Eidgenössische Technische Hochschule Zürich
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Advanced Field Theory
Assignment 5

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Exercise 5.1 Goldberger-Treiman relation

(Peskin-Schroeder pages 671 and 672)
The matrix element of the axial isospin current in the nucleon can be written in terms of form factors as

$$\langle N|J^{\mu 5a}(q)|N\rangle = \bar{u}(p')\left[\gamma^\mu\gamma^5 F_1^5(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}\gamma^5 F_2^5(q^2) + q^\mu\gamma^5 F_3^5(q^2)\right]\tau^a u(p)$$

with $q = p - p'$ and $\tau^a = \sigma^a/2$.
Working in the limit of vanishing quark masses and assuming conservation of the axial vector current, show that

$$g_A =: F_1^5(0) = \lim_{q^2\to 0}\frac{q^2}{2m_N}F_3^5(q^2)$$

where m_N is the mass of the nucleon. The low-energy pion-nucleon interaction is conventionally parametrised by the Lagrangian

$$\mathcal{L}_{\text{int}} = 2ig_{\pi NN}\pi^a\bar{N}\gamma^5\tau^a N.$$

Calculate the diagram in figure 1 using $\langle 0|J^{\mu 5a}|\pi^b(p)\rangle = -ip^\mu f_\pi\delta^{ab}e^{-ip\cdot x}$ as in the preceding exercise to show the Goldberger-Treiman relation:

$$g_A = \frac{f_\pi}{m_N}g_{\pi NN}.$$

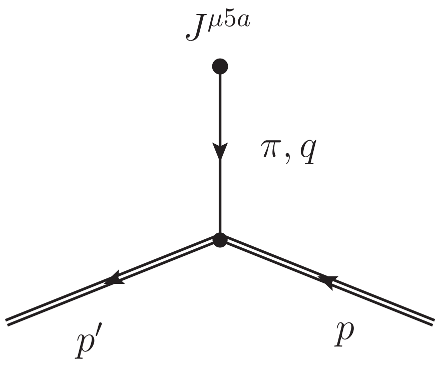

Figure 1: Scattering of an axial vector current on a nucleon via pion exchange.

Exercise 5.2 Gell-Mann–Okubo Mass Formula and Weinberg Ratio of Quark Masses

Start from the Lagrangian of chiral perturbation theory at order p^2 as stated in the lecture:

$$\mathcal{L}_{\text{CHPT},p^2} = \frac{v^2}{4}\,\text{Tr}\left(D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\right),$$

$$U = \exp(i\sqrt{2}\Phi/v), \qquad \Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -2\frac{\eta_8}{\sqrt{6}}. \end{pmatrix}.$$

Calculate the masses of the particles by inserting $D_\mu = \partial_\mu$, $\chi = 2B\,\mathrm{diag}(m_u, m_d, m_s)$ and expanding the Lagrangian up to the second order in Φ. Verify the Gell-Mann–Okubo mass formula

$$4m_K^2 - 3m_\eta^2 - m_\pi^2 = 0$$

and the Weinberg ratio of quark masses

$$\frac{2m_K^2 - m_\pi^2}{m_\pi^2} = \frac{2m_s}{m_d + m_u}$$

where $m_\pi^2 = 1/3(m_{\pi^+}^2 + m_{\pi^-}^2 + m_{\pi^0}^2)$, $m_K^2 = 1/4(m_{K^0}^2 + m_{\bar{K}^0}^2 + m_{K^-}^2 + m_{K^+}^2)$.