4}{v^2} \qquad \rightarrow \qquad v^2 = \frac{1}{\sqrt{2}G_F} \approx (246\ \mathrm{GeV})^2\,.$$

2.6 Introducing the Mass of Fermions

In the standard model, the Lagrangian density $\mathcal{L}_{SM}$ is required to be invariant under electroweak symmetry transformations, i.e. invariant under $SU(2)_L \times U(1)_Y$ transformations. If we introduce a mass term for a fermion field directly into $\mathcal{L}_{SM}$, it must look as follows

$$\mathcal{L}_{SM,\,mass\,only} = -m\bar{\psi}\psi\,,$$

since the action $S = \int d^4x\mathcal{L}$ should be of massless and thus $\mathcal{L}$ of $[m]^4$ dimensions. In other words, $[m]^x \cdot [m]^{\frac{3}{2}} \cdot [m]^{\frac{3}{2}} \stackrel{!}{=} [m]^4$, where x is the dimension of the directly introduced mass parameter. With a few algebraic tricks, we obtain

$$\begin{aligned}
\mathcal{L}_{SM,\,mass\,only} &= -m\bar{\psi}\psi = -m\left(\bar{\psi}_L + \bar{\psi}_R\right)\left(\psi_L + \psi_R\right) \\
&= -m\left(\psi^\dagger P_L\gamma^0 + \psi^\dagger P_R\gamma^0\right)\left(P_L\psi + P_R\psi\right) \\
&= -m\left(\psi^\dagger\gamma^0 P_R + \psi^\dagger\gamma^0 P_L\right)\left(P_L\psi + P_R\psi\right) \\
&= -m\left(\bar{\psi}P_R P_L\psi + \bar{\psi}P_R P_R\psi + \bar{\psi}P_L P_L\psi + \bar{\psi}P_L P_R\psi\right) \\
&= -m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)\,,
\end{aligned}$$

where we have used that $P_L\gamma^0 = \gamma^0 P_R$ (or vice versa) and that $P_L P_R = 0$.

Considering that the weak isospin component T_3 of ψ_R (a $SU(2)$ singlet) is 0, while the one of ψ_L (a $SU(2)$ doublet) is $\pm 1/2$, we see that the obtained quantity is not invariant under $SU(2)_L \times U(1)_Y$ transformations. As a consequence, we cannot introduce masses directly into the Lagrangian.

A possible solution provides again the Higgs boson. Gauge invariance and renormalizability allows to introduce Yukawa terms of the form [5] [5]:

$$\mathcal{L}_l = -\lambda_l \bar{l}_L^i \Phi l_R^j + \text{h.c.}\,,$$

for the leptons (left chiral doublet l_L and right chiral singlet l_R, λ_l are the corresponding Yukawa couplings) and for the quarks [6]:

$$\mathcal{L}_q = -\lambda_d \bar{q}_L^i \Phi d_R^j - \lambda_u \epsilon^{ab} \bar{q}_{La} \Phi_b^\dagger u_R + \text{h.c.}\,.$$

Plugging in $\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+\eta \end{pmatrix}$ into these two expressions, we obtain mass terms from v and Yukawa interaction terms between the fermions and the Higgs boson from $\eta\,(x)$. We have a closer look at the mass terms:

$$\mathcal{L}_{l^i,\,mass} = -\underbrace{\frac{1}{\sqrt{2}}\lambda_l v}_{\equiv m_l^i}\,\bar{l}_L^i l_R^i + \text{h.c.}\,,$$

where we can identify $\frac{1}{\sqrt{2}}\lambda_l v \equiv m_{l^i}$ as a mass term defined by the vacuum expectation value v, but rescaled with a *dimensionless* Yukawa coupling λ_{l^i}.

$$\mathcal{L}_{q^i,\,mass} = -\underbrace{\frac{1}{\sqrt{2}}\lambda_{d^i} v}_{\equiv m_q^i}\,\bar{d}_L^i d_R^i - \frac{1}{\sqrt{2}}\lambda_{u^i} v \bar{u}_L^i u_R^i + \text{h.c.}\,,$$

where again, we identify $\frac{1}{\sqrt{2}}\lambda_{q^i} v \equiv m_{q^i}$ with a mass term depending on the vacuum expectation value v and a *dimensionless* coupling λ_{q^i}.

In general, it can thus be written:

$$\mathcal{L}_{f,\,mass} = -m_f \bar{f} f \left(1 + \frac{h}{v}\right) \qquad \text{with} \qquad m_f = \frac{1}{\sqrt{2}}\lambda_f v\,,$$

for any fermion f [7].

[5] The hermitian conjugate terms (h.c.) introduce couplings with the same particles, but with opposite chirality.

[6] Note: In order for the Lagrangian $\mathcal{L}$ to stay $U\,(1)$ gauge invariant, we also add a *charge conjugated* term Φ^C. We then may write

$$\Phi^C = i\sigma_2 \Phi^\dagger = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \epsilon^{ab}\Phi^\dagger\,.$$

[7] In general, it would be good to have $\lambda > \mathcal{O}\,(1)$ for being able to apply perturbation theory and thus one would naturally expect $\lambda \lesssim \mathcal{O}\,(1)$. Surprisingly, the different coupling constants λ vary a lot. The standard model is not capable of solving this mystery.

2.7 The WW-Scattering Amplitude

Just as a side note and for completeness, another vital function of the Higgs boson in the standard model should be mentioned: The Higgs boson preserves the unitarity of the WW-scattering amplitude. *Fermi's golden rule* gives the reaction rate per particle W [6]:

$$W = \frac{2\pi}{\hbar} \left|\mathcal{M}_{fi}\right|^2 \cdot \rho\left(E'\right) \,,$$

where

- $\mathcal{M}_{fi}$ is the matrix transition element (scattering amplitude) going from initial state i to final state f. It is given by $\mathcal{M}_{fi} = \langle\psi_f \left|\mathcal{H}_{int}\right| \psi_i\rangle$. $\mathcal{H}_{int}$ is the Hamiltonian describing the interaction.
- $\rho\left(E'\right)$ is the density of energy final states.
- W can be related to the cross section σ by $W = \frac{\sigma \cdot v_a}{V}$, where v_a is the volume density of the target particles a and V is the volume containing the beam particles b.

This gives the relation between the scattering amplitude $\mathcal{M}_{fi}$ to the cross section σ. For large energies, the Feynman diagrams in figure 3 go as:

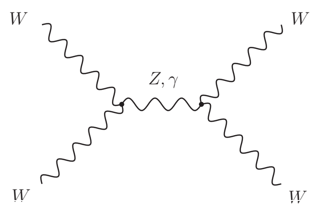
$$\left|\mathcal{M}_{fi}\right| \sim s^2 \,,$$

where s is the center of mass energy. Hence, the cross section σ should increase more and more, which is of course unphysical. In order to avoid this problem, a scalar particle with a coupling proportional $\sim m_W$ can be introduced instead of the Z and γ bosons shown in figure 3. The Higgs boson can take this role and hence preserves the unitarity of WW scattering. A similar argument can be used for the unitarity of $(\bar{f}f \rightarrow WW)$ scattering [7].

