Topic 7

The Minimal Supersymmetric Standard Model

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Table of Contents

7.1 Introduction

The main goal of this report is to give a short phenomenological introduction to the minimal Supersymmetric Standard Model, which is generally abbreviated as "MSSM". Minimal is meant in the sense that the particle contents and interactions in the known Standard Model is minimally enlarged by supersymmetry. Furthermore I need to stress, that I only treat the case of $N = 1$ supersymmetric transformations, to keep it somewhat more basic and to prevent from introducing more than the known 4 spacetime dimensions.

7.2 The Standard Model

The Standard Model of particle physics provides a very successful description of the presently known phenomena. Collider experiments, which range nowadays into the teraelectron volt (TeV) regime, confirm this highly esteemed theory without any additional structure. Still, physicists agree that the Standard Model is a work in progress, and has to be extended to successfully describe the physics beyond those energies. Not only does it not contain quantum gravitational effects, which come into play at the Planck scale $M_P = (8\pi G)^{-1/2} \sim 2.4 \cdot 10^{18}$ GeV, but also further issues, that emerge at energies between the TeV and the Planck scale, among them the notorious "hierarchy problem", indicate that the Standard Model is not a complete theory.

Even though the Standard Model is of upmost importance in discovering a complete theory of everything, which is the reason I want to give the reader a short overview over it:

7.2.1 Structure of elementary particles

The elementary fermions in the Standard Model can be divided into *quarks* and *leptons*. In each group are six particles, which can be further divided into 3 families, with similar interaction properties. In addition there exists for each particle a so called *anti-particle*, with the same properties as the original one, except for the electrical charge, which has opposite sign. All Fermions, minus anti-particles, with approximate mass and charge are summarized in Table 7.1.

Neutrinos only interact very lightly with matter, in contrast to the other leptons, which can be detected very easily as free particles in collider experiments. Quarks

Fermions			Mass $[\frac{\text{GeV}}{\text{c}^2}]$	Electric charge $[e]$	Weak isospin	Weak hypercharge	$SU(3)_C$ - repr.
Leptons	Electron	e	$511 \cdot 10^{-3}$	-1	$-\frac{1}{2}$	-1	$\mathbf{1}$
	e-Neutrino	ν_e	0	0	$\frac{1}{2}$	-1	$\mathbf{1}$
	Muon	μ	105.7	-1	$-\frac{1}{2}$	-1	$\mathbf{1}$
	μ-Neutrino	ν_μ	0	0	$\frac{1}{2}$	-1	$\mathbf{1}$
	Tauon	τ	1776.8	-1	$-\frac{1}{2}$	-1	$\mathbf{1}$
	τ-Neutrino	ν_τ	0	0	$\frac{1}{2}$	-1	$\mathbf{1}$
Quarks	Up	u	2.55	$+\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\mathbf{3}$
	Down	d	5.04	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	$\mathbf{3}$
	Charm	c	1270	$+\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\mathbf{3}$
	Strange	s	105	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	$\mathbf{3}$
	Top	t	$171.3 \cdot 10^3$	$+\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\mathbf{3}$
	Bottom	b	$4.2 \cdot 10^3$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	$\mathbf{3}$

Table 7.1: Properties of elementary fermions with spin $\frac{1}{2}\hbar$ in two times three generations without anti-particles.

can in nature only be found in bound states of multiple quarks, because they have so called *color charges* in addition to electrical charge. The color charge can take one of these three values: red (r), green (g) and blue (b) or their *anti-color*, anti-red ($\overline{r}$), anti-green ($\overline{g}$) and anti-blue ($\overline{b}$). Free particles composed of quarks, so called *hadrons* always have color charge zero. The simplest possibility to compose a hadron is to put two quarks together with color and anti-color. This leads to the *mesons*. Another possibility is the composition of three quarks with each a different color or anti-color. This gives us the *baryons*.

7.2.2 Structure of elementary interactions

The Standard Model is the particle theory of three[1] out of four known fundamental interactions and of the elementary particle that take part in these interactions. Mathematically it is a gauge theory of the strong interaction, represented by the Lie group $SU(3)_C$ and the electroweak interaction, represented by $SU(2)_L \times U(1)_Y$. These gauge groups lead in the unifying Standard Model to

[1] of the strong, electromagnetic and weak interactions

the following gauge group: $SU(3)_C \times SU(2)_L \times U(1)_Y$, which describes all three fundamental forces together as one gauge theory. The particles in the Standard model are given as fields, that fall into different representations of the various symmetry groups of the Standard Model.

Interactions are given by exchange particles, so called *gauge bosons*. Each of the interactions in the Standard Model can be described as an exchange of a gauge boson. Table 7.2 lists all the existing and experimentally confirmed gauge bosons together with their masses and charges.

Interaction (coupling to)	Bosons	Mass $[\frac{\text{GeV}}{\text{c}^2}]$	Charge (EM or Color)	(relative) Coupling Constant (1 GeV)
electromagnetic (EM charge)	Photon γ	0	0	$\alpha_{EM} \sim \frac{1}{137}$
weak (weak charge)	W-Boson W^-	80.398	$-e$	$\alpha_W \sim \frac{1}{31.7}$
	W-Boson W^+	80.398	$+e$	
	Z-Boson Z^0	91.1876	0	
strong (color)	Gluon g	0	$\mid r\overline{g}\rangle$, $\mid r\overline{b}\rangle$, $\mid g\overline{r}\rangle$ $\mid g\overline{b}\rangle$, $\mid b\overline{r}\rangle$, $\mid b\overline{g}\rangle$ $\frac{1}{2}(\mid r\overline{r}\rangle - \mid g\overline{g}\rangle)$ $\frac{1}{\sqrt{6}}(\mid r\overline{r}\rangle + \mid g\overline{g}\rangle - 2 \mid b\overline{b}\rangle)$	$\alpha_S = 1$

Table 7.2: Properties of the elementary gauge bosons with spin $1\hbar$

Strong interaction

The strong interaction describes the interaction between particles carrying color charge. Thus the only candidates are the quarks and the gluons. This kind of force is the strongest one of the three in the Standard Model, but it only reaches up to a distance of $\sim 10^{-15}$ m. The strong interaction is in fact responsible for holding together the quarks in particles like the neutron or the proton, and also assures that the nucleus of an atom is stable.

Mathematically poses the strong force a problem, because of the fact that the coupling constant is quite big, one cannot properly treat it in a perturbative approach. Thus we cannot calculate loop feynman diagrams for the strong interaction, because the series expansion in the order of the coupling constant will not converge. Luckily, the coupling constant decreases with increasing energy, and thus, in collider experiments a perturbative approach still provides very good results.

Weak interaction

The weak interaction is much inferior to the strong interaction. It describes interactions between particles with weak charge, which is mostly the weak isospin[2]. The weakness of this force comes from the heaviness of the exchange particles. In every weak reaction, energy is needed to produce that heavy particle, which reduces the reaction probability drastically. This force is responsible for the flavor change of quarks, which makes the well-known beta-decay possible.

Electromagnetic interaction

The electromagnetic interaction describes the interaction between electrically charged particles. It is expressed in the famous $\frac{1}{r}$-law. The exchange particle is the photon.

Like afore mentioned, does the strong interaction coupling constant decrease with increasing energy. Because the weak and the electromagnetic interactions increase with increasing energy, it would be nice, that at some point all the forces would be similarly strong. If this is the case, as in grand unified theories or in the MSSM, it is called *gauge coupling unification.*

7.2.3 The Higgs boson

The Higgs boson is a massive scalar elementary particle that is predicted to exist by the Standard Model. The existance of the Higgs boson is postulated as a means of resolving inconsistencies in the Standard Model of particle physics. It is predicted to be the only elementary bosonic particle in the Standard Model, but it has not been experimentally found the way the theory suggests it to be.

The Higgs boson field, which comes about as a consequence of the famous Higgs mechanism, has a non-zero vacuum expectation value (VEV). The VEV is supposed to be responsible for the masses of particles in the Standard Model. This is because the acquisition of a non-zero VEV spontaneously breaks the electroweak gauge symmetry. The fermion masses are then obtained out of the Yukawa term in the Lagrangian density, which is induced by the Higgs boson field. The masses

[2]The isospin is not actually measurable, being only a mathematical construction

for the gauge bosons on the other hand are obtained from the kinetic term of the Higgs field. This is in fact the simplest method that is capable of giving masses to the gauge bosons, while remaining compatible with gauge theories.

7.3 Theoretical Motivation

7.3.1 Hierarchy problem

First and foremost there is the infamous “Hierarchy problem”, also known as the “weak scale instability problem”. This is mainly a motivation for supersymmetry itself, and not especially for the MSSM. One can derive the whole supersymmetric theory by posing a solution to the hierarchy problem. I am going to paraphrase the problem, and drop any calculations and deeper considerations. For the interested reader I can recomend [2].

In general a hierarchy problem occurs, when fundamental parameters, such as coupling constants and masses of some Lagrangian are vastly different from the parameters measured. In this particular case, the underlying question is, why the weak force is so incredibly stronger than gravity. More technically, it dedicates itself to the question why the electroweak scale is so much smaller than the Planck scale. The hierarchy problem is not really a difficulty with the Standard Model itself, but rather a strange sensitivity of the Higgs potential to new physics in almost any imaginable extension of the Standard Model. Considering the Higgs boson as proposed in the Standard Model, the problem then is that the squared Higgs boson-mass, m_H^2 receives enormous quntum corrections from the virtual effects of every particle that couples directly or indirectly to the Higgs field.

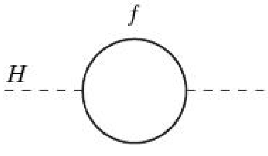

Figure 7.1: One-loop quantum corrections to the Higgs squared mass parameter m_H^2 due to a Dirac fermion

For example if one considers a Dirac fermion f with mass m_f coupling to the Higgs field with a term in the Lagrangian of the form $-\lambda_f H\overline{f}f$, then the Feynman diagram in Figure 7.1 yields a correction of the form:

$$\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{UV}^2 + \ldots, \tag{7.1}$$

where Λ_{UV} is an ultraviolet momentum cutoff. It can be interpreted as the least energy scale at which new physics enters to alter the high-energy behavior of the theory. The problem arises, if Λ_{UV} is of the order of M_P; then this quantum correction to m_H^2 is some 30 orders of magnitude larger than the required value for m_H^2. Massive particles in the Standard Model, obtaining its masses from the VEV of the Higgs boson, are thus directly or indirectly sensitive to the UV cutoff. So the entire mass spectrum of the Standard Model is crucially sensitive to Λ_{UV}. However, picking Λ_{UV} not too large, one still has to introduce some new physics at that scale, that not only alters the propagators in the loop, but actually cuts off the loop integral.

