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Advanced Field Theory
Assignment 8

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Exercise 8.1 Parity conservation in QCD

Consider the QCD Lagrangian with massive fermions

$$\mathcal{L}^{\mathrm{QCD}} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu,a} + \bar{q}_i\left(i\not{D} + m_i\right)q_i + \theta\frac{g^2}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{\mu\nu,a}$$

where all the masses m_i are real (i.e. we have chosen $\theta = \bar{\theta}$).
In a volume V, the Euclidean path-integral formula for the ground-state energy is then

$$e^{-VE} = \int \mathcal{D}A\mathcal{D}\bar{q}\mathcal{D}q \exp\left(-\int \mathrm{d}^4x_E\ \mathcal{L}^{\mathrm{QCD}}_E\right)$$

where the subscript E reminds of the fact that we are in euclidean space.

1. Integrating out the fermions, show that the energy is minimised for $\theta = 0$.
2. Show that this argument holds for any parity-breaking operator, not only the θ-term (and in particular for the axion).

Proceed as follows:

- Show that the integrand in the euclidean path integral is given by

$$\exp\left[-\int \mathrm{d}^4x_E\left(\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} + \bar{q}_i(\not{D} + m_i)q_i + \theta\frac{g^2}{32\pi^2}i\epsilon^{\mu\mu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}\right)\right]$$

- Integrate out the fermions, resulting in the fermion determinant $\det(\not{D} + m)$. Use $\gamma_5\gamma_\mu = -\gamma_\mu\gamma_5$ to show that the nonzero eigenvalues of $i\not{D}$ are paired, from this show that the fermion determinant is positive.
- Determine the minimum of the energy as a function of θ.