Solid State Theory
Exercise 8

Exercise 8.1 Van Leeuwen Theorem

Proof Van Leeuwen's theorem that here is no diamagnetism in classical physics.
Hint: With $H(p_1, \dots, p_N; q_1, \dots, q_N)$ the Hamiltonian of the N-particle system without a magnetic field the Hamiltonian with applied magnetic field B is given by $H(p_1 - e/cA_1, \dots, p_N - e/cA_N; q_1, \dots, q_N)$, where $B = \nabla \times A$ and $A_i = A(q_i)$.
Hint 2: The magnetization can be calculated using

$$M = \left\langle -\frac{\partial H}{\partial B} \right\rangle = \frac{1}{\beta} \frac{\partial \log Z}{\partial B}, \tag{1}$$

with Z the partition function of the system in the magnetic field.

Exercise 8.2 Landau Diamagnetism

Calculate the orbital part of the magnetization of the free electron gas in 3D in the limits $T \to 0$, $H \to 0$. In addition, show that the magnetic susceptibility at $T = 0$ and $H = 0$ is given by

$$\chi = \frac{1}{3} \chi_P, \tag{2}$$

where χ_P is the Pauli (spin-)susceptibility.
Hint: Calculate the free energy (Eq. (3.105) in the script) at $T = 0$ to second order in H using the Euler-Maclaurin formula,

$$\sum_0^{n_0} f(n) = \int_{-1/2}^{n_0+1/2} f(n) dn - \frac{1}{24} \left[f'(n_0 + 1/2) - f'(-1/2) \right]. \tag{3}$$

Exercise 8.3 Landau Levels in Graphene

Graphene is two-dimensional graphite; i.e., the C-atoms are arranged on a two-dimensional hexagonal lattice. The latter is not a Bravais-lattice, but a triangular lattice with a diatomic basis. Consequently, the reciprocal lattice, which is hexagonal as well, has two inequivalent points called K- and K'-points (see Fig. 1). The two atoms per unit cell

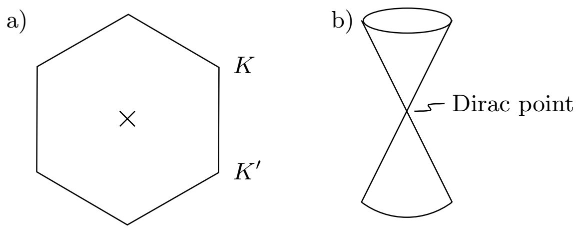

Figure 1: a) First Brillouin zone of graphene with K- and K'-points. b) Band structure of graphene at the K- and K'-points: the Dirac cones.

create a valence- and a conduction band which cross linearly in one point (called the

Dirac point) at the K- and K'-points and form the so-called Dirac cones (see Fig. 1). In undoped graphene, the Fermi energy is exactly at the Dirac point.
To a good approximation, the spectrum in graphene is linear at the Fermi energy and described by the Hamiltonian

$$\mathcal{H} = v_F(p_x\sigma_x\chi_0 + p_y\sigma_y\chi_z). \tag{4}$$

Here, the Pauli matrices σ act on a pseudo-spin and the Pauli matrices χ refer to the inequivalent points K and K'. Subsequently, it is enough to consider only one of the K- or K'-points; i.e., consider only $\mathcal{H} = v_F(p_x\sigma_x + p_y\sigma_y)$.

a) Using the Peierls-substitution $\mathbf{p} \to \mathbf{p} - (e/c)\mathbf{A}$, find the Landau levels in graphene for a magnetization perpendicular to the plane (ignore the Zeeman-term).

   Hint: Take the "square" of the Schrödinger equation.

b) Determine the degeneracy of the Landau levels.

c) Will the magnetization of graphene oscillate when changing the magnetic field? What is the dependence of the ground state energy and the magnetization on a small magnetic field?