The same problem arises if one considers scalar particles, even if there is no direct coupling between the Standard Model Higgs boson and the particle. So if there is actually a Higgs boson[3] in the form the Standard Model predicts it, one comes automatically about the hierarchy problem in one or the other form, when considering an extension of the SM. One solution to this problem is that some strange effect or symmetry leads to a striking cancellation between the various contributions to the squared Higgs mass correction, Δm_H^2. Such an effect would bring about a somewhat intriguing symmetry between fermionic and bosonic particles, which is exactly what supersymmetry is all about.

7.3.2 Gauge coupling unification

Even though there are many that label it as coincidence, in my eyes gauge coupling unification is one of the strongest motivation for the MSSM. As mentioned in section 7.2, gauge coupling unification is nothing else, but the idea, that at some high energy, most likely shortly after the big bang, all four fundamental forces were of the same strength[4] and thus indistinguishable. However, as in the ordinary Standard Model and the MSSM one neglects gravity, we constrain ourselves to only three of the forces, dropping gravity. Still, unification should be acheived even for the three forces considered in the Standard model. Shockingly though, if we naturally extend the gauge coupling constant in the Standard Model to higher energies, theoretical calculations lead to only approximate unification of the coupling constants. However in the case of the MSSM, one finds

[3]there are models in which one assumes that there exists no fundamental Higgs boson, and thereby avoids this problem

[4]i.e. their coupling constants were of the same order of magnitude

that indeed unification should theoretically occur.

Assume now, that we have already found the Lagrangian of the MSSM with its soft symmetry breaking part[5]. We consider this Lagrangian at some very large energy scale, Q_0. If we used the Lagrangian to compute masses and cross-sections for experiments at ordinary energies near the electroweak scale, the results would involve large logarithms of order $\log(Q_0/m_Z)$ coming from loop diagrams. But these large logarithms can be resummed using renormalization group (RG) equations, by treating the couplings and masses appearing in the Lagrangian as running parameters. Then the 1-loop RG equations for the Standard Model gauge couplings g_i, $i = 1, 2, 3$ are:

$$\beta_{g_a} := \frac{d}{dt} g_a = \frac{b_a g_a^3}{16\pi^2}, \quad \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{cases} (\frac{41}{10}, -\frac{19}{6}, -7)^T & \text{SM} \\ (\frac{33}{5}, 1, -3)^T & \text{MSSM} \end{cases} \tag{7.2}$$

where $t = \log(Q/Q_0)$, Q being the RG scale. The normalization for g_1 is chosen to agree with the canonical covariant derivative for grand unification of the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ into $SU(5)$. Hence in terms of the electroweak gauge couplings g and g' with $e = g \sin\theta_W = g' \cos\theta_W$ one has $g_2 = g$ and $g_1 = \sqrt{5/3}g'$, where θ_W is the *Weinberg angle*, that measures the relationship between the couplings of the electroweak gauge group $SU(2)_L \times U(1)_Y$. Defining the quantities $\alpha_a = g_a^2/4\pi$ gives us parameters whom's reciprocals run linearly with RG scale at one-loop order. It is important to note, that the MSSM coefficients in (7.2) are larger because of the extra MSSM particles in loops.

Figure 7.2 compares the RG evolution of α_a^{-1} for the Standard Model and the MSSM, including 2-loops effects. The theoretical error for the MSSM comes about because of the ignorance on the masses of the supersymmetry particles in the MSSM. Furthermore one can make out a kink in the solid lines, which indicates that supersymmetry particles enter the scenery and change the parameters in the RG equations. So it seems indeed, that the MSSM includes just the right particle content to ensure that the gauge couplings can unify at a scale $M_U \sim 2 \cdot 10^{16}$ GeV. Figure 7.2 shows us indeed the pursued unification of the gauge couplings. However in numbers, the result is even more astounding. If we calculate the Weinberg angle, which has been measured extremely accurately, we get:

[5] c.f. (7.26)

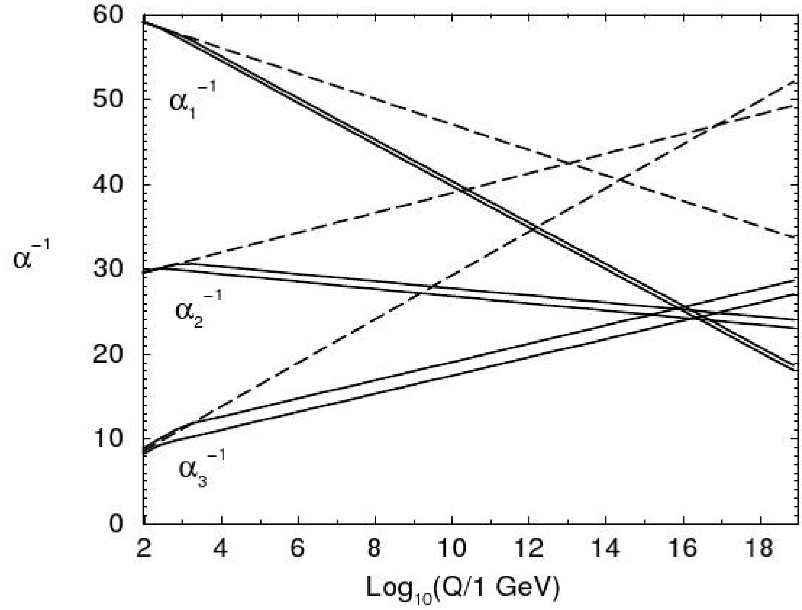

Figure 7.2: Illustration of the gauge coupling unification in the MSSM (solid lines). For comparison there is the inverse gauge couplings for the Standard Model (dashed lines)

$$\sin^2\theta_W \approx \begin{cases} 0.2100 \pm 0.0026 & \text{SM} \\ 0.2335 \pm 0.0017 & \text{MSSM} \\ 0.2316 \pm 0.0002 & \text{experimentally} \end{cases} \tag{7.3}$$

So we see that the measured Weinberg angle is in fact within the margin of error of the one calculated via the MSSM.

While the apparent unification of gauge couplings might be just an accident, it may also be taken as a strong hint in favor of a grand unified theory or superstring models, both of which can naturally accomodate gauge coupling unification below the Planck scale.

7.3.3 Dark matter

Lastly, the MSSM (with R-parity) gives a very good candidate for a dark matter particle. This strange particle must have some crucial properties, to be fancied a candidate for dark matter. Astrophysical discoveries suggest that dark matter is undetectable by emitted or scattered electromagnetic radiation and it cannot be decomposed any further. Its existence throughout the universe is assumed due to strong gravitational effects on stars, galaxies and the structure formation.

In the MSSM, on which we impose R-parity[6], we have a good candidate that satisfies theoretically most of the properties mentioned above: the lightest supersymmetric particle (LSP). It is assumed that the LSP interacts only weakly with ordinary matter and, being the lightest supersymmetric particle, it does not decay any further due to R-parity. Hence it should be stable, and if electrically neutral, would barely interact with matter. Even though being a very interesting approach, there are also many other models, that claim to have found the "dark matter particle". So, the dark matter argument is more of a nice additional property the MSSM has, than a prove of its validity.

7.4 Particle content of the MSSM

7.4.1 Supersymmetry Algebra and supermultiplets from a phenomenological viewpoint

Now let us consider a supersymmetric transformation. That means we have an operator, let us call it Q, that transforms a bosonic state into a fermionic state and vice versa:

$$Q \mid \text{BOSON}\rangle \propto \mid \text{FERMION}\rangle \text{ and } Q \mid \text{FERMION}\rangle \propto \mid \text{BOSON}\rangle. \tag{7.4}$$

Q and $Q^\dagger$ are then called *fermionic operators*. We note that the forms of Q and $Q^\dagger$ are highly restricted by mathematical considerations and one can derive the so called *supersymmetry Algebra* for Q and $Q^\dagger$:

$$\{Q_\alpha, Q^\dagger_{\dot\alpha}\} = -2\sigma^\mu_{\alpha\dot\alpha} P_\mu \tag{7.5}$$

$$\{Q_\alpha, Q_\beta\} = \{Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\} = 0 \tag{7.6}$$

$$[P^\mu, Q_\alpha] = [P^\mu, Q^\dagger_{\dot\alpha}] = 0. \tag{7.7}$$

Single-particle states of the supersymmetric theory, being physical states, fall into irreducible representations of the supersymmetry algebra, which are called *supermultiplets*. Any such object contains fermionic and bosonic states, known as *superpartners*. These superpartners are quite similar and differ in an unbroken supersymmetry only in their spin quantum number by $\pm\frac{1}{2}$.

[6] c.f. section 7.8

From (7.7) we get that $-P^2 = -P^\mu P_\mu$ commutes with Q and $Q^\dagger$, which implies that the superpartners inhabiting the same supermultiplet must have the same eigenvalues of $-P^2$ and thus should have the same masses. Experimentally this has not been observed and therefore we conclude that the symmetry has to be broken. A close analysis shows furthermore that Q and $Q^\dagger$ also commute with the generators of gauge transformations. This implies that the particles in the same supermultiplet must be in the same representation of the gauge group and thus must have the same charge, weak isospin and color degree of freedom. Similarly one can prove that each supermultiplet contains equal fermionic and bosonic degrees of freedom, $n_F = n_B$.

These considerations can be used to show that there are only two kind of supermultiplets in a realistic extension for the Standard Model. Any other renormalizable possibility for a supermultiplet to satisfy $n_F = n_B$ is reducible to a combination of the following two:

First off we treat the so-called *chiral supermultiplet*, being the simplest possibility for a supermultiplet fulfilling $n_F = n_B$ in 4 dimensions. We take a single right- or left-handed Weyl fermion ($\Rightarrow n_F = 2$, for two helicity states) and two real scalars, which we can pack into one complex scalar, as superpartners ($\Rightarrow (n_B)_{TOTAL} = 1 + 1 = 2$). This combination provides for convenient formulations of the supersymmetry algebra.

For the next simplest possibility of a supermultiplet, we consider a spin-1 vector boson, which has to be a massless[7] gauge boson in a renormalizable theory ($\Rightarrow n_B = 2$). As its superpartner we are obliged to take a massless spin-$\frac{1}{2}$ Weyl fermion, so that the theory stays again renormalizable. This supermultiplet is called a *gauge supermultiplet*, because gauge bosons and thus their fermionic partner, so called *gauginos*, transform in the adjoint representation of the gauge group. The adjoint representation being invariant under conjugation implies that the right- and left-handed components of the gaugino must have the same gauge transformation.

