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Computational Quantum Physics
Exercise 12

FS 10
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Problem 12.1 Field Theories - 4d Ising Model

Find the critical coupling for the ϕ^4 theory in the infinite coupling limit.
The continuum action for the ϕ^4 theory is given by

$$S = \int d^4x \left(\frac{1}{2}(\partial_\mu \phi_0)^2 + \frac{1}{2} m_0 \phi_0^2 + \frac{g_0}{4!} \phi_0^4 \right). \tag{1}$$

On the lattice we replace $\int d^4x$ by $a^4 \sum_x$ and $\partial_\mu \phi$ by $\frac{1}{a}(\phi(x+a) - \phi(x))$. Replacing the bare parameters using the relations

$$a\phi_0 = \sqrt{2\kappa}\phi \tag{2}$$

$$a^2 m_0^2 = \frac{1-2\lambda}{\kappa} - 8 \tag{3}$$

$$g_0 = \frac{6\lambda}{\kappa^2}, \tag{4}$$

we arrive at the lattice action

$$S = \sum_x \left(-2\kappa \sum_{\hat{\mu}} \phi(x)\phi(x+\hat{\mu}) + \phi(x)^2 + \lambda(\phi^2(x) - 1)^2 - \lambda \right), \tag{5}$$

where $\lambda = 0$ corresponds to the free field, $\lambda = \infty$ to the infinite coupling limit.
The action S has an explicit symmetry $\phi \leftrightarrow -\phi$. However, this symmetry can be spontaneously broken at a second order phase transition. At this transition the correlation length ξ/a diverges, or equivalently, for fixed physical length ξ, our lattice spacing a goes to zero and we approach the continuum limit.
The goal is to show triviality of this model, even for infinite λ.

- Show that $\lambda = \infty$ corresponds to the Ising model in four dimensions.
- Extend the program you implemented for the Wolff algorithm in the last exercise to four dimensions.
- Measure e.g. the magnetization squared or the susceptibility to find the critical coupling for the Ising model. You should get a value close to 0.075.
- Implement improved estimators (Section 7.2.3 of the script).
- Measure the correlation functions and compute the renormalized coupling and renormalized mass.
  - measure $\chi_2 = \sum_x \phi(0)\phi(x)$ and $\chi_4 = \sum_{xyz} \phi(0)\phi(x)\phi(y)\phi(z)$.
  - measure $\mu_2 = \sum_x \phi(0)x^2\phi(x)$, where x is the minimum Euclidean distance.
  - compute the renormalized coupling $g_R = \frac{64\chi_4}{\mu_2^2}$ in the symmetric phase where $\langle\phi\rangle = 0$.
  - compute the renormalized mass m_R : $(am_R)^2 = \frac{8\chi_2}{\mu_2}$
  - plot g_R versus am_R and show the triviality of this model.