Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Advanced Field Theory
Assignment 6

Spring 2010
C. Anastasiou
T. Gehrmann

Exercise 6.1 Semileptonic Tau Decay: $\tau^+ \to \bar{\nu}_\tau \pi^+$

Look at the partial width of the semileptonic decay of the tau: $\tau^+ \to \bar{\nu}_\tau \pi^+$. This process is related to $\pi^+(p) \to \mu^+(k)\nu_\mu(q)$ treated in exercise 4. Start from

$$\sum_{\text{spins}} \left|\mathcal{M}_{\pi^+\to\mu^+\nu_\mu}\right|^2 = 8\, G_F^2 f_\pi^2 \left(2(q\cdot p)(k\cdot p) - p^2(q\cdot k)\right),$$

cross the lepton to the initial state, the pion to the final state. You will arrive at

$$\Gamma = \frac{1}{8\pi} G_F^2 f_\pi^2 m_\ell^3 \left(1 - \frac{m_\pi^2}{m_\ell^2}\right)^2.$$

Exercise 6.2 Semileptonic Tau Decay: $\tau^+ \to \bar{\nu}_\tau \rho^+$

The ρ meson is an isospin triplet of massive spin 1 particles. To calculate $\tau^+ \to \bar{\nu}_\tau \rho^+$, we parametrize the matrix element of the vector current between the vacuum and the rho as

$$\left\langle 0 \,|\, J^{\mu\, a} \,|\, \rho_\lambda^b(p) \right\rangle = g_\rho \epsilon_\lambda^\mu(p) e^{-ipx} \tag{1}$$

where a and b are isospin indices, λ is the polarisation of the ρ and p its momentum. g_ρ is the rho decay constant.
Using equation (1) and the polarisation sum for a massive spin 1 particle

$$\sum_\lambda \epsilon_\lambda^\mu(p)\epsilon_\lambda^{\nu\,*}(p) = -g^{\mu\nu} + \frac{p^\mu p^\nu}{m^2}$$

show that the decay width of this process is

$$\Gamma = \frac{1}{8\pi} G_F^2 g_\rho^2 \frac{m_\tau^3}{m_\rho^2} \left(1 - \frac{m_\rho^2}{m_\tau^2}\right)^2 \left(1 + 2\frac{m_\rho^2}{m_\tau^2}\right).$$