## Exercise 5.1 Goldberger-Treiman relation

(Peskin-Schroeder pages 671 and 672)

The matrix element of the axial isospin current in the nucleon can be written in terms of form factors as

$$\langle N|J^{\mu 5a}(q)|N\rangle = \bar{u}(p') \left[\gamma^{\mu}\gamma^{5}F_{1}^{5}(q^{2}) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}\gamma^{5}F_{2}^{5}(q^{2}) + q^{\mu}\gamma^{5}F_{3}^{5}(q^{2})\right]\tau^{a}u(p)$$

with q = p - p' and  $\tau^a = \sigma^a/2$ .

Working in the limit of vanishing quark masses and assuming conservation of the axial vector current, show that

$$g_A =: F_1^5(0) = \lim_{q^2 \to 0} \frac{q^2}{2m_N} F_3^5(q^2)$$

where  $m_N$  is the mass of the nucleon. The low-energy pion-nucleon interaction is conventionally parametrised by the Lagrangian

$$\mathcal{L}_{\rm int} = 2ig_{\pi NN}\pi^a \bar{N}\gamma^5 \tau^a N.$$

Calculate the diagram in figure 1 using  $\langle 0|J^{\mu 5a}|\pi^b(p)\rangle = -ip^{\mu}f_{\pi}\delta^{ab}e^{-ip\cdot x}$  as in the preceding exercise to show the Goldberger-Treiman relation:

$$g_A = \frac{f_\pi}{m_N} g_{\pi NN}.$$



Figure 1: Scattering of an axial vector current on a nucleon via pion exchange.

## Exercise 5.2 Gell-Mann–Okubo Mass Formula and Weinberg Ratio of Quark Masses

Start from the Lagrangian of chiral perturbation theory at order  $p^2$  as stated in the lecture:

$$\mathcal{L}_{\mathrm{CHPT},p^2} = \frac{v^2}{4} \operatorname{Tr} \left( D_{\mu} U D^{\mu} U^{\dagger} + \chi U^{\dagger} + \chi^{\dagger} U \right),$$

$$U = \exp(i\sqrt{2}\Phi/v), \qquad \Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -2\frac{\eta_8}{\sqrt{6}}. \end{pmatrix}.$$

Calculate the masses of the particles by inserting  $D_{\mu} = \partial_{\mu}$ ,  $\chi = 2B \operatorname{diag}(m_u, m_d, m_s)$  and expanding the Lagrangian up to the second order in  $\Phi$ . Verify the Gell-Mann–Okubo mass formula

$$4m_K^2 - 3m_\eta^2 - m_\pi^2 = 0$$

and the Weinberg ratio of quark masses

$$\frac{2m_K^2 - m_\pi^2}{m_\pi^2} = \frac{2m_s}{m_d + m_u}$$

where  $m_{\pi}^2 = 1/3(m_{\pi^+}^2 + m_{\pi^-}^2 + m_{\pi^0}^2), m_K^2 = 1/4(m_{K^0}^2 + m_{\bar{K}^0}^2 + m_{K^-}^2 + m_{K^+}^2).$