

Exercise 10.1 Renormalization of $\lambda\phi^4$ theory

Once again let's consider the $\lambda\phi^4$ theory:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

Write up to two loop order all the (connected) diagrams contributing to the 2, 3 (if any) and 4-point function and compute the superficial degree of divergence for each of them.

Are the counterterms for the mass and coupling constant (as in exercise 8.1) sufficient to cancel all the divergences at two loop order?

Exercise 10.2 Renormalization of Yukawa theory

Now consider the Yukawa theory, involving a scalar field ϕ and a fermion field ψ :

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \bar{\psi}(i\partial\!\!\!/ - M)\psi - ig\bar{\psi}\gamma^5\psi\phi$$

- a) Write all the diagrams contributing to the 2, 3 and 4-point function up to one loop and compute their superficial degree of divergence.

In particular, show that a divergent 4ϕ term is generated at one loop. This means that the theory can not be renormalized unless one includes a scalar self interaction $\frac{\lambda}{4!}\phi^4$ and a counterterm of the same form (it is then of course possible to adjust the renormalised value of λ to zero, but that is not a natural choice, since the counterterm will still be nonzero).

Are any further interactions required?

- b) Compute the divergent part of each counterterm to the one loop order (warning: not like in the $\lambda\phi^4$ theory, the divergent diagrams may depend on the external momenta already at one loop, so you may want to introduce counterterms for the kinetic terms).