Quantum Field Theory II
Exercise Sheet 1

Exercise 1.1 Structure constants of $SU(N)$

Elements U of $SU(N)$ can be represented by $U(\theta) = e^{ig\theta^a T^a}$ where the T^a's are generators of the group.

a) Show that the generators form a Lie algebra, i.e.

$$\left[T^a, T^b\right] = if^{abc}T^c$$

Hint: Consider two independent group elements $U, U' \in SU(N)$ and compute the product $U'^{-1}U^{-1}U'U$.

b) Show that the structure constants f^{abc} are fully antisymmetric and real.

Exercise 1.2 QCD Lagrangian

The QCD Lagrangian is

$$\mathcal{L} = \bar{\psi}\left(i\not{D} - m\right)\psi - \frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu}$$

where $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$ and $D_\mu = \partial_\mu - igA^a_\mu T^a$.

a) Expand all terms in the QCD Lagrangian using the explicit expressions in terms of the gauge field for the covariant derivative and the gauge field strength. Interpret pictorially the various terms as interactions between particles.

b) Write a Lagrangian invariant under $SU(N)$ gauge transformations for a scalar field. This case appears in supersymmetric theories for the scalar partners of quarks. Sketch the interactions.

Exercise 1.3 Noether current and conserved charge

Find the Noether current and conserved charge due to the invariance under the $U(1)$ gauge transformation of the QED Lagrangian

$$\mathcal{L} = \bar{\psi}\left(i\not{D} - m\right)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu = \partial_\mu - igA_\mu$.