**ETH**
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

**Quantum Field Theory II**
**Exercise sheet 7**

FS 10
Prof. D. Wyler

### Exercise 7.1   Bosonic propagators

In the lecture you have learned how to derive the propagator for massless gauge bosons in $R_\xi$ gauges, which are defined by the gauge condition $G(A) - w(x)$ with $G(A) = \partial_\mu A^{a,\mu}$ and by multiplying the generating functional $Z$ by a factor $C = \int \mathcal{D}w \exp(-i \int d^4x \frac{w(x)^2}{2\xi})$. This is equivalent to adding a term $-\frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2$ to the Lagrangian.

In this exercise you will consider two different cases. In a) you will treat a massive vector boson while in b) you will impose a different gauge condition, called the axial gauge.

a) Find the inverse of the operator:

$$\left[\left(-\partial^2 + M^2\right)g_{\mu\nu} + \left(1 - \frac{1}{\xi}\right)\partial_\mu\partial_\nu\right] \ .$$

This will be the case of a massive gauge-boson such as $W, Z$.

b) Find the gauge boson propagator in an axial gauge

$$G(A) = \eta_\mu A^{\mu a},$$

where $\eta$ is a light-like vector $(\eta^2 = 0)$. Proceed analogously to the derivation of the bosonic propagator as you have seen in the lecture and set $\xi = 1$.

*Hint:* Go to momentum space and choose a suitable Ansatz for the propagator.
The propagator must fulfill

$$\Delta_{F,\mu\nu}^{ab}\left(\Delta_F^{(-1)}\right)^{\nu\lambda,bc} = \delta^{ac}\delta_\mu^\lambda \ .$$

### Exercise 7.2   A three-point function in QCD

Calculate the three-point function $\langle 0|TA_\mu^a(x_1)\bar\psi_i(x_2)\psi_j(x_3)|0\rangle$ to first order in the coupling strength $g$, using the path integral formalism.

*Hints:*

- Start from eq (271) on p. 63

- Use your knowledge from ex. 3.4 and 4.1.

- You may want to think ahead and only calculate those pieces that won't vanish when setting the source terms to zero.

## Exercise 7.3   BRST Jacobian

Show that the path integral is invariant under BRST transformations, i.e. show that the Jacobian of the transformation is unity.

a) Start with a 2 dimensional integral $I = \int dy d\eta \, f(y, \eta)$, where $y$ is a bosonic degree of freedom while $\eta$ is fermionic. Consider a transformation

$$y = x + \lambda a(x, \xi) \qquad \eta = \xi + \lambda b(x, \xi) \ ,$$

where $\lambda$ is Grassmannian, i.e. $\lambda^2 = 0$.

Rewrite $I$ in terms of the new variables $x$ (bosonic) and $\xi$ (fermionic). What is the Jacobian?

*Hints:*

- Write the Jacobian as $J = 1 + \lambda j(x, \xi)$.
- Integrate partially and use the identity

$$\int d\xi \frac{df(\xi)}{d\xi} g(\xi) = \mp \int d\xi \frac{dg(\xi)}{d\xi} f(\xi) \ ,$$

  where the sign is $-$ if $f$ is commuting and $+$ if it is anti-commuting. $\xi$ is anti-commuting.

b)* Now derive the Jacobian for the measure $\mathcal{D}A_\mu^a \mathcal{D}\eta \mathcal{D}\bar{\eta} \mathcal{D}\psi \mathcal{D}\bar{\psi}$ under a BRST transformation and show that it is unity.

*Hints:*

- Write down the transformation matrix for a BRST transformation and use a generalization of a) to derive the Jacobian.

- You might want to solve this exercise using properties of supermatrices (see exercise sheet 5).