

Leading order gluon fusion in realistic composite Higgs models with $SO(5)$ symmetry

Masters Thesis

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Abstract

This masters thesis starts with a complete calculation of the cross section of leading order gluon fusion. Then we discuss a composite Higgs model with a global $SO(5)$ symmetry broken at energy scale f and two different fermionic contents. We will then apply composite Higgs models to leading order gluon fusion and show that the cross section depends on the fermionic content and the parameter f .

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1 Introduction

With the large hadron collider (LHC), starting operation hopefully this year, we want to complete the experimental test of the standard model (SM), namely finding the Higgs boson. The LHC will be the first experiment that can probe energies well above electroweak symmetry breaking (EWSB) scale (~ 174 GeV) and may thus force the SM to be revisited. A very likely possibility for a revision of EWSB would be that the Higgs boson is not a fundamental scalar, but a composite state of a strongly coupled theory. At lower energy scales it would be the remnant of a broken global symmetry. So far particle physics theories are constrained by electroweak precision test (EWPT), which is in extraordinarily good agreement with the SM. However the compositeness of the Higgs could explain the *hierarchy problem*¹. A Higgs from a broken global symmetry is quite a general framework and it has been argued that the different phenomenologies of the various possible models are not very different [7], [1]. It was also shown in previous articles that these models are best calculable in 5 dimensions [6]. In this article we are therefore going to consider an example with a global $SO(5)$ symmetry broken at scale f to its custodially symmetric subgroup $SO(4)$. We will only look at the low energy description of this theory and mainly focus on the Higgs-top system, because this is expected to cause the best accessible changes compared to SM phenomenology. Another feature of the model which we have to specify is the fermionic content in $SO(5)$. We will look at two different possibilities there. The first one has a minimal fermionic content, where we extend the left handed top-bottom doublet to a five-plet of $SO(5)$ as done in [1]. The second uses a non-minimal fermionic content, where the fermionic composites span the full representation of $SO(5)$, which mix with the top sector after the global $SO(5)$ breaking [8].

The hardest test for the composite model to pass is EWPT. It has been shown that the $SO(5)$ model with minimal fermionic content is not fully compatible with EWPT [1]. With non-minimal fermionic content however, the model is compatible with EWPT and no fine tuning is needed [4].

This masters thesis is organized the following way. In section 2 we will show a full calculation of LO gluon fusion cross section in the SM. Then in section 3 we will discuss the composite Higgs model with minimal fermionic content and focus especially on the implications on LO gluon fusion. In section 4 we will do the same thing for non-minimal fermionic content and in section 5 we will conclude and discuss if these composite models could be tested at the LHC.

¹The hierarchy between the electro weak scale ($\sim 10^2$ GeV) and the scale of new physics, which is believed to be the GUT scale $\Lambda \sim 10^{16}$ GeV is very large. For fermions this is not a problem because the counterterms in the renormalization go only with the logarithm of the scale Λ . For bosons however, this is a problem, because the counterterms go with Λ^2 and naturally this would mean for the Higgs boson, that $m_h \sim \Lambda$. In the SM however $m_h \sim 100$ GeV and this is only possible with fine tuning of the parameters. In the composite Higgs model the Higgs boson consists of elementary fermions, whose mass would not be blown up by counterterms.

2 Gluon fusion cross section

2.1 Introduction

The most relevant way for Higgs production at the LHC is gluon fusion, where two colliding protons radiate of two gluons, which produce a top loop and then the top-antitop pair couples to a Higgs (see figure 1). In the loop we could also have another quark than the top, however the Yukawa coupling between the Higgs and two quarks is proportional to the mass of the quark and the top is by far the heaviest. Thus we will simply neglect other quark loops.

Gluon fusion in the SM has already been calculated to second order (NNLO) [5]. In this Master thesis we are only going to look at leading order (LO) gluon fusion. In section 3 and 4 we are going to modify this calculation and we therefore need to understand how it works in detail. It is remarkable that LO cross section is more than a factor 2 lower than NLO and NNLO cross section and therefore not useful for precise predictions at the LHC. However in our further analysis we only need to get a qualitative feeling how gluon fusion changes with new physics and it would be way out of the scope of this thesis if we would try to look at the effect of composite models on NLO or even NNLO gluon fusion.