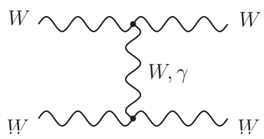
2.8 Summary

The Higgs boson in the standard model thus has three prominent functions:

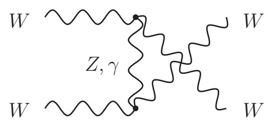
1. introduces the gauge boson mass,
2. introduces the fermion mass,
3. and is needed for a perturbative unitary gauge theory to be valid up to high energies. This includes the preservation of the unitarity of WW-scattering.



(a) s-channel.



(b) t-channel.



(c) u-channel.

Figure 3: The three channels for WW-scattering. All have a scattering amplitude $\mathcal{M}_{fi} \sim s^2$.

3 The Higgs Mechanism in a SUSY Context

3.1 Overview on SParticles and Labelling

The following conventions are used [2]:

- All particles carrying a tilde are superpartners to the particles, i.e. $\tilde{f}$ to f for fermions, $\tilde{b}$ to b for bosons, etc..
- $L_i = \begin{pmatrix} L_{\nu_{e_i}} \\ L_{e_i} \end{pmatrix}$, a superfield indexing the three generations with i and containing the fields l_{iL} and $\tilde{l}_{iL}$.
- $\bar{E}_i$, containing e^C_{iR} and $\tilde{e}^*_{iR}$.
- $Q_i = \begin{pmatrix} Q_{u_i} \\ Q_{d_i} \end{pmatrix}$, a superfield containing all q_{iL} and $\tilde{q}_{iL}$.
- $\bar{U}_i$ containing u^C_{iR} and $\tilde{u}^*_{iR}$.
- $\bar{D}_i$ containing d^C_{iR} and $\tilde{d}^*_{iR}$.

3.2 Introducing the Higgs-Doublet

We have to distinguish between the Higgs current or gauge eigenstates and the Higgs mass eigenstates:

1. The current or gauge eigenstates are represented conveniently by two *complex Higgs doublets*:

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} \qquad \text{and} \qquad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} ,$$

   where the upper indices label the electromagnetic charge of the Higgs boson. In total, there are $4 \times 2 = 8$ degrees of freedom.

2. The mass eigenstates consist of 5 Higgs bosons:

   - 2 *charged* Higgs boson: H^+ and H^-
   - 2 *neutral CP even* Higgs bosons: H and h. By convention h is chosen to be the lighter one.
   - 1 *neutral CP odd* Higgs boson: A.

   In total, we count only 5 degrees of freedom. 3 have been absorbed by the gauge bosons $W^\pm$ and Z and this way give them a mass, just as in the standard model.

3.2.1 Why Two Higgs Doublets

On the contrary to the standard model, two Higgs doublets are needed in the MSSM. This is for the following reasons:

- In the SM, quark masses can be generated by the Higgs doublet for u-type quarks. To obtain d-type masses then simply requires H^C, where the charge conjugation is given by $H^C = i\sigma^2 H^*$. For the MSSM, this will yield a problem with the superpotential W as it should stay analytic. In order to produce u-type as well as d-type quarks, we thus need two doublets.

- Keeping the summed hypercharge equal to zero is needed for an anomaly free theory. In the SM, the summed hypercharge is naturally zero, i.e. $\sum_f Y_f = 0$ for each generation of fermions. If we introduce new fermions in the form of Higgsinos, they need to fulfill the same relation. We thus need one Higgs doublet with $Y = +1$ and one with $Y = -1$.

- A similar problem as the one described in the first point also exists for the masses of the charginos, which arise from mixtures between gauginos and higgsinos.

  The Higgsinos can be introduced as left chiral fields, where right chiral ones are obtained by hermitian conjugation. From the used conventions, there is $\tilde{H}^-_{dL}$ and $\tilde{H}^+_{uL}$, but *no* $\tilde{H}^+_{dL}$ or $\tilde{H}^-_{uL}$. This requires both doublets in order to be able to construct all charginos with mass.

3.3 Higgs Potential

As a quick repetition, we introduce the complete MSSM Lagrangian [4]:

$$\mathcal{L}_{MSSM} = \mathcal{L}_{gauge} + \mathcal{L}_{matter} + \mathcal{L}_D + \mathcal{L}_W + \mathcal{L}_{soft}\,, \tag{3}$$

with the following components:

$$
\begin{aligned}
\mathcal{L}_{gauge} &= -\tfrac{1}{4}G^{a\mu\nu}G^{a}_{\mu\nu} - \tfrac{1}{4}W^{a\mu\nu}W^{a}_{\mu\nu} - \tfrac{1}{4}B^{\mu\nu}B_{\mu\nu} \\
&\quad + Tr\left(\bar{\tilde{g}} i \not{D} \tilde{g}\right) + Tr\left(\bar{\tilde{W}} i \not{D} \tilde{W}\right) + \tfrac{1}{2}\bar{\tilde{B}} i \not{\partial} \tilde{B} \\
\mathcal{L}_{matter} &= \textstyle\sum_{\psi=f,\,\tilde{H}_i} \bar{\psi} i \not{D} \psi + \sum_{\phi=\tilde{f},\,H_i} |D_\mu \phi|^2 + i \sum_{\psi,\,\phi,\,V} \frac{g^V}{\sqrt{2}} \left(\bar{\psi}_L T^a \tilde{V}^a \phi - \bar{\tilde{V}}^a T^a \psi_L \phi^*\right) \\
\mathcal{L}_D &= -\tfrac{1}{2}\textstyle\sum_{a,\,V} \left|D^V_a\right|^2 \quad \text{with} \quad D^V_a = -g^a \phi^*_i T^a_{ij} \phi_j \\
\mathcal{L}_W &= -\textstyle\sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2 - \frac{1}{2}\sum_{ij} \bar{\psi}^C_{iL} \frac{\partial^2 W}{\partial\phi_i \partial\phi_j} \psi_{jL} + \text{h.c.} \\
\mathcal{L}_{soft} &= -\tfrac{1}{2}\textstyle\sum_i M_i \bar{\lambda}^a_i \lambda^a_i - m^2_{H_d} |H_d|^2 - m^2_{H_u} |H_u|^2 \\
&\quad + B\mu\epsilon^{ij}\left(H^i_d H^j_u + \text{h.c.}\right) - m^2_{\tilde{Q}}\left(\tilde{u}^\dagger_L \tilde{u}_L + \tilde{d}^\dagger_L \tilde{d}_L\right) \\
&\quad - M^2_{\tilde{u}} \tilde{u}^\dagger_R \tilde{u}_R - M^2_{\tilde{d}} \tilde{d}^\dagger_R \tilde{d}_R + \left(\tilde{l} \text{ terms}\right) \\
&\quad + \tfrac{g}{\sqrt{2} m_W} \epsilon_{ij} \left(\tfrac{m_d}{\cos\beta} A_d H^i_d \tilde{Q}^j \tilde{d}^\dagger_R + (\tilde{u} \text{ terms}) + \left(\tilde{l} \text{ terms}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
W &= W_R + W_{\not{R}} \\
W_R &= -\epsilon_{ij}\mu H^i_d H^j_u + \epsilon_{ij}\left(\lambda_L H^i_d \tilde{L}_j \tilde{e}^C + \lambda_d H^i_d \tilde{Q}_j \tilde{d}^C + \lambda_u H^i_u \tilde{Q}_j \tilde{u}^C\right) \\
W_{\not{R}} &= \epsilon_{ij}\left(\lambda \tilde{L}_i \tilde{L}_j \tilde{e}^C + \lambda' \tilde{L}_i \tilde{Q}_j \tilde{d}^C\right) + \lambda'' \tilde{u}^C \tilde{d}^C \tilde{d}^C \,,
\end{aligned}
$$