These remarks lead us to the conclusion that in a supersymmetric extension of the Standard Model each known fundamental particle[8] is either in a chiral or a gauge supermultiplet and has a supersymmetric partner with a spin differing by $\pm\frac{1}{2}$. We further find that only chiral supermultiplets can contain the Standard Model

[7] at least as long as the gauge symmetry is not broken

[8] being a realistic theory, these particles are in an irreducible representation

fermions, because otherwise their right- and left-handed parts would transform equally under Lorentz transformations, which is obviously not true.

7.4.2 Chiral Supermultiplets

First off let us consider a Standard Model fermion. As afore mentioned, its superpartner will then be a spin-0 particle. The nomenclature is such that one prepends a "-s"to the Standard Model fermion name. This gives us for example the *squark* or the *slepton* as the superpartners of the quark, lepton respectively. But as the left- and right-handed pieces of the quarks and leptons are two seperate Weyl fermions[9], each, the left- as well as the right-handed pieces do have its own complex scalar partner. One denotes this complex spin-0 partner by "$\tilde{x}$", where x refers to the Standard Model particle. For example if we consider the left- and right-handed electron e_L and e_R, their superpartner would be denoted as $\tilde{e_L}$, $\tilde{e_R}$ respectively. It is important though to remember that the "-L"and the "-R"do not refer to some sort of helicity of the selectron, which has none at all, but only to the one of their superpartner.

There is the exception of the neutrino: In the Standard Model, the neutrinos are always left-handed, if one neglects their very small masses. Therefore, in the MSSM there is only one complex supersymmetric particle corresponding to the different neutrinos: These are the $\tilde{\nu_e}$, $\tilde{\nu_\tau}$ and $\tilde{\nu_\mu}$.

The Standard Model Higgs particle requires a special treatment as well. Having spin 0, it is obvious that the Higgs boson must be in a chiral supermultiplet. It turns out that one chiral supermultiplet is not enough, but one needs two seperate Higgs supermultiplets to embed the Standard Model satisfactory into supersymmetry. This grounds on several facts. One reason is that if there were only one Higgs chiral supermultiplet, careful calculations would result in a gauge anomaly for the electroweak gauge symmetry. Furthermore do we need both Higgs supermultiplets to give the appropriate masses to the Standard Model quarks and leptons in a supersymmetric theory. We denote the two Higgs particles as H_u and H_d, to point out that the Higgs vacuum expectation value gives masses to the up-, down-quark respectively. The nomenclature is chosen such that for the spin-$\frac{1}{2}$ superpartner, one appends a "-ino"to the name of the Standard Model particle. Thus we get the *Higgsinos* as the fermionic superpartners of the Higgs scalars. Below we argue why the Higgsinos cannot be a Standard Model particle.

[9] which transform differently under $SU(2)_L$-gauge transformation

Now we have found all of the chiral supermultiplets of a minimal supersymmetric viable extension of the Standard Model. They are summarized in Table 7.3, classified according to their representation under the Standard Model gauge group, $SU(3)_C \times SU(2)_L \times U(1)_Y$:

Names		spin 0	spin $\frac{1}{2}$	$SU(3)_C \times SU(2)_L \times U(1)_Y$
squarks, quarks	Q	$(\tilde{u_L}, \tilde{d_L})$	(u_L, d_L)	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
(×3 families)	$\overline{u}$	$\tilde{u}_R^*$	$u_R^\dagger$	$(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$
	$\overline{d}$	$\tilde{d}_R^*$	$d_R^\dagger$	$(\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
sleptons, leptons	L	$(\tilde{\nu}, \tilde{e_L})$	(ν, e_L)	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
(×3 families)	$\overline{e}$	$\tilde{e}_R^*$	$e_R^\dagger$	$(\mathbf{1}, \mathbf{1}, 1)$
Higgs, Higgsinos	H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$
	H_d	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

Table 7.3: Chiral supermultiplets in the MSSM. The spin-0 fields are complex scalars, and the spin-$\frac{1}{2}$ fields are left-handed two-component Weyl fermions.

In Table 7.3, only the first family representative for the quarks and leptons is shown. Therefore, to obtain all particles, a family index has to be added to the chiral supermultiplet names, and we get: Q_i, $\overline{u_i}$, $\overline{d_i}$, ... (for $i = 1, 2, 3$). The second column in Table 7.3 denotes the symbol for the whole chiral supermultiplets, for example does Q stand for the $SU(2)_L$ supermultiplet containing $\tilde{u_L}$ and u_L, as well as $\tilde{d}_L$ and d_L. Furthermore we used the standard convention that all chiral supermultiplets are defined in terms of left-handed Weyl spinors.

From Table 7.3 we see that the supermultiplet H_d has the same Standard Model gauge quantum numbers as the left-handed sleptons and leptons L_i. However it is not possible to take a neutrino and a Higgs scalar to be superpartners, which would imply that the sneutrino and the Higgs boson were the same particle. This is because careful analysis shows that many problems would result, such as lepton-number non-conservation and a mass for at least one of the neutrinos. Thus, all of the superpartners of the Standard Model particles are indeed new particles and cannot be identified with some other Standard Model state.

7.4.3 Gauge Supermultiplets

The vector bosons of the Standard Model, together with their superpartners, the gauginos, obviously reside in gauge supermultiplets. The nomenclature is still the same as above. So we have as spin-$\frac{1}{2}$ supersymmetry color octet partner of the gluon, the *gluino*. The electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ is associated with the spin-1 gauge bosons $W^{\pm}$, W^0 and B^0. Their superpartners are then the *winos* and the *bino*, denoted by $\widetilde{W^{\pm}}$, $\widetilde{W^0}$ and $\widetilde{B^0}$ respectively. After electroweak symmetry breaking, the W^0 and the B^0 gauge eigenstates mix to give the mass eigenstates Z^0 and γ. The corresponding $\widetilde{W^0}$, $\tilde{B}^0$ mixtures are called *zino*, *photino*, and they are denoted by $\tilde{Z}^0$, $\tilde{\gamma}$ respectively. All the gauge supermultiplets of a minimal supersymmetric extension of the Standard Model are summarized in Table 7.4:

Names	spin $\frac{1}{2}$	spin 1	$SU(3)_C \times SU(2)_L \times U(1)_Y$
gluino, gluon	$\tilde{g}$	g	$(\mathbf{8}, \mathbf{1}, 0)$
winos, W bosons	$\widetilde{W^{\pm}}\ \widetilde{W^0}$	$W^{\pm}\ W^0$	$(\mathbf{1}, \mathbf{3}, 0)$
bino, B boson	$\tilde{B^0}$	B^0	$(\mathbf{1}, \mathbf{1}, 0)$

Table 7.4: Gauge supermultiplets in the MSSM.

Surprisingly, none of the superpartners of the Standard Model particle have been discovered as of now. This leads to the conclusion that the supersymmetry must be broken, because, for example $\tilde{e_L}$ and $\tilde{e_R}$ would have masses equal to the electron mass[10]. But such particles would have been easy to detect. So, clearly supersymmetry is a broken symmetry.

7.5 General supersymmetric Lagrangian

Now we turn our attention to more theoretical considerations. In this section it is our goal to find a Lagrangian for supersymmetry which is as general as possible and which contains the known Standard Model interactions. For the sake of brevity, I mostly neglect derivations. For the interested reader I can recommend [5] or ....

The Lagrangian of a realistic minimal supersymmetric $N = 1$ extension of the Standard Model is constrained by many factors, such as renormalizibiliy and the

[10] $m_e \sim 0.511 MeV$

condition that the particle content must be minimally increased from that of the Standard Model, and many more. Furthermore we set upon the Lagrangian the condition that the action must be invariant under supersymmetry transformations. This leads us to the supersymmetry algebra[11] for the supersymmetry transformations.

7.5.1 The Wess-Zumino model

First off consider the simplest supersymmetric Lagrangian for a free chiral supermultiplet, which consists only of the kinetic terms of the scalar and fermionic fields without interactions. This is called the *Wess-Zumino model*. For this Lagrangian to bring upon an action invariant under supersymmetry on- as well as off-shell[12], we need to introduce an *auxiliary field*, a complex scalar field F, such that $\mathcal{L}_{auxiliary} = F^*F$. The physical reason to introduce such an auxiliary field, which is only a "book-keeping device"to make supersymmetry close off-shell, is to make sure that the number of bosonic degrees of freedom match the number of fermionic degrees of freedom on- and off-shell.

	DOF for φ	DOF for ψ	DOF for F	Total
ON-SHELL	2	2 (Spin)	0	$n_B = n_F = 2$
OFF-SHELL	2	4 ($\mathbb{C}$ 2-component object)	2	$n_B = n_F = 4$

Table 7.5: Real degrees of freedom for the bosonic field, φ, and the fermionic one, ψ.

In this simple supersymmetric model, we can split up the Lagrangian the following way:

$$\mathcal{L}_{free} = \mathcal{L}_{fermionic} + \mathcal{L}_{scalar} + \mathcal{L}_{auxiliary}, \tag{7.8}$$

where:

$$\mathcal{L}_{fermionic} = -\partial^\mu \varphi^* \partial_\mu \varphi \tag{7.9}$$
$$\mathcal{L}_{scalar} = \imath \psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi. \tag{7.10}$$

This Lagrangian produces indeed an invariant action as explicit calculations, done in detail in [5], show.

[11] c.f. equation (7.5) - (7.7)
[12] i.e. such that it does not satisfy classical equations of motions necessarily

7.5.2 Interactions of chiral supermultiplets

Since we want to consider a realistic theory, we obviously need to include interactions between the particles. In this second step we want to construct a comprehensive theory of masses and non-gauge interactions for particles in a chiral supermultiplet. Still maintaining the supersymmetry-invariance of the Lagrangian on- and off-shell, we get high restrictions on the form of non-gauge couplings. We introduce an index i to indicate that we sum over different particles. Thus we get the following free-part of the Lagrangian:

$$\mathcal{L}_{free} = -\partial^\mu \varphi^{*i} \partial_\mu \varphi_i + \imath \psi^{\dagger i} \overline{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i, \tag{7.11}$$

where φ_i is the complex scalar field and ψ_i is the left-handed Weyl fermion in the supermultiplet indexed by i. By power-counting, we can only have field content with total mass dimension less than or equal to four. The invariance under supersymmetry transformation poses further constraint, leading to a non-gauge interaction Lagrangian part of the form:

$$\mathcal{L}_{interaction} = \left(-\frac{1}{2} W^{ij} \psi_i \psi_j + W^i F_i \right) + c.c., \tag{7.12}$$

where W^{ij} and W^i are polynomials in the scalar fields φ_i and φ^{*i} of degrees 1, 2 respectively. Imposing supersymmetry-invariance on $\mathcal{L}_{interaction}\big|_{\psi\text{-spinor}}$, the above Lagrangian constrained to terms containing the ψ-spinor, one finds that $W^{ij} = W^{ji}$ and that W^{ij} must be an analytic function in φ. This leads to the most general form for W^{ij}:

$$W^{ij} = \frac{\delta^2 W}{\delta \varphi_i \delta \varphi_j}, \tag{7.13}$$

where W is called the *superpotential* and can be written as:

$$W = \frac{1}{2} M^{ij} \varphi_i \varphi_j + \frac{1}{6} y^{ijk} \varphi_i \varphi_j \varphi_k. \tag{7.14}$$

Here M^{ij} is the symmetric mass-matrix for the fermionic field and y^{ijk} is the totally symmetric Yukava coupling of the scalar field φ_k and two fermions. The superpotential is not a potential in the ordinary physical sense, but just an analytic function of φ_i of that unfortunate name.

