

Exercise 1.1 Dimensional Regularisation

Prove the following identities:

$$\int \frac{d^d p_E}{(2\pi)^d} \frac{1}{(p_E^2 + \Delta)^n} = \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(n - \frac{d}{2})}{\Gamma(n)} \Delta^{\frac{d}{2} - n}$$

$$\int \frac{d^d p_E}{(2\pi)^d} \frac{p_E^2}{(p_E^2 + \Delta)^n} = \frac{1}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(n - \frac{d}{2} - 1)}{\Gamma(n)} \Delta^{1 + \frac{d}{2} - n}.$$

You will need to make use of the integral representation of the beta function

$$\int_0^1 dx x^a (1-x)^b = B(a+1, b+1) = \frac{\Gamma(a+1)\Gamma(b+1)}{\Gamma(a+b+2)}$$

and of the integral over the surface of the d -sphere

$$\int d^{d-1} \Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}.$$

Exercise 1.2 Feynman Parameters

Prove the formula for the Feynman parametrisation of n propagators by induction:

$$\frac{1}{D_1^{a_1} \dots D_n^{a_n}} = \frac{\Gamma(a_1 + a_2 + \dots + a_n)}{\Gamma(a_1)\Gamma(a_2)\dots\Gamma(a_n)}$$

$$\times \int_0^1 dx_1 \dots \int_0^1 dx_n \frac{\delta(1 - x_1 - x_2 - \dots - x_n) x_1^{a_1-1} x_2^{a_2-1} \dots x_n^{a_n-1}}{[x_1 D_1 + \dots + x_n D_n]^{a_1 + \dots + a_n}}.$$

One way verify the identity for $n = 2$ is by direct integration, for this one can use the hypergeometric integral

$$\int_0^1 dx x^a (1-x)^b (1-zx)^c = \frac{\Gamma(a+1)\Gamma(b+1)}{\Gamma(a+b+2)} {}_2F_1(-c, a+1, a+b+2; z)$$

$$= \frac{\Gamma(a+1)\Gamma(b+1)}{\Gamma(a+b+2)} \sum_{k=0}^{\infty} \frac{\Gamma(-c+k)}{\Gamma(-c)} \frac{\Gamma(a+1+k)}{\Gamma(a+1)} \frac{\Gamma(a+b+2)}{\Gamma(a+b+2+k)} \frac{z^k}{k!}$$

Exercise 1.3 Generalisation of γ_5 in d dimensions

In four dimensions, γ_5 can be defined by its two properties:

- (1) $\text{Tr}(\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_5) = \epsilon_{\mu\nu\rho\sigma} \text{Tr} \mathbf{1}$
- (2) $\{\gamma_\mu, \gamma_5\} = 0$

Show that those two properties cannot be maintained simultaneously in d dimensions, by showing that (2) leads to

$$(d-2)(d-4) \text{Tr}(\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_5) = 0$$