where

- W is the superpotential containing a R-parity conserving and an R-parity breaking part, labelled with W_R and $W_{\not{R}}$ respectively [8].
- $\not{D}$ is the slashed notation for $\gamma^\mu D_\mu$.
- $iD_\mu = i\partial_\mu - g_s G^a_\mu \frac{\lambda^a}{2} - g W^a_\mu \frac{\sigma^a}{2} - g' B_\mu \frac{Y}{2}$.
- The mass parameter μ in a supersymmetric context is identified with the *higgsino mass*.

[8] R-parity in the MSSM takes the role of baryon and lepton number conservation: $\Delta B = 0$ and $\Delta L = 0$.

We now focus on the *scalar tree level potential* including exclusively Higgs terms. In its most general form, it is given by

$$\begin{aligned} V_{higgs} &= V_{higgs,\,SUSY} + V_{higgs,\,soft} \\ &= \underbrace{\frac{1}{8}\left(g'^2 + g^2\right)\left(|H_d|^2 - |H_u|^2\right)^2}_{(A)} + \underbrace{\frac{g^2}{2}\left|H_d^\dagger H_u\right|^2}_{(B)} \\ &\quad + \underbrace{|\mu|^2\left(|H_d|^2 + |H_u|^2\right)}_{(C)} + V_{higgs,\,soft} \end{aligned} \tag{4}$$

with

$$V_{higgs,\,soft} = \underbrace{m_{H_d}^2\,|H_d|^2 + m_{H_u}^2\,|H_u|^2 + \left(m_{12}^2 H_d \cdot H_u + \text{h.c.}\right)}_{(D)}$$

and

$$m_{12}^2 = B\mu\,,$$

with B a scale factor.

(A): The terms proportional to $\left(g'^2 + g^2\right)$ are D-term contributions from $\mathcal{L}_D$. The D-terms namely are $-\frac{1}{2}\sum_{a,\,V}\left|D_a^V\right|^2$ with $D_a^V = -g^a\phi_i^* T_{ij}^a \phi_j$.

(B): This term has the same origin as the (A) terms, but mixes two different field components of the Higgs doublets.

(C): The terms proportional to $|\mu|^2$ arise from F-terms. This is from $\left|\frac{\partial W}{\partial \phi_i}\right|^2$ terms, which are contained in $\mathcal{L}_W$ by differentiating the superpotential W once and squaring the result.

(D): This term was introduced into the soft symmetry breaking term of the Lagrangian in order to give masses to the two Higgs doublets.

Now, we introduce the vacuum expectation values of the two Higgs doublets, which *minimize the potential* V_{higgs}. To keep it simple, we make use of the freedom to choose a $SU(2)_L$ gauge and set $H_u^+|_{min} = 0$ without loss of generality. If $\partial V_{higgs}/\partial H_u^+ = 0$ should be valid at the minimum, it automatically follows that also $H_d^-|_{min} = 0$. We thus can assign the following vacuum expectation values:

$$\langle H_u^0 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_u \end{pmatrix} \qquad \text{and} \qquad \langle H_d^0 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_d \\ 0 \end{pmatrix}$$

Near the minimum, this choice simplifies the potential V_{higgs} to [9]

$$
\begin{aligned}
V^0_{higgs} &= \tfrac{1}{8}\left(g'^2+g^2\right)\left(\left|H^0_d\right|^2-\left|H^0_u\right|^2\right)^2 \\
&\quad + \left(m^2_{H_d}+|\mu|^2\right)\left|H^0_d\right|^2+\left(m^2_{H_u}+|\mu|^2\right)\left|H^0_u\right|^2-m^2_{12}\left(H^0_dH^0_u+\text{h.c.}\right)
\end{aligned}
$$

3.4 Higgs Mechanism in the MSSM

As a general remark in advance: the spontaneous symmetry breaking in the MSSM follows an analogous path to the one in the SM, just in a more complex way: there are two Higgs doublets and the scalar Higgs potential V_{higgs} contains more terms. First, let us introduce the following quantities:

- A connection between the vacuum expectation values and the mass of the Z gauge boson can be obtained by evaluating $\left|D_\mu\langle H^0_d\rangle\right|^2 + \left|D_\mu\langle H^0_u\rangle\right|^2$ analogously to what we did in the SM to get the gauge boson masses:

$$
m^2_Z = \frac{g^2+g'^2}{4}\left(\langle H_u\rangle^2+\langle H_d\rangle^2\right) = \frac{g^2+g'^2}{4}\left(v^2_u+v^2_d\right) \equiv \frac{g^2+g'^2}{4}v^2\,.
$$

- The ratio between the two vacuum expectation values [10]:

$$
\frac{v_u}{v_d} \equiv \tan\beta\,.
$$

- And combining the last two equations:

$$
v_u = v\sin\beta \qquad \text{and} \qquad v_d = v\cos\beta\,.
$$

As in the SM, we first try to find bounds on the parameter μ^2 and the mass terms. This will yield two boundary conditions.

In the potential V^0_{higgs}, there are *quartic* and *quadratic* parts. We notice that for $H^0_u = \pm H^0_d$ the quartic part vanishes. This direction is called the D-flat direction, since the terms in the potential coming from the D-terms vanish. In order for the potential to be bounded from below, the quadratic

[9] The negative sign in front of the term including m^2_{12} arises from $\epsilon_{du} = -1$.

[10] The fact that we want v_u and v_d to be real valued limits β: $0 \leq \beta \leq \pi/2$. Current theoretical wisdom suggests that $1 \leq \tan\beta \leq 60$.

terms should be positive along the D-flat direction. This leads to the bound [11]:

$$m_{H_u}^2 + m_{H_d}^2 + 2\,|\mu|^2 \stackrel{!}{>} 2\left|m_{12}^2\right| \qquad \textit{(convexity)}\,. \tag{5}$$

The quadratic part of V_{higgs}^0 can be written as [12]:

$$V_{higgs}^{0,\,quad.} = \left(H_d^{0*}, \quad H_u^0\right) \begin{pmatrix} m_{H_d}^2 + |\mu|^2 & -m_{12}^2 \\ -m_{12}^2 & m_{H_u}^2 + |\mu|^2 \end{pmatrix} \begin{pmatrix} H_d^0 \\ H_u^{0*} \end{pmatrix}\,.$$

Since the vacuum expectation values v_u and v_d should be non-zero, at least one of the eigenvalues of the mass squared matrix should be *negative*. From equation 5, we know that the trace of the mass matrix has to be *positive*. This implies that its determinant has to be negative for a spontaneous symmetry breakdown [13]:

$$m_{12}^4 > \left(m_{H_d}^2 + |\mu|^2\right)\left(m_{H_u}^2 + |\mu|^2\right) \qquad \textit{(non-trivial minimum)}\,. \tag{6}$$

If this condition is not fulfilled, the minimum will be stable for $H_u = H_d = 0$, i.e. we have a trivial minimum and there will be no symmetry breaking.