Similarly, we get for W^i, this time the Lagrangian of equation (7.12) is constrained to terms containing spacetime derivatives, $\mathcal{L}_{interaction}\big|_\partial$, the following identity, which supports our choice of name for W^i and W^{ij}:

$$W^i = \frac{\delta W}{\delta \varphi_i}. \tag{7.15}$$

We conclude that the most general non-gauge interaction for chiral supermultiplets is determined by a single function, which is analytic in the complex scalar fields. The auxiliary fields F_i and F^{*i} can be eliminated using the classical equations of motion, $F_i = -W_i^*$ and $F^{*i} = -W^i$. This is a consequence of the Euler-Lagrange equation applied to $(\mathcal{L}_{interaction} + \mathcal{L}_{free})\big|_{F_i}$, which, having no kinetic terms, turns out to be satisfied trivially.

These considerations lead to the general supersymmetric non-broken Lagrangian for a chiral supermultiplet with non-gauge interaction:

$$\begin{aligned}\mathcal{L}_{chiral} = &- \partial^\mu \varphi^{*i} \partial_\mu \varphi_i - V(\varphi, \varphi^*) + \imath \psi^{\dagger i} \overline{\sigma}^\mu \partial_\mu \psi_i - \frac{1}{2} M^{ij} \psi_i \psi_j - \frac{1}{2} M^*_{ij} \psi^{\dagger i} \psi^{\dagger j} \\ &- \frac{1}{2} y^{ijk} \varphi_i \psi_j \psi_k - \frac{1}{2} y^*_{ijk} \varphi^{*i} \psi^{\dagger j} \psi^{\dagger k},\end{aligned} \tag{7.16}$$

where $V(\varphi, \varphi^*) = W^k W_k^* \geq 0$ is the scalar potential for the theory. Evaluating the equations of motions from (7.16), we find that the fermions and bosons satisfy the same wave equation, with exactly the same squared mass matrix. This leads to a collection of chiral supermultiplets, which contain a mass-degenerate complex scalar and Weyl fermion, confirming our assumption in the phenomenological overview over the particle content in chapter 7.4.

7.5.3 Lagrangian for gauge supermultiplets

Consider now gauge supermultiplets, consisting of a massless gauge boson field, A_μ^a and a two-component Weyl fermion gaugino, λ^a, where the index a runs over the adjoint representation of the gauge group[13].

Similarly to the case of the chiral supermultiplet we do not have the same degrees of freedom on- and off-shell[14], and thus have to introduce one real bosonic auxiliary field, called D^a, in order for supersymmetry to be consistent off-shell. This field can again be removed on-shell using its equation of motion.
This leads to a Lagrangian density for a gauge supermultiplet of the form:

$$\mathcal{L}_{gauge} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \imath \lambda^{\dagger a} \overline{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2} D^a D^a, \tag{7.17}$$

[13] i.e. $a = 1, \ldots, 8$ for $SU(3)_C$, $a = 1, 2, 3$ for $SU(2)_L$ and $a = 1$ for $U(1)_Y$

[14] c.f. Table 7.6

	DOF for A_μ	DOF for λ	DOF for D	Total
ON-SHELL	2 (2× fermionic)	2 (2× bosonic)	0	$n_B = n_F = 2$
OFF-SHELL	3 (−1 due to gauge)	4 (2× $\mathbb{C}$ fermionic)	1	$n_B = n_F = 4$

Table 7.6: Real degrees of freedom for gauge supermultiplets.

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$ is the usual Yang-Mills field strength, g the gauge coupling constant for the considered gauge theory, f^{abc} the structure constants and $D_\mu \lambda^a = \partial_\mu \lambda^a + g f^{abc} A^b_\mu \lambda^c$ the covariant derivative of the gaugino field.

7.5.4 Supersymmetric gauge interaction

Now we are prepared to consider a general Lagrangian density for a supersymmetric theory with chiral as well as gauge supermultiplets and their interactions. We assume now that the chiral supermultiplets transform under the gauge group in a representation with hermitian matrices $(T^a)_i^j$ such that: $[F^a, T^b] = \imath f^{abc} T^c$. As afore mentioned, supersymmetry and gauge transformations commute, which implies that the scalar, fermion, and auxiliary fields must be in the same representation of the gauge group. This together with the condition of renormaliziblity lead to the full Lagrangian density for a renomalizable supersymmetric theory, with only one gauge group, of the form:

$$\begin{aligned}\mathcal{L} =& \mathcal{L}_{chiral}(\partial_\mu \longrightarrow D_\mu) + \mathcal{L}_{gauge} \\ &- \sqrt{2} g (\varphi^* T^a \psi) \lambda^a - \sqrt{2} g \lambda^{\dagger a} (\psi^\dagger T^a \varphi) + g (\varphi^* T^a \varphi) D^a. \end{aligned} \tag{7.18}$$

In (7.18), $\mathcal{L}_{chiral}(\partial_\mu \longrightarrow D_\mu)$ denotes the chiral supermultiplet Lagrangian of equation (7.16), where the ordinary derivatives are replaced everywhere by the gauge-covariant derivatives:

$$\begin{array}{lclcl} \partial_\mu \varphi_i & \longrightarrow & D_\mu \varphi_i & = & \partial_\mu \varphi_i - \imath g A^a_\mu (T^a \varphi)_i \\ \partial_\mu \varphi^{*i} & \longrightarrow & D_\mu \varphi^{*i} & = & \partial_\mu \varphi^{*i} + \imath g A^a_\mu (\varphi^* T^a)^i \\ \partial_\mu \psi_i & \longrightarrow & D_\mu \psi_i & = & \partial_\mu \psi_i - \imath g A^a_\mu (T^a \psi)_i. \end{array}$$

Furthermore $\mathcal{L}_{gauge}$ is the Lagrangian density for gauge supermultiplets as given in equation (7.17). The first two terms in the second line in (7.18) constitute a direct coupling of gauginos to matter fields. One can interpret this as the "supersymmetrization"of the usual gauge boson couplings to matter fields. The

Figure 7.3: (a) depicts the coupling of a gaugino to a chiral fermion and a complex scalar, (b) is "supersymmetrized"version of (a). Both vertices have the same coupling constant g.

two vertices are depicted in figure 7.3. The last term in (7.18) combined with the $D^a D^a/2$-term in $\mathcal{L}_{gauge}$ gives the equation of motion: $D^a = -g(\varphi^* T^a \varphi)$. Plugging this into equation (7.18), one finds that the complete scalar potential is:

$$V(\varphi, \varphi^*) = \underbrace{F^{*i} F_i}_{\text{F-term}} + \underbrace{\frac{1}{2} \sum_a D^a D^a}_{\text{D-term}} = W_i^* W^i + \frac{1}{2} \sum_a g_a^2 (\varphi^* T^a \varphi)^2. \tag{7.19}$$

Here we have written the sum $\sum_a$ to cover the case that the gauge group has several different factors with gauge couplings g_a. In the MSSM we have the three couplings g_3, g and g' corresponding to the $SU(3)_C$, $SU(2)$ and the $U(1)$ gauge groups respectively. We note that the F-terms are fixed by Yukawa couplings and fermion mass terms and the D-terms are fixed by the gauge interactions.

Summarizing, we have found that in a renormalizable supersymmetric field theory, the interactions and masses of all particles are determined just by their gauge transformation properties and by the holomorphic superpotential W. Often when treating supersymmetry, one comes across so called *superfields*, which are objects that contain as components all of the bosonic, fermionic and auxiliary fields within the corresponding supermultiplet[15]. Therefore W is often sait to be a function of chiral superfields instead of the bosonic fields. In terms of superfields, one can rewrite equation (7.14) in the following way:

$$W = \frac{1}{2} M^{ij} \Phi_i \Phi_j + \frac{1}{6} y^{ijk} \Phi_i \Phi_j \Phi_k, \tag{7.20}$$

which leads in fact to the same physics. It is notable that in any given theory, only a few of the parameters M^{ij} and y^{ijk} are non-zero. This is because the entries of the mass matrix can only be non-zero for i and j such that Φ_i and Φ_j

[15] i.e. $\Phi_i \supset (\varphi_i, \psi_i, F_i)$

transform under the gauge group in representations that are conjugates of each other. Similarly for y^{ijk}, the entries can only be non-zero if Φ_i, Φ_j and Φ_k transform in representations that can combine to form a singlet. This completes the discussion of an unbroken, realistic supersymmetric theory, and hands us nearly all the parts to the puzzle of the MSSM.

Now we turn our attention to the interactions that are implied by the superpotential. Restricting the general Lagrangian to terms in which the superpotential turns up, and expanding it via equation (7.14), we get:

$$\begin{aligned}\mathcal{L}\big|_W = &-\frac{1}{2}M^{ij}\psi_i\psi_j - \frac{1}{2}M^*_{ij}\psi^{\dagger i}\psi^{\dagger j} - \frac{1}{2}y^{ijk}\varphi_i\psi_j\psi_k - \frac{1}{2}y^*_{ijk}\varphi^{*i}\psi^{\dagger j}\psi^{\dagger k} \\ &- M^*_{ik}M^{kj}\varphi^{*i}\varphi_j - \frac{1}{2}M^{in}y^*_{jkn}\varphi_i\varphi^{*j}\varphi^{*k} - \frac{1}{2}M^*_{in}y^{jkn}\varphi^{*i}\varphi_j\varphi_k \\ &- \frac{1}{4}y^{ijn}y^*_{kln}\varphi_i\varphi_j\varphi^{*k}\varphi^{*l}. \end{aligned} \tag{7.21}$$

The interactions implied by the superpotential (7.21) are shown in figures 7.4 and 7.5. Those in figure 7.4 are all determined by the dimensionless parameters y^{ijk}. It is important, that for each Yukawa coupling of $\varphi_i\psi_j\psi_k$ with strength y^{ijk}, there must be equal couplings of $\varphi_j\psi_i\psi_k$ and $\varphi_k\psi_i\psi_j$, because y^{ijk} is completely symmetric under interchange of any two of its indices. Hereby it is noteworthy that the relationship between the Yukawa interactions in figures 7.4 (a), (b) and the scalar interaction of figure 7.5 (c) is exactly of the special type needed to cancel the quadratic divergences in quantum corrections to scalar masses.