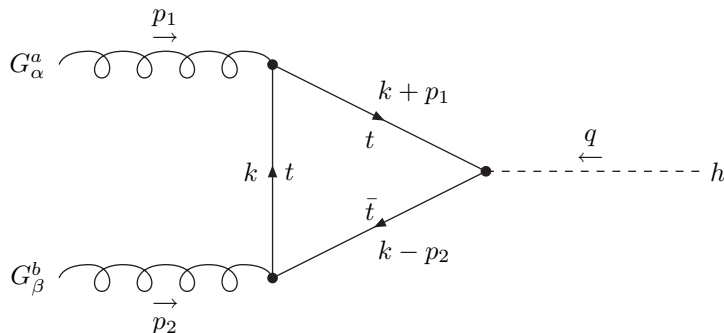


Figure 1: First diagram for LO gluon Fusion.

2.2 Feynman rules

There are two relevant diagrams for gluon fusion (see figures 1 and 2). We will first calculate the contribution of the first diagram and then show that the second contributes the same. Starting with figure 1 we can immediately apply Feynman rules. We will use the notation from [2].

We start at the Yukawa vertex and amputate the gluon propagators. The matrix element uncontracted with polarizations is

$$\mathcal{M}_1^{\alpha\beta} = \int_{-\infty}^{\infty} \frac{d^d k}{(2\pi)^d} \left(-\frac{i}{\sqrt{2}} y_t \delta^{lj} \right) \left(\frac{i(\not{p}_1 + \not{k} + m_t)}{(p_1 + k)^2 - m_t^2} \right) (-ig_s \gamma^\alpha T_{ji}^a) \cdot \quad (1)$$

$$\cdot \left(\frac{i(\not{k} + m_t)}{k^2 - m_t^2} \right) (-ig_s \gamma^\beta T_{il}^b) \left(\frac{i(\not{k} - \not{p}_2 + m_t)}{(k - p_2)^2 - m_t^2} \right), \quad (2)$$

where d is the number of dimensions. We take the trace over closed fermion lines

$$\left(\frac{y_t g_s^2}{\sqrt{2}}\right) T_{ji}^a T_{il}^b \delta^{lj} \cdot \frac{Tr[(\not{p}_1 + \not{k} + m_t)\gamma^\alpha(\not{k} + m_t)\gamma^\beta(\not{k} - \not{p}_2 + m_t)]}{[(p_1 + k)^2 - m_t^2][k^2 - m_t^2][(k - p_2)^2 - m_t^2]}. \quad (3)$$

Now for the generators of $SU(3)$, we use the identity $Tr[T^a T^b] = \frac{1}{2}\delta^{ab}$. The matrix element has then the form

$$\mathcal{M}_1^{\alpha\beta} = \left(\frac{y_t g_s^2}{2\sqrt{2}}\right) \delta^{ab} \int_{-\infty}^{\infty} \frac{d^d k}{(2\pi)^d} \frac{m_t \cdot Tr[\dots]}{[(p_1 + k)^2 - m_t^2][k^2 - m_t^2][(k - p_2)^2 - m_t^2]}. \quad (4)$$

Expanding the trace and keeping in mind that odd products of γ -matrices vanish, we only get odd powers of m_t inside the trace and can take out a factor m_t

$$Tr[\dots] = Tr[\underbrace{\not{p}_1 \gamma^\alpha \not{k} \gamma^\beta + \not{p}_1 \gamma^\alpha \gamma^\beta \not{k}}_{\not{p}_1 \gamma^\alpha \{ \gamma^\beta, k \}}] - Tr[\not{p}_1 \gamma^\alpha \gamma^\beta \not{p}_2] + \quad (5)$$

$$+ Tr[\underbrace{\not{k} \gamma^\alpha \not{k} \gamma^\beta + \gamma^\alpha \not{k} \gamma^\beta \not{k}}_{Tr[2\not{k} \gamma^\alpha \not{k} \gamma^\beta]}] + Tr[\not{k} \gamma^\alpha \gamma^\beta \not{k}] - \quad (6)$$

$$- Tr[\not{k} \gamma^\alpha \gamma^\beta \not{p}_2 + \gamma^\alpha \not{k} \gamma^\beta \not{p}_2] + m_t^2 \cdot Tr[\gamma^\alpha \gamma^\beta]. \quad (7)$$

Now we use the following identities for γ -matrices:

- $\{ \gamma^\alpha, \gamma^\beta \} = 2g^{\alpha\beta}$ (8)