It should be noted that for $m_{H_u}^2 = m_{H_d}^2$ *not* both conditions 5 and 6 can be fulfilled, at least at tree level. This is the supersymmetry invariant limit, which indicates that supersymmetry breaking and electroweak breaking are closely related in the MSSM, i.e. first the supersymmetry has to be broken so that the electroweak symmetry can be broken.

Again, as in the standard model, we try to find relations between the vacuum expectation value v and the parameters in the potential V_{higgs}. We do so by using the conditions for minima:

[11] Because of quantum corrections and renormalization group evolution $m_{H_u}^2$, $m_{H_d}^2$ and m_{12}^2 become running quantities, but equation 5 has to hold at all scales.

[12] The mass matrix is defined as:

$$m_{lm}^2 = \left\langle \frac{\partial^2 V_{higgs}}{\partial\phi_l \partial\phi_m} \right\rangle\,,$$

as described in section 3.5.

[13] Note: To find the eigenvalues λ of a 2×2 matrix $\mathbb{M}$ we can use:

$$det\,(\mathbb{M} - \lambda\mathbb{I}_{2\times 2}) = \lambda^2 - Tr\,(\mathbb{M}) + det\,(\mathbb{M})$$

and thus:

$$\lambda_{1,\,2} = \frac{1}{2}\left(Tr\,(\mathbb{M}) \pm \sqrt{Tr\,(\mathbb{M})^2 - 4det\,(\mathbb{M})}\right)\,.$$

$$\frac{\partial V_{higgs}}{\partial v_d} = \frac{\partial V_{higgs}}{\partial v_u} = 0\,,$$

which yields:

$$m^2_{H_d} + |\mu|^2 = m^2_{12}\frac{v_u}{v_d} - \frac{1}{8}\left(g'^2 + g^2\right)\left(v_d^2 - v_u^2\right)\,,$$
$$m^2_{H_u} + |\mu|^2 = m^2_{12}\frac{v_d}{v_u} + \frac{1}{8}\left(g'^2 + g^2\right)\left(v_d^2 - v_u^2\right)\,.$$

These two equations can be transformed to the following two, eliminating B and $|\mu|$ [14] in favour of $\tan\beta$:

$$-2B\mu = -2m^2_{12} = \left(m^2_{H_d} - m^2_{H_u}\right)\tan 2\beta + m_Z^2 \sin 2\beta\,,$$
$$|\mu|^2 = (\cos 2\beta)^{-1}\left(m^2_{H_u}\sin^2\beta - m^2_{H_d}\cos^2\beta\right) - \frac{1}{2}m_Z^2\,.$$

3.5 Introducing Higgs Masses at Tree Level

In order to find expressions for the different Higgs masses, we look for the mass squared matrix of the Higgs scalars. It can be obtained from the quadratic part of the original potential V_{higgs} from equation 4 by differentiating twice. This includes cumbersome calculations, which are not shown here. At the end, the mass terms will enter as

$$V_{higgs} = \frac{1}{2}m^2_{lm}\phi_l\phi_m \qquad \text{with} \qquad m^2_{lm} = \left\langle \frac{\partial^2 V_{higgs}}{\partial\phi_l\partial\phi_m}\right\rangle\,, \tag{7}$$

where m^2_{lm} is the 8×8 mass matrix and $\phi_{l,m}$ is the notation of any real or imaginary part of a Higgs component field. Evaluating the mass matrix, we see that it breaks up into *four* 2×2 *matrices*, which we will have a closer look at.

3.5.1 Neutral Goldstone and CP Odd Higgs

If we consider only the *neutral and imaginary* parts of the Higgs field components and use the vacuum expectation values, we have chosen earlier:

$$\langle H_d\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_d \\ 0 \end{pmatrix} \qquad \text{and} \qquad \langle H_u\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_u \end{pmatrix}\,,$$

we obtain a mass matrix of the form:

$$m_{\Im m\, H^0_{u,d}} = \frac{m^2_{12}}{v_d v_u}\begin{pmatrix} v_u^2 & v_d v_u \\ v_d v_u & v_d^2 \end{pmatrix}\,.$$

[14] Note: The sign of μ still remains a free parameter.

By diagonalizing this matrix, we obtain the two mass terms in form of the eigenvalues:

$$
\begin{aligned}
m_{G^0}^2 &= 0\,, \\
m_A^2 = \frac{m_{12}^2}{v_d v_u}\left(v_d^2 + v_u^2\right) &= \frac{2m_{12}^2}{\sin 2\beta}\,,
\end{aligned}
$$

where the first mass eigenvalue belongs to a *neutral Goldstone boson* and the second one to a *neutral CP odd* scalar, which is labelled as the *neutral CP odd Higgs boson A*. The corresponding mass eigenstates are:

$$
\begin{aligned}
\frac{G^0}{\sqrt{2}} &= -\Im m\, H_d^0 \cos\beta + \Im m\, H_u^0 \sin\beta\,, \\
\frac{A}{\sqrt{2}} &= \Im m\, H_d^0 \sin\beta + \Im m\, H_u^0 \cos\beta\,,
\end{aligned}
$$

where β is the mixing angle [15]. Analogous to the standard model, the G^0 degree of freedom can be gauged into a longitudinal degree of freedom for the Z^0 boson, which will give it a mass term.

3.5.2 Charged Goldstones and Higgs

We collect all *charged Higgs components*, which yields in a compact complex 2×2 matrix representing a 4×4 part of m_{lm}^2:

$$
\begin{aligned}
m_{H_{u,d}^{\pm}} &= \begin{pmatrix} m_{H_d}^2 + |\mu|^2 + \frac{1}{8}\left(g'^2 + g^2\right)\left(v_d^2 - v_u^2\right) + \frac{1}{4}g^2 v_u^2 & m_{12}^2 + \frac{1}{4}g^2 v_d v_u \\ m_{12}^2 + \frac{1}{4}g^2 v_d v_u & m_{H_u}^2 + |\mu|^2 - \frac{1}{8}\left(g'^2 + g^2\right)\left(v_d^2 - v_u^2\right) + \frac{1}{4}g^2 v_d^2 \end{pmatrix} \\
&= \left(\frac{m_{12}^2}{v_d v_u} + \frac{1}{4}g^2\right)\begin{pmatrix} v_u^2 & v_d v_u \\ v_d v_u & v_d^2 \end{pmatrix}.
\end{aligned}
$$