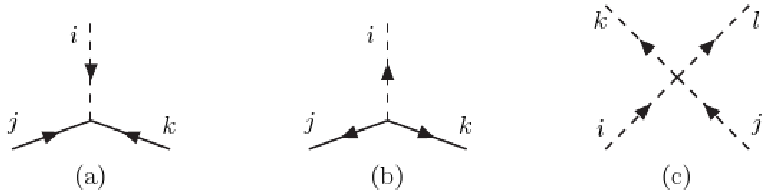

Figure 7.4: Dimensionless non-gauge interaction vertices, determined by couplings y^{ijk}, y_{ijk}, and $y^{ijn}y^*_{kln}$ for (a), (b), (c) respectively.

For completion, I show figure 7.6, that depicts all the gauge interactions in a supersymmetric theory. Thereby it is important to notice, that figure (i) goes with the gauge coupling, g, even though there are no gauge particles present.

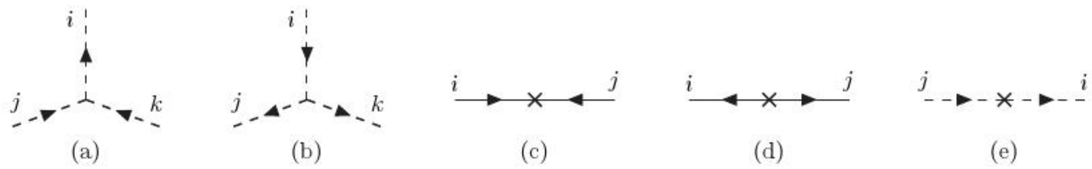

Figure 7.5: Dimensionful couplings determined by the (scalar)^3 interaction vertex $M^*_{in}y^{jkn}$, the conjugate interaction $M^{in}y^*_{jkn}$, the fermion mass term M^{ij}, the conjugate fermion mass term M^*_{ij} and the scalar squared-mass term $M^*_{ij}M^{kj}$ for (a), (b), (c), (d), and (e) respectively.

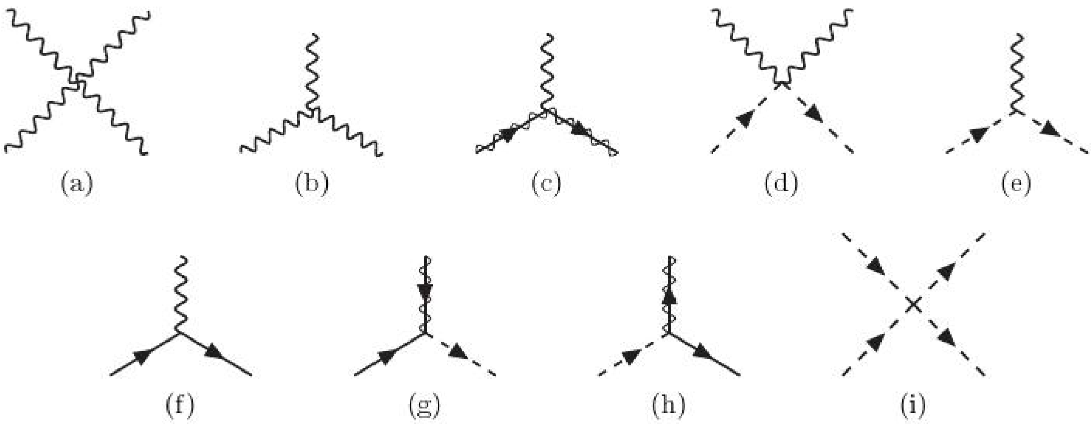

Figure 7.6: Supersymmetric gauge interaction vertices.

7.5.5 Soft supersymmetry breaking interactions

As of now, general supersymmetry is a theory that was mostly constructed to solve the problem of quadratic divergences, which it does successfully. As afore mentioned a realistic phenomenological model must contain supersymmetry breaking. To still pose a valid solution to the hierarchy problem, we can only consider *soft symmetry breaking*[16], because otherwise the relations between the dimensionless couplings that hold in an unbroken supersymmetric theory are not valid anymore. But exactly those relations are essential to cancel the higher order correction terms to the squared Higgs scalar mass. Furthermore, it is expected by theoretical considerations, that supersymmetry should be an exact symmetry, that is broken spontaneously[17]. This leads to the desired property that super-

[16]i.e. of positive mass dimension of the coupling constant

[17]i.e. the underlying model has a Lagrangian that is invariant under supersymmetry, but has a vacuum state that is not.

symmetry is hidden at low energies.

For lack of other constraints, except the gauge invariance, the renormalizability and the fact, that we require soft supersymmetry breaking, for the explicit symmetry breaking part of the Lagrangian, we “parametrize our ignorance”by plainly adding all possible terms that satisfy the three above restrictions:

$$\mathcal{L}_{soft} = -\left(\frac{1}{2}M_a\lambda^a\lambda^a + \frac{1}{6}a^{ijk}\varphi_i\varphi_j\varphi_k + \frac{1}{2}b^{ij}\varphi_i\varphi_j\right) + c.c. - (m^2)^i_j\varphi^{j*}\varphi_i, \quad (7.22)$$

where M_a is the gaugino mass for each gauge group, $(m^2)^i_j$ and b^{ij} are the scalar squared-mass terms, and a^{ijk} are the (scalar)^3 couplings. All other possible terms can be absorbed into the other parts of the Lagrangian by redefining their coupling constant. The terms in $\mathcal{L}_{soft}$ can give masses to all of the scalars and gauginos in a theory, even if the gauge bosons and fermions in chiral supermultiplets are massless.

Now we have all the knowledge to consider the MSSM as a realistic, phenomenological softly broken supersymmetric extension of the Standard Model.

7.6 The superpotential and supersymmetric interactions in the MSSM

7.6.1 The superpotential

The superpotential for the MSSM is:

$$W_{MSSM} = \overline{u}\mathbf{y_u}QH_u - \overline{d}\mathbf{y_d}QH_d - \overline{e}\mathbf{y_e}LH_d + \mu H_uH_d. \quad (7.23)$$

Here H_u, H_d, Q, L, $\overline{u}$, $\overline{d}$ and $\overline{e}$ are the chiral superfields corresponding to the chiral supermultiplets in Table 7.3. Alternatively they can be thought of as the corresponding scalar fields, but then one would have to add tildes, which would lead to confusing notation. Secondly it is notable that $\mathbf{y_u}$, $\mathbf{y_d}$ and $\mathbf{y_e}$, the Yukawa coupling parameters are 3×3 matrices. Furthermore all of the gauge and family indices are suppressed in equation (7.23). The last term in the equation is traditionally called the *μ-term.* It can be written out as $\mu(H_u)_\alpha(H_d)_\beta\epsilon^{\alpha\beta}$, where $\epsilon^{\alpha\beta}$ is used to tie together the $SU(2)_L$ weak isospin indices $\alpha,\beta = 1,2$ in a gauge-invariant way. Similarly, the term $\overline{u}\mathbf{y_u}QH_u$ can be written out as $\overline{u}^{ia}(\mathbf{y_u})^j_iQ_{j\alpha a}(H_u)_\beta\epsilon^{\alpha\beta}$, where $i = 1,2,3$ is a family index, and $a = 1,2,3$ is a color index which is lowered in the **3** and raised in the $\overline{\mathbf{3}}$ representation of the gauge group $SU(3)_C$.

Terms of the form $H^*_\alpha H_\alpha$, $\alpha = u, d$, are forbidden, because the superpotential is analytic in the chiral superfields. Therefore, one can interpret the μ-term as a supersymmetric version of the Higgs boson mass in the Standard Model. Furthermore it is clear now, why there must be two Higgs bosons in a supersymmetric theory. For terms of the form $\overline{\alpha} Q H^*_\alpha$, $\alpha = u, d, e$, are not allowed, again because the superpotential has to be holomorphic in the superfields. Hence we need both, H_u as well as H_d to give Yukawa couplings, and thus masses, to all of the quarks and leptons.

However it is noteworthy, that generally the superpotential could include also other terms than the ones in (7.23), and it would still be analytic and gauge invariant in the chiral superfields. They were not included, because they would violate either baryon or lepton number conservation, which must be highly suppressed from an experimental point of view[18].

7.6.2 Supersymmetric interactions

Now that we know the form of the superpotential, we can start discussing the allowed interactions between the particles in the MSSM. First off it is useful to consider a simple approximation, that helps us finding the most important interactions. Since the top quark, bottom quark and tau lepton are the heaviest fermions in the Standard Model, we can approximate the Yukawa coupling parameters, by only considering the $(3,3)$-family components to be important:

$$\mathbf{y_u} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix} \qquad \mathbf{y_d} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix} \qquad \mathbf{y_e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}. \tag{7.24}$$

In a next step we write out the superpotential of equation (7.23) in terms of this approximation and with seperate $SU(2)_L$ weak isospin components:

$$\begin{aligned} W_{MSSM} \approx\ & y_t(\bar{t}tH_u^0 - \bar{t}tH_u^0 - \bar{t}bH_u^+) - y_b(\bar{b}tH_d^- - \bar{b}bH_d^0) \\ & - y_\tau(\overline{\tau}\nu_\tau H_d^- - \overline{\tau}\tau H_d^0) + \mu(H_u^+ H_d^- - H_u^0 H_d^0), \end{aligned} \tag{7.25}$$

[18] c.f. section 7.8

where: $Q_3 = (t, b)$, $L_3 = (\nu_\tau, \tau)$, $H_u = (H_u^+, H_u^0)$, $H_d = (H_d^0, H_d^-)$, $\overline{u}_3 = \overline{t}$, $\overline{d}_3 = \overline{b}$, $\overline{e}_3 = \overline{\tau}$.

The Yukawa interactions y^{ijk} are completely symmetric in a general supersymmetric theory. Therefore we know that $\mathbf{y_u}$, $\mathbf{y_d}$ and $\mathbf{y_e}$ imply not only Higgs-quark-quark and Higgs-lepton-lepton couplings as in the Standard Model, but also squark-Higgsino-quark and slepton-Higgsino-lepton interactions. Figure 7.7 shows some of the interactions involving the top-quark Yukawa coupling y_t. Note that t_L and $t_R^\dagger$ have been used in place of their synonyms t and $\overline{t}$. For each of the three interactions, there is another with $H_u^0 \to H_u^+$ and $t_L \to -b_L$ corresponding to the second part of the first term in equation (7.25).