- $Tr[\gamma^\mu \gamma^\nu] = 4g^{\mu\nu}$ (9)

- $Tr[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma] = 4(g^{\mu\nu} g^{\rho\sigma} + g^{\mu\sigma} g^{\nu\rho} - g^{\mu\rho} g^{\nu\sigma})$ (10)

The final result for the trace is

$$Tr[\dots] = 4[2p_1^\alpha k^\beta - 2k^\alpha p_2^\beta + g^{\alpha\beta}(-p_1 p_2 - k^2 + m_t^2) - p_1^\alpha p_2^\beta + p_1^\beta p_2^\alpha + 4k^\alpha k^\beta]. \quad (11)$$

Looking at figure 2 we see that only the incoming gluons are exchanged. This means we have to make the following exchanges in the matrix element

- $\alpha \leftrightarrow \beta$
- $p_1 \leftrightarrow p_2$
- $a \leftrightarrow b$

The contribution from the second diagram is then

$$\mathcal{M}_2^{\alpha\beta} \sim \int_{-\infty}^{\infty} \frac{d^d k}{(2\pi)^d} \frac{4(2p_2^\beta k^\alpha - 2k^\beta p_1^\alpha + g^{\beta\alpha}(-p_2 p_1 - k^2 + m_t^2) - p_2^\beta p_1^\alpha + p_2^\alpha p_1^\beta + 4k^\beta k^\alpha)}{[(p_1 + k)^2 - m_t^2][k^2 - m_t^2][(k - p_2)^2 - m_t^2]}. \quad (12)$$

This integral is the same as the integral for first diagram. If we substitute $k \mapsto -k$, the volume element $d^d k$ does not change and we get exactly the same contribution as for the first

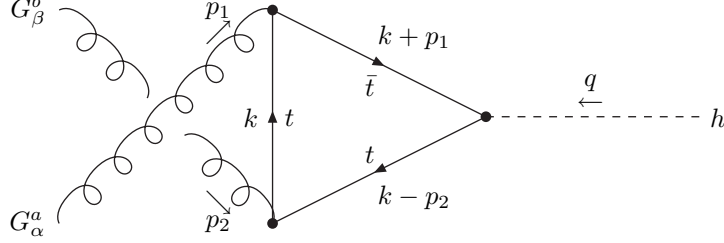


Figure 2: Second diagram for LO gluon Fusion.

diagram. For the whole matrix element we therefore have to take a factor 2. The matrix element, uncontracted with polarizations then reads

$$\mathcal{M}^{\alpha\beta} = \mathcal{M}_1^{\alpha\beta} + \mathcal{M}_2^{\alpha\beta} = \left(4m_t \cdot \frac{y_t g_s^2}{\sqrt{2}}\right) \delta^{ab} \int_{-\infty}^{\infty} \frac{d^d k}{(2\pi)^d} f^{\alpha\beta}(k), \quad (13)$$

where

$$f^{\alpha\beta}(k) = \frac{2p_1^\alpha k^\beta - 2k^\alpha p_2^\beta + g^{\alpha\beta}(-p_1 p_2 - k^2 + m_t^2) - p_1^\alpha p_2^\beta + p_1^\beta p_2^\alpha + 4k^\alpha k^\beta}{[(p_1 + k)^2 - m_t^2][k^2 - m_t^2][(k - p_2)^2 - m_t^2]}. \quad (14)$$

2.3 Tensor reduction

We look at the numerator of $f^{\alpha\beta}(k)$ and we want to get rid of the Dirac indices in k . We will apply *tensor reduction*, this means that we contract the terms in the numerator that depend on k , i.e.

$$2p_1^\alpha k^\beta - 2k^\alpha p_2^\beta + 4k^\alpha k^\beta. \quad (15)$$

The most general ansatz for a numerator without Dirac indices in k is

$$A(k)g^{\alpha\beta} + B(k)p_1^\alpha p_1^\beta + C(k)p_1^\alpha p_2^\beta + D(k)p_2^\alpha p_1^\beta + E(k)p_2^\alpha p_2^\beta, \quad (16)$$

where $A(k), B(k), C(k), D(k), E(k)$ are unknown functions of k . To find these, we need to contract with five different tensors to get five equations. Keep in mind that p_1 and p_2 are the momenta of the gluons and since they are massless they satisfy $p_i^2 = m^2 = 0$.

