Quantum Field Theory II
Exercise sheet 11

Exercise 11.1 The Standard Model Higgs mechanism

As the last exercise of this semester, you will have a brief encounter with the *Higgs mechanism* (or sometimes Brout-Englert-Higgs mechanism or even Higgs-Brout-Englert-Guralnik-Hagen-Kibble mechanism). It was proposed almost 50 years ago, but experiments have failed to find evidence for the validity of the theory so far. The *Large Hadron Collider* at CERN was specifically designed to discover the particle associated with the Higgs mechanism, the *Higgs boson*.

a) First of all, let us see why we need the Higgs mechanism at all. As an example we consider the first lepton generation, i.e. the electron and its neutrino. The gauge group is $SU(2) \times U(1)$. From experiment, we know that there are no *right-handed neutrinos* in nature (neglecting neutrino oscillations). Therefore, right- and left-handed leptons come in different structures. The left-handed electron and neutrino form a doublet w.r.t the gauge group $SU(2)$:

$$\psi_L := \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \mapsto \begin{pmatrix} \nu'_L \\ e'_L \end{pmatrix} = e^{i\theta^a T^a} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

where the $T^a = \sigma^a/2$ are the generators of $SU(2)$ (σ^a are the Pauli matrices). The right-handed electron, on the other hand, forms a singlet under $SU(2)$, i.e. it is invariant:

$$\psi_R := e_R \mapsto e'_R = e_R$$

Under $U(1)$, the two structures transform as

$$\psi_L \mapsto \psi'_L = e^{-i\theta}\psi_L, \quad \psi_R \mapsto \psi'_R = e^{-2i\theta}\psi_R$$

If we now naively tried to give a mass to the electron, we would include the following term in the Lagrangian:

$$\mathcal{L} \ni -m\bar{e}e = -m(\bar{e}_L e_R + h.c.)$$

Why is this term not allowed?

b) To assign mass in a gauge-invariant way, we now introduce a complex *scalar field* H coupling to the fermions as follows:

$$\mathcal{L} \ni y\bar{\psi}_L H \psi_R + h.c.$$

where y is a coupling constant. How does H have to transform under $SU(2)$ and $U(1)$?

c) Now that you've seen that H is a $SU(2)$ doublet, find the gauge transformation i.e. the $SU(2)$ matrix U such that

$$UH = U\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} 0 \\ H_r \end{pmatrix}$$

where H_r is real. This is called the *unitary gauge*.

d) Now comes the crucial part. We add (in addition to the usual kinetic terms) a potential for the scalar field:

$$\mathcal{L} \supset -\mu^2(H^\dagger H) - \frac{\lambda}{4}(H^\dagger H)^2$$

For $\mu^2 > 0$, the potential is minimal for $H_r = 0$ and the vacuum expectation value (VEV) of the Higgs field is $v := \langle H_r \rangle = 0$. Show that $v \neq 0$ for $\mu^2 < 0$.

e) Choosing $\mu^2 < 0$, we can now write $H_r = v + h$ (with $\langle h \rangle = 0$), where h corresponds to the physical Higgs boson everybody is looking for. Expand the term from (b) and show that we've assigned a mass to the electron in a gauge invariant way. What is the mass expressed in terms of v and y? In addition, we obtain an interaction between the Higgs boson and the electron. What is the interaction's coupling strength expressed in m_e (the electron mass) and v?

f)* The Higgs mechanism can do even more! We turn our attention towards the Higgs field's kinetic term we've neglected so far. It is:

$$\mathcal{L} \ni \frac{1}{2}(D_\mu H)^\dagger (D^\mu H), \quad \text{where} \quad D_\mu = \partial_\mu - igW_\mu^a T^a - \frac{ig'}{2} B_\mu$$

where W_μ^a and B_μ are the $SU(2)$ resp. $U(1)$ gauge fields. Write H as in part (e) and expand the kinetic term (explicitly inserting the $SU(2)$ generators). The physical gauge bosons are defined as:

$$\begin{aligned} W_\mu^\pm &:= \frac{1}{\sqrt{2}}(W_\mu^1 \pm iW_\mu^2) \\ Z_\mu &:= \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu \\ A_\mu &:= \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu \end{aligned}$$

where $\tan\theta_W = g'/g$. Find the masses for all of them expressed in terms of v, g and $\cos\theta_W$.