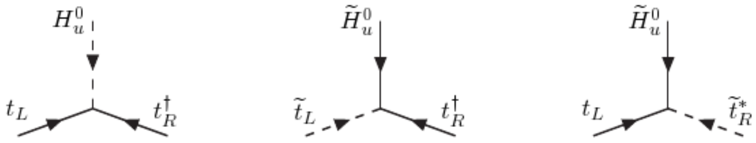

Figure 7.7: The top-quark Yukawa coupling (a) and its "supersymmetrizations"(b) and (c), which are all of strength y_t.

The above approximation of the superpotential, (7.25), shows the most important interactions in the MSSM, determined by W. It is an illustrative example on how remarkably economic supersymmetry is; there are many interactions determined by only a few parameters.

However, the dimensionless interactions determined by the superpotential are usually not the most important ones of direct interest for phenomenology. This is because the Yukawa couplings are known to be very small except the y_t, y_b and y_τ. Instead, processes for the superpartners in the MSSM are dominated by the supersymmetric interactions of gauge coupling strength.

7.7 Soft supersymmetry breaking in the MSSM

7.7.1 Soft supersymmetry breaking Lagrangian

To complete the description of the MSSM, we need to specify the soft supersymmetry breaking terms. For this purpose, we use the recipe found in chapter 7.5, about the general soft symmetry breaking lagrangian, to find:

$$
\begin{aligned}
\mathcal{L}_{soft}^{MSSM} = &-\frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\tilde{W}\tilde{W} + M_1\tilde{B}\tilde{B} + c.c\right) \\
&-\left(\tilde{\bar{u}}\mathbf{a_u}\tilde{Q}H_u - \tilde{\bar{d}}\mathbf{a_d}\tilde{Q}H_d - \tilde{\bar{e}}\mathbf{a_e}\tilde{Q}H_d + c.c.\right) \\
&-\tilde{Q}^\dagger\mathbf{m^2_{\tilde{Q}}}\tilde{Q} - \tilde{L}^\dagger\mathbf{m^2_{\tilde{L}}}\tilde{L} - \tilde{\bar{u}}\mathbf{m^2_{\tilde{\bar{u}}}}\tilde{\bar{u}}^\dagger - \tilde{\bar{d}}\mathbf{m^2_{\tilde{\bar{d}}}}\tilde{\bar{d}}^\dagger - \tilde{\bar{e}}\mathbf{m^2_{\tilde{\bar{e}}}}\tilde{\bar{e}}^\dagger \\
&-m^2_{H_u}H_u^*H_u - m^2_{H_d}H_d^*H_d - (bH_uH_d + c.c.).
\end{aligned}
\tag{7.26}
$$

Here, M_i, $i = 1, 2, 3$, are the gluino, wino and bino mass terms, the adjoint representation gauge indices are suppressed on the wino and gluino fields, and the gauge indices on all of the chiral supermultiplet fields are dropped. The second line in (7.26) contains the (scalar)^3 couplings. Each of $\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$, $\mathbf{m^2_{\tilde{Q}}}$, ..., $\mathbf{m^2_{\tilde{\bar{e}}}}$ is a complex 3×3 matrix in family space. The third line consists of squark and slepton mass terms of the $(m^2)_i^j$ type in equation (7.22). The matrices in the third line, $\mathbf{m^2_{\tilde{Q}}}$, ..., $\mathbf{m^2_{\tilde{\bar{e}}}}$ must be hermitian so that the Lagrangian is real. In the last line there are the supersymmetry-breaking contributions to the Higgs potential.

Looking at the soft symmetry-breaking part of the MSSM-Lagrangian it is obvious that there is a vast amount of new parameters that were not present in the ordinary Standard Model. In fact, if one counts them, there are 105 new parameters for masses, phases and mixing angles in the MSSM-Lagrangian that cannot be rotated away by redefining the phases and flavor basis for the quark and lepton supermultiplets. Hence supersymmetry-breaking seems to introduce an enormous amount of arbitrariness in the Lagrangian.

However not all hope is lost. In fact experimental evidence indicates that some powerful organizing principle must constrain the amount of new parameters introduced by the $\mathcal{L}_{soft}^{MSSM}$-part of the theory. This is because most of the new parameters in (7.26) imply flavor mixing or CP violating processes of the types that are experimentally prohibited. Other experimental discoveries indicate that there are further, even stronger constraints on the form of the soft supersymmetry-breaking part of the Lagrangian. As a consequence there are several models on how to implement these discoveries and how to minimize the amount of unknown parameters. Even though there is a considerable disagreement among theorists, as to what the specific model should actually be, most of these models are indicative of an assumed underlying simplicity or symmetry of the Lagrangian at some very high energy scale. I want to introduce shortly two of the most successful models, the *minimal supergravity*, *mSUGRA* and the *gauge mediated supersymmetry* *breaking, GMSB*:

7.7.2 Minimal supergravity

Mainly there are two assumption that motivate the minimal supergravity mediated supersymmetry breaking model. First off, we take experimental constraints into account, which lead to *soft supersymmetry breaking universality*, the hypothesis, that all mass matrices in (7.26) are approximately proportional to the unit matrix, that triple scalar couplings are proportional to the Yukawa matrices and that the breaking parameters introduce no complex phases. This universality hypothesis is summarized in the following equations:

$$\mathbf{m}_{\tilde{\mathbf{Q}}^2} = m_Q^2\mathbf{1}, \quad \mathbf{m}^{\mathbf{2}}_{\bar{\mathbf{u}}} = m_{\bar{u}}^2\mathbf{1}, \quad \mathbf{m}^{\mathbf{2}}_{\bar{\mathbf{d}}} = m_{\bar{d}}^2\mathbf{1}, \quad \mathbf{m}^{\mathbf{2}}_{\bar{\mathbf{e}}} = m_{\bar{e}}^2\mathbf{1}, \quad \mathbf{m}_{\tilde{\mathbf{L}}^2} = m_L^2\mathbf{1}, \tag{7.27}$$

$$\mathbf{a_u} = A_{u0}\mathbf{y_u}, \quad \mathbf{a_d} = A_{d0}\mathbf{y_d}, \quad \mathbf{a_e} = A_{e0}\mathbf{y_u}, \tag{7.28}$$

and:

$$\arg(M_1) = \arg(M_2) = \arg(M_3) = \arg(A_{u0}) = \arg(A_{d0}) = \arg(A_{e0}) = 0. \tag{7.29}$$

These assumptions lie on the basis of minimal supergravity. MSUGRA assumes that breaking occurs through a coupling to gravity in its simplest form. Because gravity is colour-blind it is justified, that the breaking mass matrices are proportional to the unit matrix. Furthermore it assumes unification at a high energy scale. Therefore, with the RG equations, one can calculate back from the point of unification to get the masses at the electroweak scale. This leaves just four parameters and a sign: m_0, $m_{1/2}$, A_0, $\tan\beta$ and the sign of μ, where:

$$m_{1/2} = M_1 = M_2 = M_3 \tag{7.30}$$

$$m_0^2 = m_Q^2 = m_{\bar{u}}^2 = m_{\bar{d}}^2 = m_{\bar{e}}^2 = m_L^2 = m_{H_1}^2 = m_{H_2}^2 \tag{7.31}$$

$$A_0 = A_{u0} = A_{d0} = A_{e0} \tag{7.32}$$

$$\tan\beta = \frac{\langle H_u^0\rangle}{\langle H_d^0\rangle} \tag{7.33}$$

and μ is the parameter in (7.23).

However, mSUGRA involves also weakly motivated assumptions. We know even in the Standard Model of non-zero parameters, such as the off-diagonal components and the CP-violating phase in the quark mixing matrix. Even though those parameters are small, they are still there and non-zero. So the case for strict soft supersymmetry breaking universality is not very strong. However, it is still approximately well satisfied and should be valid up to a certain point of accuracy. Furthermore is the assumption of gauge unification, without the context of a grand unified theory, mainly aesthetical.

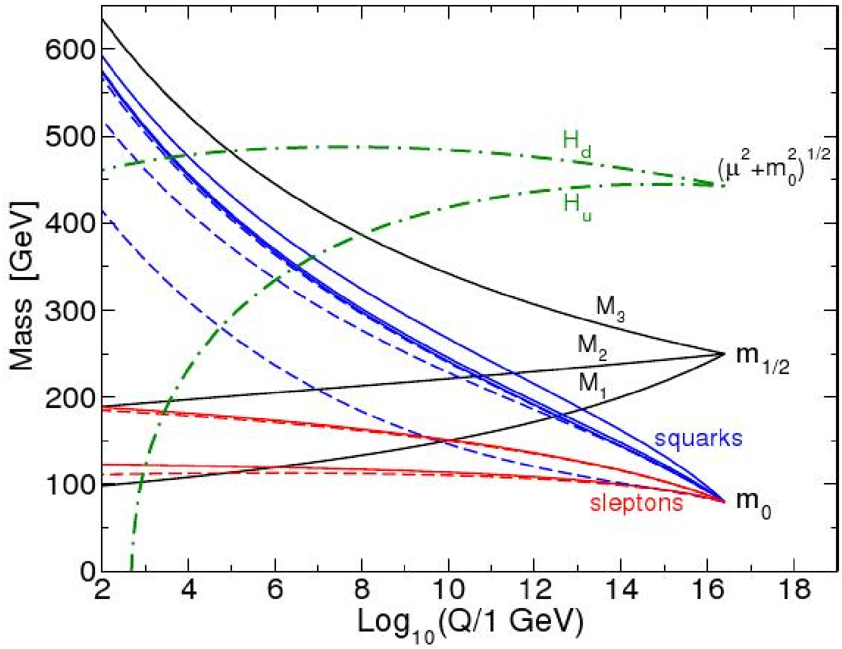

Figure 7.8: RG evolution of scalar and gaugino mass parameters in the MSSM together with the mSUGRA assumptions imposed at $Q_0 = 2.5 \times 10^{16}$ GeV.

Figure 7.8 shows the RG evolution of scalar and gaugino masses in a typical model based on minimal supergravity. The parameters were chosen to be: $m_0 = 80$ GeV, $m_{1/2} = 250$ GeV, $A_0 = -500$ GeV, $\tan\beta = 10$ and $\mu > 0$. It is notable, that $\mu^2 + m_{H_u}^2$ runs negative, providing for electroweak symmetry breaking. At the electroweak scale, the values of the Lagrangian soft parameters can be used to extract the physical masses and cross-sections of the particles, and other observables such as dark matter abundances.