We see that this is the same matrix as for the CP odd Higgs apart from the prefactor and thus obtain mass terms of the form:

$$
\begin{aligned}
m_{G^\pm}^2 &= 0\,, \\
m_{H^\pm}^2 = \left(\frac{m_{12}^2}{v_d v_u} + \frac{1}{4}g^2\right)\left(v_d^2 + v_u^2\right) &= m_A^2 + m_Z^2\,.
\end{aligned}
$$

The first term is identified with *two charged Goldstone bosons* $G^\pm$, the latter one with *two charged Higgs bosons* $H^\pm$. From now, we can treat m_A^2

[15] As in the standard model, we can find the rotation angle from gauge eigenstates to mass eigenstates by making an ansatz and thus obtaining a value for $\tan\beta_0$, with $\tan\beta_0$ the rotation angle for the neutral imaginary Higgs components. It is not clear from the beginning that $\tan\beta_0 = \tan\beta$, but it turns out to be true and thus has been labelled accordingly.

as input parameter eliminating m_{12}^2. By finding the eigenstates of the mass matrix, we can obtain the mass diagonal fields:

$$\begin{aligned} G^{\pm} &= -\cos\beta H_d^{\pm} + \sin\beta H_u^{\pm}\,, \\ H^{\pm} &= \sin\beta H_d^{\pm} + \cos\beta H_u^{\pm}\,, \end{aligned}$$

As discussed in the standard model, the two $G^{\pm}$ degrees of freedom can be gauged into longitudinal degrees of freedom for the $W^{\pm}$ bosons.

3.5.3 Neutral CP Even Higgs

The last components to consider are the *real* parts of the *neutral* Higgs fields. We find:

$$m^2_{\Re e\, H^0} = \frac{1}{2}\begin{pmatrix} 2m_{H_d}^2 + 2\left|\mu\right|^2 + \frac{1}{4}\left(g'^2+g^2\right)\left(3v_d^2 - v_u^2\right) & -2m_{12}^2 - \frac{1}{2}v_d v_u\left(g'^2+g^2\right) \\ -2m_{12}^2 - \frac{1}{2}v_d v_u\left(g'^2+g^2\right) & 2m_{H_u}^2 + 2\left|\mu\right|^2 + \frac{1}{4}\left(g'^2+g^2\right)\left(3v_u^2 - v_d^2\right) \end{pmatrix}$$

This matrix is then expressed in terms of m_A^2 and m_Z^2 and we obtain the masses:

$$m_{H,h}^2 = \frac{1}{2}\left(m_A^2 + m_Z^2 \pm \left(\left(m_A^2+m_Z^2\right)^2 - 4m_Z^2 m_A^2\cos^2 2\beta\right)^{1/2}\right),$$

and the corresponding eigenstates:

$$\begin{aligned} \frac{H}{\sqrt{2}} &= \left(\Re e\, H_d^0 - \frac{v_d}{\sqrt{2}}\right)\cos\alpha + \left(\Re e\, H_u^0 - \frac{1}{\sqrt{2}}v_u\right)\sin\alpha\,, \\ \frac{h}{\sqrt{2}} &= -\left(\Re e\, H_d^0 - \frac{v_d}{\sqrt{2}}\right)\sin\alpha + \left(\Re e\, H_u^0 - \frac{1}{\sqrt{2}}v_u\right)\cos\alpha\,. \end{aligned}$$

The state h is identified with the lighter of the two Higgs. Since they both have the same charge and are both CP even, they can only be distinguished by their mass.

How the angles α and β are related, can be best seen in a graphical representation shown in figure 4. Mathematically it is described by:

$$\cos 2\alpha = -\frac{m_A^2 - m_Z^2}{m_H^2 - m_h^2}\cos 2\beta\,. \tag{8}$$

From the bound that $0 \leq \beta \leq \pi/2$, we obtain a bound for α: $-\pi/2 \leq \alpha \leq 0$.

Furthermore, we can interprete the mass eigenstates h as *a fluctuation along the shallow direction* in figure 5. H is then correspondingly *a fluctuation along the steep direction.*

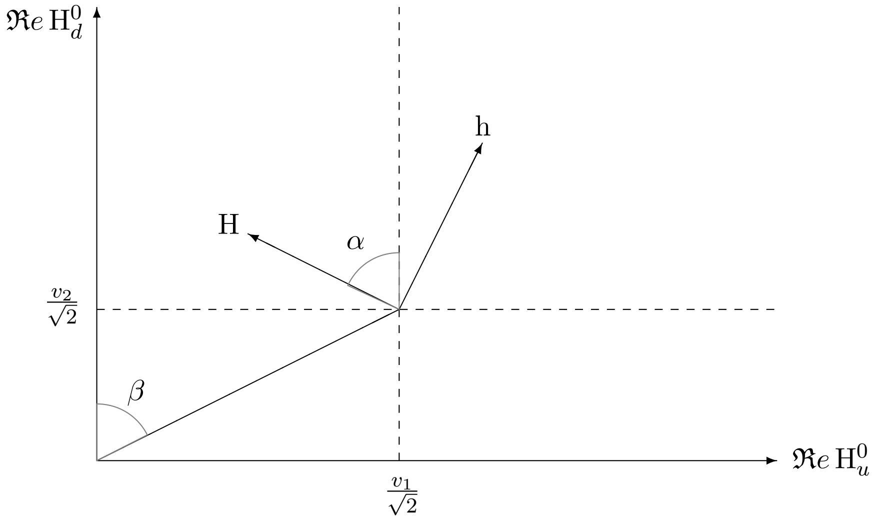


Figure 4: The rotation angles β and α in a plot $\Re e\, H_u^0$ versus $\Re e\, H_d^0$. A value for $\tan\beta > 1$ is chosen, which is consistent with expected values.

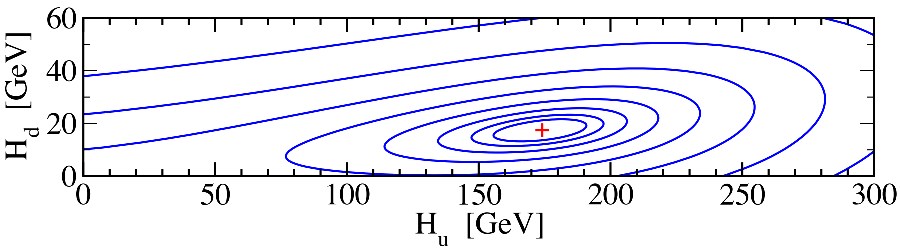


Figure 5: The same plot as shown in figure 4, displaying the potential through equipotential lines. Typical values for $\tan\beta$ have been chosen, i.e. $\tan\beta \approx -\cot\alpha \approx 10$. The mass eigenstate of h is a fluctuation along the shallow direction of the potential. The mass eigenstate for H is conversely fluctuating along the steep direction. Source: [1].

3.5.4 Relations and Constraints

The previous section indicates that the Higgs mass spectrum is completely controlled by two parameters, which can be chosen to be m_A and $\tan\beta$. There can be found several constraints, from which only the most important ones are listed here:

- $m_W \leq m_{H^\pm}$.
- $m_h \leq m_Z \leq m_H$.
- $m_h \leq m_A \leq m_H$.
- m_h is bounded from above, m_H, $m_{H^\pm}$ and m_A can be large.