- Contracting with $g^{\alpha\beta}$ gives

$$2(p_1 \cdot k) - 2(p_2 \cdot k) + 4k^2 = A(k) \cdot d + C(k)(p_1 \cdot p_2) + D(k)(p_1 \cdot p_2). \quad (17)$$

- Contracting with $p_1^\alpha p_1^\beta$ gives

$$-2(p_1 \cdot p_2)(k \cdot p_1) + 4(k \cdot p_1)^2 = E(k)(p_1 \cdot p_2)^2. \quad (18)$$

- Contracting with $p_1^\alpha p_2^\beta$ gives

$$4(k \cdot p_1)(k \cdot p_2) = A(k)(p_1 \cdot p_2) + D(k)(p_1 \cdot p_2)^2. \quad (19)$$

- Contracting with $p_2^\alpha p_1^\beta$ gives

$$2(p_1 \cdot p_2)(k \cdot p_1) - 2(p_1 \cdot p_2)(k \cdot p_2) + 4(k \cdot p_1)(k \cdot p_2) = A(k)(p_1 \cdot p_2) + C(k)(p_1 \cdot p_2)^2. \quad (20)$$

- Contracting with $p_2^\alpha p_2^\beta$ gives

$$2(p_1 \cdot p_2)(k \cdot p_2) + 4(k \cdot p_2)^2 = B(k)(p_1 \cdot p_2)^2. \quad (21)$$

Solving this system of equations we get the five unknown parameters and the numerator of $f^{\alpha\beta}(k)$ can then be written without Dirac indices in k

$$\begin{aligned} & g^{\alpha\beta} \left[\frac{4}{d-2} \left(k^2 - \frac{2(k \cdot p_1)(k \cdot p_2)}{(p_1 \cdot p_2)} \right) + (m_t^2 - k^2 - (p_1 \cdot p_2)) \right] + \\ & + p_1^\alpha p_1^\beta \left[\frac{4(k \cdot p_2)^2}{(p_1 \cdot p_2)^2} + \frac{2(k \cdot p_2)}{(p_1 \cdot p_2)} \right] + \\ & + p_1^\alpha p_2^\beta \left[\frac{1}{d-2} \left(4d \frac{(k \cdot p_1)(k \cdot p_2)}{(p_1 \cdot p_2)^2} + \frac{2k \cdot (p_1 - p_2 - 2k)}{(p_1 \cdot p_2)} \right) - 1 \right] + \\ & + p_2^\alpha p_1^\beta \left[\frac{4d}{d-2} \left(\frac{(k \cdot p_1)(k \cdot p_2)}{(p_1 \cdot p_2)^2} - \frac{k^2}{p_1 \cdot p_2} \right) + 1 \right] + \\ & + p_2^\alpha p_2^\beta \left[\frac{4(k \cdot p_1)^2}{(p_1 \cdot p_2)^2} - \frac{2(k \cdot p_1)}{(p_1 \cdot p_2)} \right]. \end{aligned} \quad (22)$$

2.4 Ward identity

At some point we have to contract our matrix element with the polarization vectors

$$\varepsilon_{1,\alpha} \varepsilon_{2,\beta} \mathcal{M}^{\alpha\beta}. \quad (23)$$

The *Ward identity* says that for massless particles if the polarization ε_μ is longitudinal, which means in the same direction as the momentum k_μ of the particle, the matrix element vanishes, i.e

$$\varepsilon_\mu \mathcal{M}^\mu(k) = k_\mu \mathcal{M}^\mu(k) = 0. \quad (24)$$

In our case we have two massless gluons, which are longitudinally polarized. The Ward identity gives us thus the condition

$$p_{1,\alpha} p_{2,\beta} \left(A(k) g^{\alpha\beta} + B(k) p_1^\alpha p_1^\beta + C(k) p_1^\alpha p_2^\beta + D(k) p_2^\alpha p_1^\beta + E(k) p_2^\alpha p_2^\beta \right) = 0. \quad (25)$$

Note that terms like $\varepsilon_{1,\alpha} p_1^\alpha = (\varepsilon_1 \cdot p_1)$ vanish. We get

$$p_{1,\alpha} p_{2,\beta} \left(A(k) g^{\alpha\beta} + D(k) p_2^\alpha p_1^\beta \right) = 0 \quad \Rightarrow \quad A(k) = -D(k)(p_1 \cdot p_2). \quad (26)$$