Figure 7.9 shows a qualitative mass spectrum of the MSSM obtained from the mSUGRA assumption. The values of the four and a half unknown parameters is chosen exactly as in figure 7.8. It is notable, that the LSP is indeed the bino-like neutralino[19], $\tilde{N}_1$. We have additionally wino-like $\tilde{N}_2$ and $\tilde{C}_1^{\pm}$ and higgsino-like $\tilde{N}_3$, $\tilde{N}_4$ and $\tilde{C}_2^{\pm}$. However, it is important, that the mass spectrum is very sensible

[19] c.f. section **??**

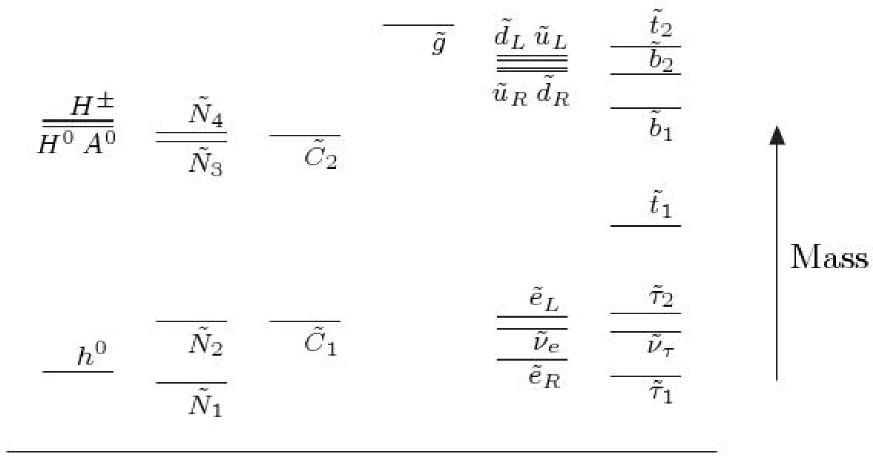

Figure 7.9: Sample of a schematic mass spectra for the undiscovered particle in the MSSM for mSUGRA.

to the input parameters, and thus one has to treat the example featured in figure 7.9 as nothing more than an example that is very unlikely to be true.

To summarize, one should be aware that the most important property of the mSUGRA model is its predictive power. A theory with too many parameters is simply impractical, and cannot make any predictions.

7.7.3 Gauge mediated supersymmetry breaking

In gauge-mediated supersymmetry breaking (GMSB), the ordinary gauge interactions are responsible for the appearance of soft supersymmetry breaking in the MSSM. The basic idea is to introduce some new chiral supermultiplets, called *messengers*, that couple to the ultimate source of supersymmetry breaking, the so-called *hidden sector*, and also couple indirectly to the MSSM through the ordinary gauge bosons and gaugino interactions. Here, the gravitational communication between the MSSM and the source of supersymmetry breaking are treated as relatively unimportant, and are thus negligible. The basic idea of GMSB is sketched in figure 7.10.

Phenomenologically it is important to notice that via the GMSB, the amount of unknown parameters in $\mathcal{L}_{soft}$ can be reduced to 6 parameters, which makes it a very powerful tool. In the GMSB, not the neutralino becomes the LSP, but the gravitino.

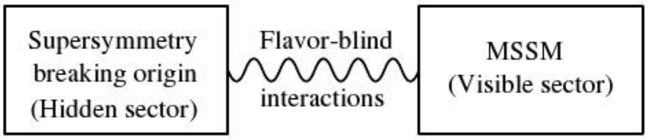

Figure 7.10: Presumed schematic structure for supersymmetry breaking.

To conclude, it is important to note that it is easy to imagine that the essential physics of supersymmetry breaking is not captured by either of the above scenarios. However for the sparticle search, they are extremely important, because they predict somewhat measurable quantities, and one gets phenomenological consequences, that could be found in collider experiments in the near future.

7.8 Matter- and R-Parity

As mentioned before could the superpotential of (7.23) include also other terms, such that it would still be renormalizable, analytic and gauge invariant in the chiral superfields. But they all would violate either the baryon number (B) or the total lepton number (L). The most general gauge-invariant and renormalizable superpotential would also include terms of the form:

$$W_{\Delta L=1} = \frac{1}{2}\lambda^{ijk} L_i L_j \overline{e}_k + \lambda'^{ijk} L_i Q_j \overline{d}_k + \mu'^{i} L_i H_u \tag{7.34}$$

$$W_{\Delta B=1} = \frac{1}{2}\lambda''^{ijk} \overline{u}_i \overline{d}_j \overline{d}_k, \tag{7.35}$$

where family indices $i = 1, 2, 3$ have been restored. Table 7.7 lists the assignments of baryon and lepton number for each particle in the MSSM. It follows therefore, that the terms given in (7.34) violate lepton number by 1 unit and the ones in (7.35) violate baryon number by 1 unit.

	Q_i	$\overline{u}_i$	$\overline{d}_i$	L_i	$\overline{e}_i$	remaining particles
baryon number, $B =$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	0	0
lepton number, $L =$	0	0	0	1	-1	0

Table 7.7: Baryon and lepton number assignments to each particle.

Experimental evidence strongly suggests, that B- and L-violating processes are highly suppressed in nature, therefore it seems kind of obvious to neglect such terms in the superpotential. The strongest experimental indication comes from the non-observation of proton decay, which would violate both B and L-number by 1 unit. If both, λ' and λ'' couplings were non-zero and unsuppressed, then the lifetime of the proton would be extremely short. In contrast, the decay time of the proton into lepton and meson final states is experimentally known to be higher than 10^{32} years. This indicates, that some of the components of λ' and λ'' are extremely small. Many other processes also give strong restrictions on the violation of lepton and baryon numbers.

To justify the superpotential as given in (7.23), one introduces a new symmetry, which has the effect of eliminating the possibility of B- and L-violating terms in the renormalizable superpotential. This symmetry is called *R-parity* or equivalently *matter-parity.*

Matter parity is a multiplicatively conserved quantum number defined as:

$$P_M = (-1)^{3(B-L)}, \tag{7.36}$$

for each particle in the MSSM. The symmetry principle is that a candidate term in the Lagrangian is allowed only if the product of P_M for all of the fields in it is equal to $+1$. If matter-parity is being enforced upon the MSSM-Lagrangian, it is obvious that the terms in (7.34) and (7.35) are not allowed, and the superpotential is indeed of the form given in (7.23). L and B number are violated by non-perturbative electroweak effects. However, matter parity can be implemented as an exact symmetry, because one expects that baryon number and total lepton number violation can occur in tiny amounts, due to non-renormalizable terms in the Lagrangian.

It is useful to state matter parity in terms of R-parity, which is defined as:

$$P_R = (-1)^{3(B-L)+2s}, \tag{7.37}$$

where s is the spin of the particle. Matter- and R-parity are equivalent because the product of $(-1)^{2s}$ for the particles involved in any interaction vertex in a theory that conserves angular momentum is always equal to $+1$. Notably, as opposed to matter-parity, R-parity does not commute with supersymmetry, because particles in a supermultiplet do not have the same R-parity. However, the R-parity assignment is a very useful tool for phenomenological considerations, because all the Standard Model particle and the Higgs bosons do have R-parity $+1$, whereas the particles added through the imposition of supersymmetry have

R-parity -1. These particles with uneven R-parity are then called *sparticles*. So, if R-parity is exactly conserved, then every interaction vertex has to contain an even number of $P_R = -1$ particles. So, in collider experiments sparticles can only be produced in even numbers.

The implementation of R-parity strongly suggests the existence of dark matter as the *lightest supersymmetric particle*[20], or *LSP*. This is because if the LSP is electrically neutral, it interacts only weakly with ordinary matter and thus makes a great candidate for non-baryonic dark matter as required by cosmological discoveries. Furthermore does the MSSM equipped with R-parity suggest that each sparticle other than the LSP must eventually decay into a state that contains an odd number of LSP′s, which would explain the huge amount of dark matter in the universe.

Even though the postulation of parity is phenomenologically well-motivated by proton decay and the hope that the LSP will turn out to be a "dark matter particle", it is noteworthy that the MSSM would not suffer any internal inconsistencies without R-parity. Furthermore one can doubt that parity is exactly conserved, because in the Standard Model every discrete symmetry is inexact[21]. Fortunately one can show that it is sensible to formulate matter parity as a discrete symmetry that is exactly conserved.

7.9 SUSY particles in the MSSM

?? In this section, we shall give a brief introduction to the physics of the various supersymmetry particle states in the MSSM. The discussion is complicated by mixing phenomena. In particluar, after $SU(2)_L \times U(1)_Y$ breaking, mixing will in general occur between any two, or more fields which have the same colour, charge and spin. I will treat especially the so-called *neutralinos* and the *charginos*.

7.9.1 Neutralinos

Consider the sector consisting of the neutral higgsinos $\tilde{H}_u^0$ and $\tilde{H}_d^0$, and the neutral gauginos $\tilde{B}$ and $\widetilde{W^0}$. In absence of electroweak symmetry breaking, the bino and wino fields would have masses given by the soft supersymmetry-breaking mass

[20] the lightest particle with R-parity $P_R = -1$
[21] c.f. C-, P- and T-Symmetry

terms in (7.26). However, bilinear combinations of one of $(\tilde{B}, \widetilde{W^0})$ with one of $(\tilde{H}_u^0, \tilde{H}_d^0)$ are generated, when the neutral scalar Higgs fields acquire a VEV. Such terms will appear as non-zero off-diagonal entries in the 4×4 mass matrix for the four fields, and therefore they will cause mixing. After the mass matrix is diagonalized, the resulting four neutral mass eigenstates are called *neutralinos*, and denoted by $\tilde{N}_i$, $i = 1, 2, 3, 4$. By convention, these are labeled in ascending order, such that:

$$m_{\tilde{N}_1} < m_{\tilde{N}_2} < m_{\tilde{N}_3} < m_{\tilde{N}_4}.$$

The lightest neutralino, $\tilde{N}_1$ is usually assumed to be the LSP, unless there is a lighter gravitino or R-parity is not conserved, because it is the only MSSM particle that can make a good dark matter candidate.

