Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Advanced Field Theory
Assignment 3

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Exercise 3.1 Pseudoscalar Higgs Coupling to Gluons

In this exercise we are going to study the production of a pseudoscalar in gluon fusion. The diagram in figure 1 translates into

$$\mathcal{M}_1^{\alpha\beta} = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \left(\frac{-i}{\sqrt{2}} y_f \delta_{lj}\right) \left(-i g_s T^a_{ji}\right) \left(-i g_s T^b_{il}\right) \cdot \mathrm{Tr}\left(\frac{(\gamma^5)(i)(\not{p}_1 + \not{k} + m_f)\gamma^\alpha (i)(\not{k} + m_f)\gamma^\beta (i)(\not{k} - \not{p}_2 + m_f)}{\left((p_1+k)^2 - m_f^2\right)\left(k^2 - m_f^2\right)\left((k-p_2)^2 - m_f^2\right)}\right)$$

where m_f denotes the mass of the fermion circulating in the loop, y_f is the Yukawa coupling of the fermion in the loop (which is proportionate to the mass), g_s is the coupling of the strong interaction, T^a, T^b are colour matrices with $\mathrm{Tr}(T^a T^b) = 1/2\ \delta^{ab}$. γ^5 is present only because we are dealing with a pseudoscalar higgs. Evaluate the trace and relate the second diagram with the two gluons exchanged to the first one.
After executing the trace, you will ecounter the integral

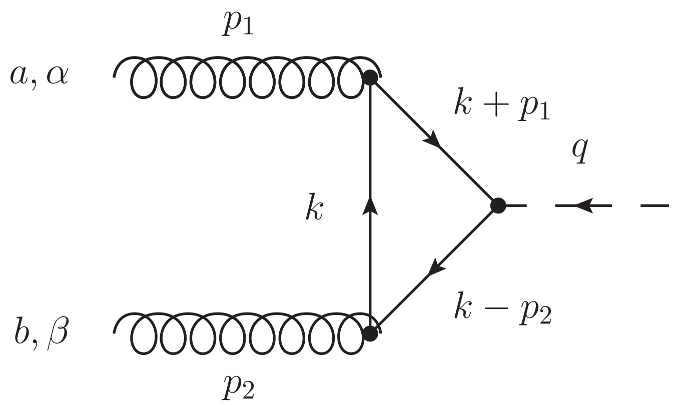

Figure 1: One of the two diagrams for $g\ g \to h$ at leading order.

$$I(p_1, -p_2) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{\left((p_1+k)^2 - m_f^2\right)\left(k^2 - m_f^2\right)\left((k-p_2)^2 - m_f^2\right)} = \frac{i}{(4\pi)^2} \frac{1}{m_h^2} f(\tau)$$

with

$$f(\tau) = \begin{cases} \arcsin^2\left(\frac{1}{\sqrt{\tau}}\right) & \tau \geq 1 \\ \frac{-1}{4}\left(\log\left[\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}\right] - i\pi^2\right)^2 & \tau < 1 \end{cases}$$

$$\tau = \left(\frac{2m_f}{m_h}\right)^2.$$

Now you should average the matrix element norm squared over gluon polarizations:

$$\left\langle |\mathcal{M}|^2 \right\rangle = \frac{1}{N_p^2 N_g^2} \sum_{\text{polarisations}} \left| \epsilon^*_{1,\alpha}(\lambda_1, p_1) \epsilon^*_{2,\beta}(\lambda_2, p_2) \mathcal{M}^{\alpha\beta} \right|$$

where N_p denotes the number of gluon polarisations (2) and N_g denotes the possible values of the colour indices a and b (8). Here you can use

$$\sum_{\text{polarisations}} \epsilon_\nu(k) \epsilon^*_\mu(k) \to -g_{\mu\nu}$$

as in QED.