We can now combine the above with Eq. (22) and get

$$\mathcal{M}^{\alpha\beta} = \left(4m_t \cdot \frac{y_t g_s^2}{\sqrt{2}} \delta^{ab} \right) A(m_t) \left(g^{\alpha\beta} - \frac{p_2^\alpha p_1^\beta}{(p_1 \cdot p_2)} \right), \quad (27)$$

where

$$A(m_t) \equiv \int_{-\infty}^{\infty} \frac{d^d k}{(2\pi)^d} \frac{A(k)}{[(p_1 + k)^2 - m_t^2][k^2 - m_t^2][(k - p_2)^2 - m_t^2]}. \quad (28)$$

Solving the integral in $A(m_t)$ is quite a lengthy and technical calculation. It can be found in appendix A. The result we are going to use for our further calculation is

$$A(m_t) = \frac{i}{(4\pi)^2} [1 + (1 + \tau)f(\tau)], \quad (29)$$

where

$$f(\tau) = \begin{cases} \arcsin^2 \frac{1}{\sqrt{\tau}}, & \tau \geq 1, \\ -\frac{1}{4} \left(\log \left[\frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}} \right] - i\pi \right)^2, & \tau < 1, \end{cases} \quad (30)$$

$$\tau = 4 \left(\frac{m_t}{m_h} \right)^2. \quad (31)$$

2.5 Average over polarizations

To get the matrix element we need to average over ingoing polarizations and gluons. This means we have to contract Eq. (27) with the polarization vectors of the gluons, sum over all polarizations and divide by the number of polarizations N_p and gluons N_g

$$|M|^2 = \sum_{pol} \left| \varepsilon_{1,\alpha}^*(\lambda_1, p_1) \varepsilon_{2,\beta}(\lambda_2, p_2) \mathcal{M}^{\alpha\beta} \frac{1}{N_p N_g} \right|^2, \quad (32)$$

where \sum_{pol} means that we take $\lambda_1 = 1, 2$ and $\lambda_2 = 1, 2$. This is a rather lengthy calculation, but a nice exercise. However the result looks very simple

$$\sum_{pol} |\varepsilon_{1,\alpha}^*(\lambda_1, p_1) \varepsilon_{2,\beta}(\lambda_2, p_2) \mathcal{M}^{\alpha\beta}|^2 = (d-2) \left(4m_t \cdot \frac{y_t g_s^2}{\sqrt{2}} \delta^{ab} \right)^2 |A(m_t)|^2. \quad (33)$$

The amplitude is then

$$|M|^2 = \frac{\delta^{aa}(d-2)}{(N_g N_p)^2} \left(\frac{4y_t g_s^2 m_t}{\sqrt{2}} \right)^2 \cdot A(m_t) \cdot A^*(m_t), \quad (34)$$

where $y_t = \frac{m_t}{v}$ is the Yukawa coupling, with $v = 174$ GeV. We now insert the following constants $d = 4$, $N_g = 8$, $N_p = 2$ and substitute $g_s^2 = 4\pi\alpha_s(\mu)$. Moreover $\delta^{aa} = Tr(\mathbb{I}_{8 \times 8}) = 8$, because we have 8 different gluons. Putting it all together we have

$$|M|^2 = \frac{1}{2} \frac{m_t^4}{v^2} (4\pi)^2 \alpha_s^2(\mu) A(m_t) \cdot A^*(m_t). \quad (35)$$

Using the results from the integration Eq. (29) we get the final and quite simple result for the matrix element squared

$$|M|^2 = \frac{1}{2} \left(\frac{1}{4\pi} \right)^2 \frac{m_t^4}{v^2} \alpha_s^2(\mu) |1 + (1 - \tau)f(\tau)|^2 \quad (36)$$

2.6 Cross section

The infinitesimal cross section for two particles with mass and momentum m_1, p_1 and m_2, p_2 and one outgoing particle with m_h, q can be easily expressed when knowing the matrix element [9]

$$d\hat{\sigma}_{gg\rightarrow h} = \frac{1}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - q) |M|^2 \frac{d^3 q}{(2\pi)^3} \frac{1}{2q_0} \quad (37)$$

We integrate this to get the gluon gluon to Higgs cross section

$$\hat{\sigma}_{gg\rightarrow h} = \int d\hat{\sigma}_{gg\rightarrow h} = \frac{\pi}{m_h^2} |M|^2 \delta(2p_1 \cdot p_2 - m_h^2). \quad (38)$$

This is the standart textbook result for LO gluon fusion. It can for example be found in [10].