3.5.5 The Decoupling Limit

In short, the decoupling limit is the limit where $m_A \to \infty$. This leads to two major effects:

1. $m_h \to m_Z\,|\cos 2\beta|$. This gives a first upper bound for the mass m_h:

   $$m_h < m_Z\,|\cos 2\beta| \leq 91.2 \text{ GeV}\,,$$

   which means that m_h has to be smaller than the Z boson mass, since $\cos 2\beta$ cannot be larger than 1. This boundary also leads to that *couplings between h and fermions or gauge boson pairs become identical to the couplings described in the standard model* [16]. Analogously, *hhh* or *hhhh* self-couplings become equal to SM couplings.

2. All other Higgs masses become uniformly heavy.

Numerically, this limit starts to become important for $m_A \geq 250$ GeV, which is not at all unrealistic: First, A has not been found in collider experiments so far and secondly, the larger m_A is, the larger can m_h be.

This is an important advantage of the MSSM compared with the SM. In the standard model, m_h is not bounded from above and there is no connection between m_h and m_Z. The MSSM provides more detailed information on where to look for the neutral CP even Higgs h.

Figure 6 shows the masses for h, H and $H^\pm$ in dependence of the CP odd Higgs' mass m_A for two different values of $\tan\beta$. For both values of $\tan\beta$ the decoupling limit can be seen. We also notice that for a large $\tan\beta$, the bound for m_h is reached very soon. For $\tan\beta = 30$, this is around $m_A = 130$ GeV.

[16] This can be seen using equation 8 and adapting the values given in table 1 for the decoupling limit.

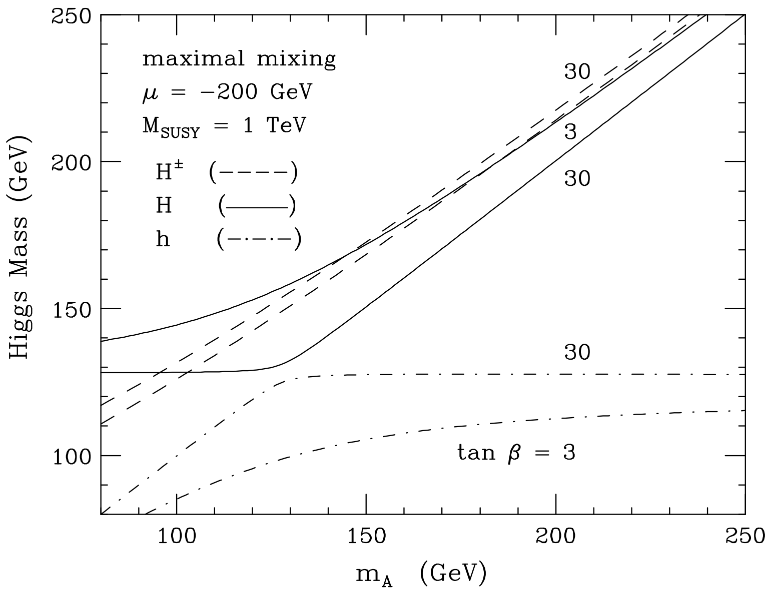


Figure 6: The four Higgs masses m_h, m_H and $m_H^{\pm}$ in dependence of the fifth Higgs mass m_A. Source: [9].

3.6 Summary of Higgs Mechanism in MSSM

We now have determined the conditions for symmetry breaking in the MSSM. It is important to notice that most of the mechanisms follow an anologous way to the one described in the standard model, i.e.

1. Symmetry Breaking: Choosing the parameters in the Higgs potential V_{higgs} according to equations 5 and 6 together with fixing the vacuum expectation value spontaneously breaks the $SU(2)_L \times U(1)_Y$ symmetry down to a $U(1)_{em}$.

2. Goldstone Bosons: This yields 3 massless Goldstone bosons, 5 Higgs bosons have mass. These are in particular H^+, H^-, H, h and A.

3. Gauge Transformation: By applying the unitary gauge, the degrees of freedom of the massless Goldstone bosons can be made into longitudinal degrees of freedom of the gauge bosons.

4. Mass of Gauge Bosons: Having a longitudinal degree of freedom implies that they obtain a mass.

3.7 Higgs-Particle Vertices

This following particle couplings appear in the Lagrangian:

1. Physical Higgs bosons coupling to standard model fermions:

   - Higgs-fermion-antifermion couplings $H\bar{f}f$.

   These couplings arise from the term $-\frac{1}{2}\sum_{ij}\bar{\psi}^C_{iL}\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\psi_{jL} + \text{h.c.}$ in the Lagrangian $\mathcal{L}_W$.

2. Physical Higgs bosons coupling to standard model gauge bosons:

   - Higgs-gauge-gauge couplings HGG.
   - Higgs-gauge-Higgs couplings HGH.
   - Higgs-Higgs-gauge-gauge couplings $HHGG$.

   They originate from the term $+\sum_{\phi=\tilde{f},H_i}|D_\mu\phi|^2$ in the Lagrangian $\mathcal{L}_{matter}$ and are the kinetic terms of the Higgs fields.

3. Higgs self couplings. The Higgs self couplings come from the trilinear and quartic terms in the Higgs potential V_{higgs}, which originate from $-\frac{1}{2}\sum_{a,V}\left|D^V_a\right|^2$ in $\mathcal{L}_D$.

An overview on the different couplings with respect to the standard model can be seen in table 1 [9]. The MSSM couplings are very similar to the SM ones, but are slightly changed by a factor determined through the angles α and β. They are determined completely by the electroweak parameters of the standard model and the angles β and α.

The Higgs-fermion-antifermion Yukawa interaction part of the Lagrangian can be written as [2]:

$$\begin{aligned}\mathcal{L}_{yuk,\,H\bar{f}f} \quad = \quad & \tfrac{g\cdot m_d}{2m_W\cos\beta}\sum_f \bar{f}_d f_d\,(H\cos\alpha - h\sin\alpha) + \tfrac{ig\cdot m_d\tan\beta}{2m_W}\sum_f \bar{f}_d\gamma_5 f_d A \\ & \tfrac{g\cdot m_u}{2m_W\sin\beta}\sum_f \bar{f}_u f_u\,(H\sin\alpha + h\cos\alpha) + \tfrac{ig\cdot m_u\cot\beta}{2m_W}\sum_f \bar{f}_u\gamma_5 f_u A \quad , \\ & \tfrac{g}{\sqrt{2}m_W}\sum_f \left(H^+\bar{f}_u\,(m_u\cot\beta P_L + m_d\tan\beta P_R)\,f_d + \text{h.c.}\right)\end{aligned}$$

where f is summed over all *leptons and quarks*.

1st line: d-type fermion-antifermion-Higgs interactions with the neutral Higgs H, h and A.

2nd line: u-type fermion-antifermion-Higgs interactions with the neutral Higgs H, h and A.

