## Exercise 6.1 Scalar QCD

Consider scalar QCD, a theory of complex scalar fields interacting with SU(3) gauge bosons. Interactions like these are present in supersymmetric extensions of the Standard Model. The Lagrangian is given by

$$\mathcal{L} = \left(D_{\mu}\phi\right)^{\dagger} \left(D^{\mu}\phi\right) - m^{2}\phi^{\dagger}\phi - \frac{1}{4}G^{\mu\nu,a}G_{\mu\nu,a} ,$$

where  $\phi$  is in a representation R of SU(3) with generators  $T_R^a$  and

$$\begin{array}{lll} G^{\mu\nu,a} &=& \partial^{\mu}A^{\nu,a} - \partial^{\nu}A^{\mu,a} + gf^{abc}A^{\mu,b}A^{\nu,c} \;, \\ D^{\mu} &=& \partial^{\mu} - igT^a_RA^{\mu,a} \;. \end{array}$$

The goal of this exercise is to derive the Feynman rules in the gauge  $\partial^{\mu}A^{a}_{\mu} = 0$ .

a) Add the gauge-fixing term for  $R_{\xi}$  gauge,  $\mathcal{L}_{gf} = -\frac{1}{2\xi} \partial^{\mu} A^{a}_{\mu} \partial^{\nu} A^{a}_{\nu}$ , the ghost Lagrangian,  $\mathcal{L}_{ghost} = -\bar{c}^{a} \partial^{\mu} D^{ab}_{\mu} c^{b}$  and source terms for  $A^{\mu,a}$  and for  $\phi$ ,  $\phi^{\dagger}$  to  $\mathcal{L}$ . Then you can write

$$Z[J_A, J_\phi, J_{\phi^{\dagger}}] \propto \int \mathcal{D}A \mathcal{D}\phi \mathcal{D}\phi^{\dagger} \mathcal{D}\bar{c} \mathcal{D}c \, \exp\left(iS_{\text{free}} + iS_{\text{int}} + iS_{\text{src}}\right)$$

where  $S_{\text{free}}$  only contains non-interacting terms quadratic in the fields and terms with external sources are collected in  $S_{\text{src}}$ . State  $S_{\text{free}}$ ,  $S_{\text{int}}$  and  $S_{\text{src}}$  explicitly. *Remember:*  $D^{ab}_{\mu} = \delta^{ab}\partial_{\mu} - igA^{c}_{\mu}(T^{c}_{A})^{ab}$ ,  $T_{A}$  being in the adjoint representation of SU(3).

- b) Rewrite  $S_{\text{free}}$  and  $S_{\text{int}}$  in momentum space.
- c) Determine the propagators from  $S_{\text{free}}$  using known results from the lecture.
- d) What are the interaction vertices?

## Exercise 6.2 BRST transformation

As seen in the previous exercise, the path integral for a non-abelian gauge theory is quadratic in the gauge-fixing term:

$$Z \sim \int \mathcal{D}A^{a}_{\mu} \mathcal{D}\overline{\psi}^{i} \mathcal{D}\psi^{i} \mathcal{D}\overline{\eta}^{a} \mathcal{D}\eta^{a} e^{i\int d^{4}x \left[\mathcal{L}_{YM} + \mathcal{L}_{fermions} + \mathcal{L}_{ghost} - \frac{1}{2\xi} (\mathcal{G}^{a}(A^{a}_{\mu}))^{2}\right]}$$

It can be made linear in the gauge-fixing term by introducing a new bosonic field  $w^a$ , and one recovers the previous path integral by integrating out the field  $w^a$ .

In this formalism, the exponent in the path integral is not gauge-invariant, but is invariant under the so-called BRST symmetry. Under this symmetry, the fields transform proportionally to an infinitesimal parameter  $\theta$  as

$$egin{array}{rcl} \delta_{ heta} A^a_\mu &=& -rac{ heta}{g} D^{ab}_\mu \eta^b \ \delta_{ heta} \eta^a &=& rac{ heta}{2} f^{abc} \eta^b \eta^c \end{array}$$

Prove that two successive BRST transformations leave the gauge field invariant:

$$\delta_{\theta_1} \delta_{\theta_2} A^a_\mu = 0$$