But at the LHC we have two colliding protons. The gluons come from the protons and we want to know how they act. The momentum distribution of partons (in our case gluons) and hadrons (protons) is described by the *parton distribution functions* (Pdf). Theoretically this has not yet been properly understood because of colour confinement at low momentum. However for high hadron momenta there are good sets of experimental data which we will use. To get the proton proton to Higgs cross section we have to integrate the cross section convoluted with the Pdfs for the two protons.

$$\sigma_{PP\rightarrow h} = \frac{\pi}{m_h^2} |M|^2 \int_0^1 Pdf(x_1) Pdf(x_2) \delta(2p_1 \cdot p_2 - m_h^2) dx_1 dx_2 \quad (39)$$

Here $x_{1,2}$ is the *Bjoerken scale factor* which tells us which fraction of the proton momentum is carried by the gluons. The relation is $p_i = P_i x_i$, where p_i is the gluon momentum and P_i is the momentum of the proton.

We want to evaluate the δ -function in the above equation. First we substitute

$$u \equiv 2p_1 \cdot p_2 - m_h^2 = 2P_1 x_1 \cdot P_2 x_2 - m_h^2 \quad (40)$$

$$\Rightarrow \frac{du}{dx_2} = 2x_1 P_1 \cdot P_2 = S x_1, \quad (41)$$

where $S = (P_1 + P_2)^2 = 2P_1 \cdot P_2$ is the center of mass energy

$$\sigma_{PP\rightarrow h} = \frac{\pi}{m_h^2} |M|^2 \int_0^1 \int_{-m_h}^{Sx_1 - m_h} Pdf(x_1) Pdf\left(\frac{u + m_h^2}{Sx_1}\right) \delta(u) dx_1 du \frac{1}{Sx_1}. \quad (42)$$

When evaluating the δ -function, we get a condition for the lower boundary of the x_1 integration. This lower boundary reflects the fact that the Higgs boson can only be produced when the gluons carry enough energy. It is also necessary because at very low Bjoerken scale the Pdfs are not well defined anymore. The final result before integrating over the Pdfs is

$$\sigma_{PP\rightarrow h} = \frac{\pi}{m_h^2} |M|^2 \int_{\frac{m_h^2}{S}}^1 \frac{dx_1}{Sx_1} Pdf(x_1) Pdf\left(\frac{m_h^2}{Sx_1}\right). \quad (43)$$

2.7 Integration and results

To get the LO proton proton to Higgs cross section we only have to integrate over the Pdfs and plug in the numbers. The integration was done on a computer using Vegas and the

data was taken from Cteq[12] and MRST[13]. Because we have only calculated LO gluon fusion, we only take the leading order data, which is not very much up to date. The numbers are taken to be $m_t = 172.4$ GeV, $v = 174$ GeV and $S = 14000^2$ GeV² which is about the maximum center of mass energy expected to be reached at the LHC. For high energies scales μ we need to consider the “running” of $\alpha_s(\mu)$. Analogous to [9] we take

$$\alpha_s(\mu) = \frac{\alpha(\mu_0)}{1 - \frac{b_s}{2\pi} \log\left(\frac{\mu_0}{\mu}\right)}, \quad (44)$$

where $\alpha(\mu_0)$ has been measured at a particular energy scale μ_0 and $b_s = 11 - \frac{2}{3}N_f$. The factor N_f is the number of flavours, which we take to be 5 according to heavy top approximation, which is usually performed when calculating gluon fusion. This means $b_s = 23/3$. The energy scale $\mu_0 = M_Z = 91.187$ GeV is the same for MRST and Cteq, but they use different values for $\alpha(M_Z)$. So for Cteq we take $\alpha(M_Z) = 0.1176$ and for MRST we take $\alpha(M_Z) = 0.130$.

In the plot below we have calculated the cross section for various Higgs masses from 100 GeV to 500 GeV. One can see that the cross section with MRST data is a little higher than for Cteq data. This is probably due to the higher $\alpha_s(\mu_0)$. The cross section decreases with higher Higgs mass until $m_h > 2m_t$, i.e. $\tau > 1$, where we see a little bump.