3rd line: interaction between charged Higgs, u-type and d-type fermions.

The terms for HGG, HGH, $HHGG$ and for the Higgs self-couplings follow analogously.

	Φ	λ_{u_i}	λ_{d_i}	$g-type$
SM	H	1	1	1
MSSM	h	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$\sin(\beta-\alpha)$
	H	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\cos(\beta-\alpha)$
	A	$1/\tan\beta$	$\tan\beta$	0

Table 1: The standard model couplings have been normalized to 1, the MSSM couplings are shown in relation to the ones in the standard model. There is no coupling between A and gauge bosons, since A is CP odd [9].

3.8 Higgs-Sparticle Vertices

Higgs-sparticle vertices describe a multitude of couplings, which only an overview is given of:

1. Higgs couplings to *neutralinos* $\tilde{\chi}^0$ and *charginos* $\tilde{\chi}^{\pm}$. Neutralinos and charginos are mass eigenstates containing Higgsino as well as gaugino current eigenstates. The couplings arise from the term $+i\sum_{\psi,\,\phi,\,V}\frac{g^a}{\sqrt{2}}\left(\bar{\psi}_L T^a\tilde{V}^a\phi-\tilde{V}^a T^a\psi_L\phi^*\right)$ in $\mathcal{L}_{matter}$.

2. Higgs couplings to *sfermions* $\tilde{f}$. These couplings come from D and F-terms in the scalar potential, i.e. from $-\frac{1}{2}\sum_{a,\,V}\left|D_a^V\right|^2$ in $\mathcal{L}_D$ and from $-\sum_i\left|\frac{\partial W}{\partial\phi_i}\right|^2$ in $\mathcal{L}_W$, but also from trilinear soft supersymmetry breaking terms $+\frac{g}{\sqrt{2}m_W}\epsilon_{ij}\left(\frac{m_d}{\cos\beta}A_d H_d^i\tilde{Q}^j\tilde{d}_R^\dagger+(\tilde{u}\text{ terms})+\left(\tilde{l}\text{ terms}\right)\right)$. In particular we have:

   - Higgs-slepton-antislepton coupling $H\tilde{l}\bar{\tilde{l}}$.
   - Higgs-squark-antisquark coupling $H\tilde{q}\bar{\tilde{q}}$.
   - Higgs-Higgs-slepton-antislepton coupling $HH\tilde{l}\bar{\tilde{l}}$.
   - Higgs-Higgs-squark-antisquark coupling $HH\tilde{q}\bar{\tilde{q}}$.

3.9 Radiative Effects

So far, we have only studied masses and couplings at tree level. Since h is the lightest Higgs boson, which imposes that radiative effects can introduce large scale changes, we will focus on this particle. Radiative effects also provide an escape why the h bosons might not have been discovered yet. Furthermore, we will constrain our thoughts to one-loop corrections for m_h. The dominating contributions come from t quarks, and its supersymmetric partners, the stop quarks $\tilde{t}_L$ and $\tilde{t}_R$, since they have large Yukawa couplings with h [17]. As we will later see, the corrections are proportional to $\sim m_t^4$.

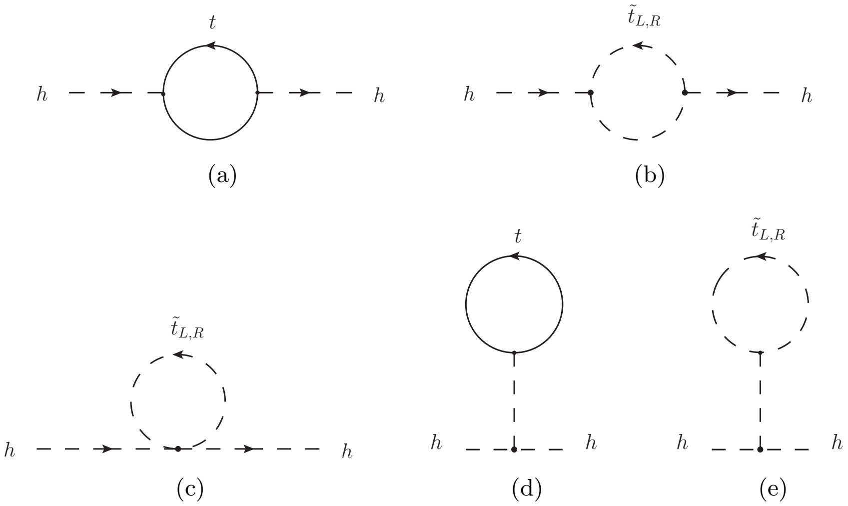


Figure 7: These loops contribute to a one-loop correction of the mass m_h. The loops 7(a) to 7(c) contribute due to the large top and stop masses. The contributions from loops 7(d) and 7(e) are rendered zero in the effective potential technique.

To get a better theoretical understanding, we consider a method called *effective potential technique* [18]:

$$V_{higgs}^{1} = V_{higgs}^{(0)}(Q) + \Delta V_{higgs}^{(1)}(Q) ,$$

with the tree level potential $V_{higgs}^{(0)}$, the correction $\Delta V_{higgs}^{(1)}$ and Q as the scale at which the couplings are renormalized. $\Delta V_{higgs}^{(1)}$ can be calculated as shown in [5]. In the following, we will only outline the proceedings distinguishing two cases with the first one being a simplification of the second one:

Corrections *without* $\tilde{t}_L$-$\tilde{t}_R$ mixing: First, we assume $m_{\tilde{t}_L} = m_{\tilde{t}_R}$ and secondly, we neglect any mixing among the two stop quarks. Further:

1. We find $m_t^2(h)$ in terms of the neutral u-type higgs current state $\left|H_u^0\right|$. Anologously $m_{\tilde{t}_L}$ and $m_{\tilde{t}_R}$ in terms of $\left|H_u^0\right|$.

[17] Since $\tilde{t}$ are the supersymmetric partners of the t quark, the Yukawa coupling of t can also be used for $\tilde{t}$, and vertices including t as well as $\tilde{t}$ are proportional to $\sim \lambda_t$.

[18] Three main methods for calculating radiative effects can be used: a) direct diagram calculation, b) renormalization group methods and c) effective potential techniques. a) and b) will not be further discussed here.

2. Plugging the obtained mass terms into $\Delta V^{(1)}_{H,t-\tilde{t}}(Q)$, the correction to the original potential is now only dependent on $t-\tilde{t}$-terms.
3. Since only the doublet H_u gets a change, we do not have to consider terms containing the H_d doublet.
4. To obtain the mass matrix, we need to derivate the new potential twice again with respect to the *real* and *imaginary* parts of the Higgs component fields.
5. The change in the mass matrix of the CP even Higgs h can then be displayed in a form $m_0 + \delta m(\epsilon_h)$, with δm the mass change depending on ϵ_h, a parameter describing the magnitude of the corrections. δm has the form

$$\delta m^2_{\mathfrak{Re}\,H^0} = \begin{pmatrix} 0 & 0 \\ 0 & \frac{\epsilon_h}{\sin^2\beta} \end{pmatrix} \quad \text{with} \quad \epsilon_h = \frac{3G_F m_t^4}{\sqrt{2}\pi^2} \ln \frac{m_{\tilde{t}}^2}{m_t^2} \, .$$

While $\tan\beta$ stays as it is, $\tan\alpha$ needs to be changed as a consequence of the changed mass matrix.