In the gauge-eigenstate basis, $\psi^0 = (\tilde{B}, \widetilde{W^0}, \tilde{H}_u^0, \tilde{H}_d^0)$, the neutralino mass part is given by:

$$\mathcal{L}_{neutralino\ mass} = -\frac{1}{2}(\psi^0)^T \mathbf{M}_{\tilde{\mathbf{N}}} \psi^0 + c.c., \tag{7.38}$$

where we have the following mass matrix:

$$\mathbf{M}_{\tilde{\mathbf{N}}} = \begin{pmatrix} M_1 & 0 & -g' v_d/\sqrt{2} & g' v_u/\sqrt{2} \\ 0 & M_2 & g v_d/\sqrt{2} & -g v_u/\sqrt{2} \\ -g' v_d/\sqrt{2} & g v_d/\sqrt{2} & 0 & -\mu \\ g' v_u/\sqrt{2} & -g v_u/\sqrt{2} & -\mu & 0 \end{pmatrix}. \tag{7.39}$$

M_1 and M_2 come directly from (7.26), the soft supersymmetry breaking part of the Lagrangian, the $-\mu$-entries are the supersymmetric higgsino mass terms as given in (7.25), and the other terms emerge from (7.18), with the Higgs scalars replaced by their VEVs:

$$v_u = \langle H_u^0 \rangle, \qquad v_d = \langle H_d^0 . \rangle$$

The mass matrix can be diagonalized by a unitary matrix $\mathbf{N}$ to obtain the mass eigenstates:

$$\tilde{N}_i = \mathbf{N_{ij}} \psi_j^0 \tag{7.40}$$

such that:

$$\mathbf{N}^*\mathbf{M}_{\tilde{\mathbf{N}}}\mathbf{N}^{-1} = \begin{pmatrix} m_{\tilde{N}_1} & 0 & 0 & 0 \\ 0 & m_{\tilde{N}_2} & 0 & 0 \\ 0 & 0 & m_{\tilde{N}_3} & 0 \\ 0 & 0 & 0 & m_{\tilde{N}_4} \end{pmatrix} \tag{7.41}$$

has positive eigenvalues $m_{\tilde{N}_j}$, $j = 1, 2, 3, 4$. In general are the terms for the masses quite complicated. However, there is the simple case in which we have the following limit:

$$\mu \gg m_Z,\ M_1 \gg m_Z, \text{ and } M_2 \gg m_Z \tag{7.42}$$

This means that the electroweak symmetry breaking effects can be viewed as a small perturbation on the neutralino mass matrix. This leads to a decoupling between the gauginos and the Higgsinos and the neutralino mass eigenstates are a "bino-like" $\tilde{N}_1 \approx \tilde{B}$, a "wino-like" $\tilde{N}_2 \approx \widetilde{W}^0$, and "Higgsino-like" $\tilde{N}_{\frac{3}{4}} \approx \frac{\tilde{H}_u^0 \pm \tilde{H}_d^0}{\sqrt{2}}$.

The above limit, leading to a bino-like neutralino LSP, often emerges from mSUGRA boundary conditions on the soft parameters, which tend to require it in order to get correct electroweak symmetry breaking.

7.9.2 Charginos

The charged analogues of neutralinos are called *charginos*. There are two positively charged ones associated with $(\widetilde{W}^+, \tilde{H}_u^+)$ and two negatively charged ones associated with $(\widetilde{W}^-, \tilde{H}_d^-)$. The mixing between the Higgsinos occurs via the μ term in (7.25), and similarly to the neutralinos, does the mixing between the charged gauginos and higgsinos occur via the last terms in (7.18), after electroweak symmetry breaking. Let us denote the eigenstates corresponding to the charginos by $\tilde{C}_1$ and $\tilde{C}_2$, such that we have again:

$$m_{\tilde{C}_1} < m_{\tilde{C}_2}$$

In the gauge-eigenstate basis $\psi^\pm = (\widetilde{W}^+, \tilde{H}_u^+, \widetilde{W}^-, \tilde{H}_d^-)$, the chargino mass terms in the Lagrangian are:

$$\mathcal{L}_{chargino\ mass} = -\frac{1}{2}(\psi^\pm)^T \mathbf{M}_{\tilde{\mathbf{C}}} \psi^\pm + c.c., \tag{7.43}$$

where:

$$\mathbf{M}_{\tilde{\mathbf{C}}} = \begin{pmatrix} \mathbf{0} & \mathbf{X}^T \\ \mathbf{X} & \mathbf{0} \end{pmatrix}, \text{ and } \qquad \mathbf{X} = \begin{pmatrix} M_2 & gv_u \\ gv_d & \mu \end{pmatrix}. \tag{7.44}$$

Diagonalizing, we get that the mass eigenstates are related to the gauge eigenstates by two unitary 2×2 matrices $\mathbf{U}$ and $\mathbf{V}$ according to:

$$\begin{pmatrix} \tilde{C}_1^+ \\ \tilde{C}_2^+ \end{pmatrix} = \mathbf{V} \begin{pmatrix} \widetilde{W^+} \\ \tilde{H}_u^+ \end{pmatrix}, \qquad \begin{pmatrix} \tilde{C}_1^- \\ \tilde{C}_2^- \end{pmatrix} = \mathbf{U} \begin{pmatrix} \widetilde{W^-} \\ \tilde{H}_d^- \end{pmatrix}, \tag{7.45}$$

such that:

$$\mathbf{U}^* \mathbf{X} \mathbf{V}^{-1} = \begin{pmatrix} m_{\tilde{C}_1} & 0 \\ 0 & m_{\tilde{C}_2} \end{pmatrix}, \tag{7.46}$$

has positive real entries $m_{\tilde{C}_i}$, $i = 1, 2$. Diagonalizing these matrices, we get the doubly degenerate eigenvalues of the matrix $\mathbf{M}_{\tilde{\mathbf{C}}}^\dagger \mathbf{M}_{\tilde{\mathbf{C}}}$, $m_{\tilde{C}_1}^2$ and $m_{\tilde{C}_2}^2$.

Again treating the limit of (7.42), we get that the chargino mass eigenstates consist of a wino-like $\tilde{C}_1^\pm$, and a higgsino-like $\tilde{C}_2^\pm$.

7.9.3 Gluinos

The gluino being a color octet fermion, cannot mix with any other MSSM particle, even if R-parity is violated. Therefore one gets a break from mixing phenomena in the MSSM. From model such as mSUGRA and GMSB follows, that the gluino mass is considerably larger than the ones of the neutralinos and charginos[22].

7.9.4 Squarks and Sleptons

The scalar partners of the SM fermions form the largest collection of new particles in the MSSM. There are altogether 21 new fields[23]: Four squark flavours and chiralities, $\tilde{u}_L$, $\tilde{u}_R$, $\tilde{d}_L$, $\tilde{d}_R$, and three slepton flavours and chiralities, $\tilde{\nu}_{eL}$, $\tilde{e}_L$, $\tilde{e}_R$ in the first family, all repeated for the other two families. These are all complex scalar fields.

In principle any scalars with the same electric charge, R-parity and colour quantum numbers can mix with each other. This would lead to 6×6 mixing problems, which are quite hard to solve analytically. However, there are again phenomenological constraints, that imply that interfamily mixing among the supersymmetry states must be very suppressed only.

[22] c.f. figure 7.9

[23] the neutrinos are treated as massless here

7.10 Experimental signals for supersymmetry

7.10.1 Hadron colliders

The Tevatron as well as the LHC have active experiments, searching for supersymmetric particles. Being hadron colliders, they best search for strongly interacting particles. Therefore most experiments focus on the production of squarks and gluinos. Assuming that R-parity is exactly satisfied, the LSP will be stable, and thus, after the squarks and gluinos decay, each chain will contain one LSP, that, if electrically neutral, should leave the detector unseen. This leads to the prediction that the MSSM will produce a so-called *missing energy* signal, because of the LSP leaving the detector. Within the next few years, the search for supersymmetry will be taken up at the LHC. This should almost certainly result in finding supersymmetry, if the theoretical assumption of supersymmetry are correct.

As an illustration of possible signatures for neutralino and chargino production, we mention the *trilepton signal* arising from the production:

$$p\overline{p}\ (\text{or } pp)\ \longrightarrow \tilde{C}_1^{\pm}\tilde{N}_2 + X \tag{7.47}$$

followed by the decays:

$$\tilde{C}_1^{\pm} \longrightarrow l'^{\pm}\nu\tilde{N}_1 \tag{7.48}$$

$$\tilde{N}_2 \longrightarrow l\bar{l}\tilde{N}_1 \tag{7.49}$$

Here the two LSPs in the final state carry away $2m_{\tilde{N}_1}$ of missing energy, which should be observed as missing transverse energy. In addition there should be three energetic isolated leptons, and little jet activity. The expected Standard Model background should be small as well. Figure 7.11 shows the complete diagram for a clean trilepton event at a hadron collider. However up to now, no events have been registered at the Tevatron. Maybe this will change if we proceed to higher energies at the LHC.

7.10.2 e^+e^- colliders

At e^+e^- colliders, all sparticles (except the gluino) can be produced in tree-level reactions:

$$e^+e^- \longrightarrow \tilde{C}_i^+\tilde{C}_j^-,\ \tilde{N}_i\tilde{N}_j,\ \tilde{l}^+\tilde{l}^-,\ \tilde{\nu}\tilde{\nu}^*,\ \tilde{q}\tilde{q}^* \tag{7.50}$$

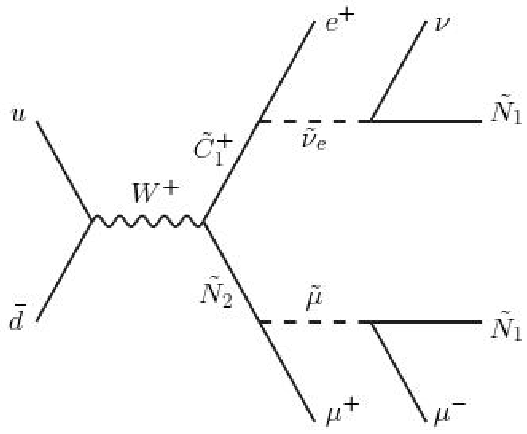

Figure 7.11: Complete Feynman diagram for a clean trilepton event at a hadron collider.

The important interactions for sparticle production are the gaugino-fermion-scalar couplings and the ordinary vector boson interactions. The cross-sections are thus determined just by the electroweak gauge couplings and the sparticle mixings.

7.11 Conclusion

We have seen how to implement supersymmetry into the well-known Standard Model, and how to extend it with the constraints that there should be minimal additional structure and particle content. This gave us the MSSM, a phenomenological viable theory that combines the confirmed Standard Model with the remarkable supersymmetry-theory. We did furthermore discuss some of the consequences of such a model, such as the particle content and their interactions, up to a point, at which a lot of arbitrariness seems to take over. Additionally have we found, that there are some strong indications that the MSSM could be a valid solution to some of the current problems of the Standard Model, such as the hierarchy problem. Further, we have seen that the MSSM leads to a remarkable result in the form of the gauge coupling unification, which makes it, one of the top-candidates for a next step into the direction of a theory of everything. However, by experiments currently held, the parameterspace for the MSSM gets more and more restricted, and it could very well be, that the LHC excludes the MSSM.

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