This yields a new upper boundary for m_h:

$$m_h < \sqrt{m_Z^2 \cos^2 2\beta + \epsilon_h} \approx 110 \text{ GeV} \, ,$$

where the value has been obtained assuming a modest m_A, i.e. $m_A > 300$ GeV. This value has already excluded experimentally. Further, we should note that this correction has a dependence on m_t^4, but also a logarithtmic dependence on $m_{\tilde{t}}^2$.

Corrections *with* $\tilde{t}_L$-$\tilde{t}_R$ mixing: For this case, we allow $\tilde{t}_L$-$\tilde{t}_R$ mixing, which is described by the off-diagonal elements of the mass matrix of $\tilde{t}_{L,R}$. Additionally, $m_{\tilde{t}_L}$ and $m_{\tilde{t}_R}$ do not necessarily have to be the same.

1. Again, we find $m_t^2(h)$ in terms of the neutral u-type higgs current state $|H_u^0|$. But now, $m_{\tilde{t}_L}$ and $m_{\tilde{t}_R}$ depend on $|H_u^0|$ *and* $|H_d^0|$.
2. Both doublets $H_{u,d}$ get changed and we need to consider both minimization conditions $\partial V^1_{higgs}/\partial H_d^0 = 0$ and $\partial V^1_{higgs}/\partial H_u^0 = 0$.
3. To obtain the mass matrix, we need to derivate the new potential twice again with respect to the *real* and *imaginary* parts of the Higgs component fields.
4. The change in the mass matrix of the CP even Higgs can then be displayed in a form $m_0+\delta m(\epsilon_h)$. The mass matrix δm containing the corrections, has now the form:

$$\delta m^2_{\mathfrak{Re}\,H^0} = \begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{pmatrix} \, .$$

All entries have obtained a correction represented by Δ_{ij} [19]. This allows to obtain an even weaker boundary for the Higgs mass m_h:

$$m_h^2 \leq m_Z^2 \cos^2 2\beta + \Delta_{11} \cos^2 \beta + \Delta_{12} \sin 2\beta + \Delta_{22} \sin^2 \beta \approx 132 \text{ GeV},$$

having chosen values in order to maximize the possible values for m_h.

There can be found less stringent bounds for m_h resorting to *next-to-minimal* supersymmetric models. The bound can hence be forced up to [1]:

$$m_h \leq 150 \text{ GeV}.$$

3.10 Higgs Particles at Collider Experiments

In the previous section, there were several methods explained that allow to weaken the bound on the Higgs mass m_h in order to explain why h has not been found as of now. This section summarizes the experimental techniques and limits.

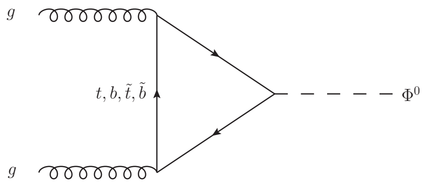
Decay: Since Higgs particles cannot be measured directly, it has to be studied into what particles the h, H, $H^{\pm}$ and A decay. In general, it is very difficult to measure corresponding processes. The dominant process is that Higgs decay into $b\bar{b}$ pairs, which are impossible to distinguish from the background. One possibility thus is to resort to the cleanest, though very rare process of Higgs decaying into $\gamma\gamma$ pairs.

Production: In a collider, the production processes of supersymmetric Higgs particles are very similar to the ones in the standard model. The differences arise from the changed coupling constants. The most dominant process is the gluon fusion, which is important for all $\tan\beta$. It is also the dominant process in the standard model. For large $\tan\beta$ however, bremsstrahlung becomes more important. This is through quark pairs annihilating to a gluon, which produces a $t\bar{t}$ or $b\bar{b}$ pair, one of which then radiates a Higgs. Both mentioned processes are shown in figure 8.

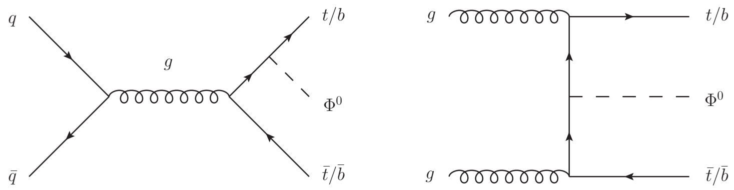
Figure 9 shows the mass m_h in dependence of different values for $\tan\beta$. Many values have already been excluded by the LEP and LEP2 experiments. The last resort seems to lie in the hope that there are large contributions from radiative corrections as well as that m_A is large enough to allow for high m_h values.

So, the Higgs particle is still not found and if we are a bit unlucky, everything written so far is useless. In other words, this might be a good point to stop.

[19] The expressions for Δ_{ij} are quite lengthy and can be found in [2].



(a) gluon fusion.



(b) bremsstrahlung.

Figure 8: The two most important production processes in the MSSM.

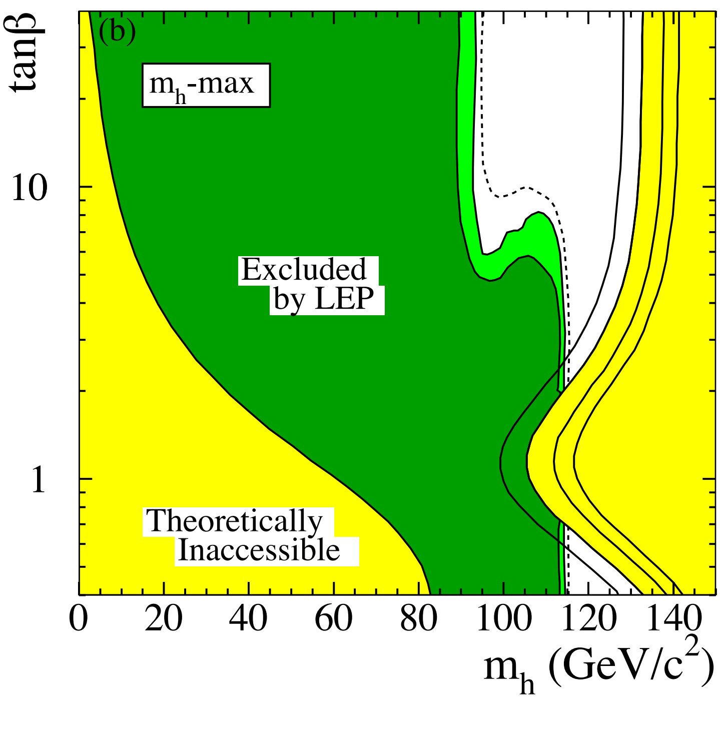


Figure 9: Limits to m_h combining theoretical considerations with experimental data. Green is the part excluded by LEP and LEP2, yellow the theoretically prohibited region. Source: [9].

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