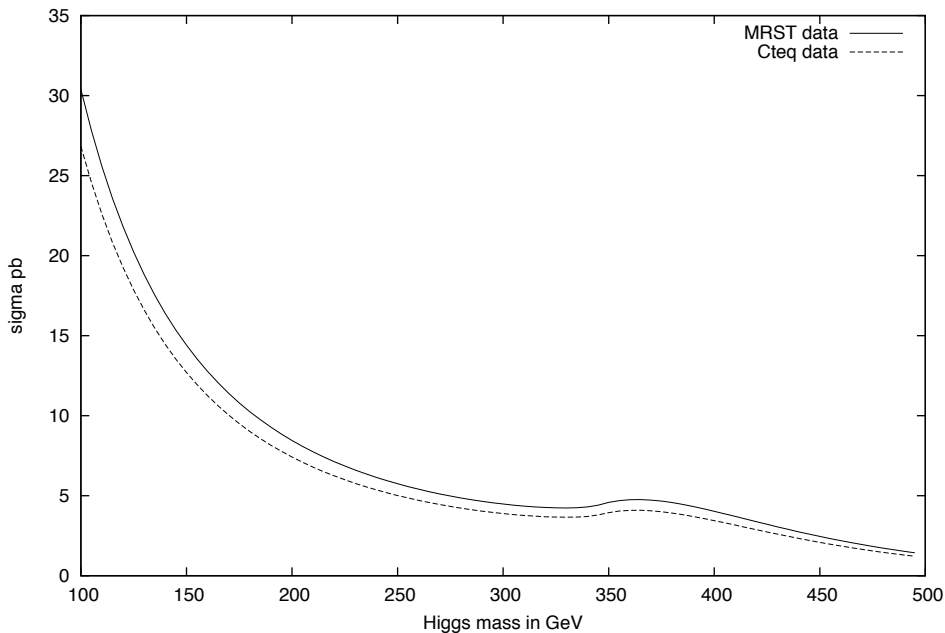


Figure 3: LO gluon fusion cross section with data from MRST (solid) and Cteq (dashed).

3 Composite Higgs model with minimal fermionic content

3.1 General implications for composite Higgs model with global $SO(5)$ symmetry

We will first look at some general features of an $SO(5)$ composite Higgs model, which are also valid for the model we are going to discuss in section 4. The low energy description is a sigma model with global $SO(5)$ symmetry broken spontaneously to $SO(4)$ at energy scale f . We introduce a scalar five-plet constraint by

$$\phi^2 = \left(\begin{array}{c} \vec{\phi} \\ \phi_5 \end{array} \right)^2 = f^2. \quad (45)$$

The energy scale f should be above the EWSB scale ($v = 174$ GeV). For the rest of our discussion we will set $f = 500$ GeV. The first four components of ϕ we call $\vec{\phi}$. The custodially symmetric subgroup $SO(4) \equiv SU(2)_L \times SU(2)_R \subset SO(5)$ acts on $\vec{\phi}$ and we can gauge this to the SM gauge group $SU(2)_L \times U(1)$. This is very convenient, because we can easily get the SM gauge bosons W^\pm , Z and γ .

3.2 The kinetic Lagrangian

The kinetic Lagrangian of the field ϕ is basically the SM kinetic Lagrangian generalized to 5 dimensions

$$\mathcal{L}_{kin} = \frac{1}{2}(D_\mu\phi)^\dagger(D_\mu\phi), \quad D_\mu\phi = \partial\phi - i(g_2W_\mu^aT_L^a - g_1B_\mu T_R^3)\phi. \quad (46)$$

The generators $T_{L,R}$ in 5 dimensions are

$$T_{L,R}^a = \begin{pmatrix} \frac{1}{2}\sigma^a & 0 & 0 \\ 0 & \frac{1}{2}\sigma^a & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (47)$$

where σ^a are the Pauli Matrices. We set $v^2 = \frac{1}{2}\langle\vec{\phi}\rangle^2$, which is totally equivalent to EWSB in the SM. With Eq. (45) we find $\phi^2 = f^2 = \vec{\phi}^2 + \phi_5^2$. After EWSB we insert the vacuum expectation value (VEV) for ϕ with a little fluctuation η

$$\phi = \begin{pmatrix} v + \eta \\ 0 \\ 0 \\ v + \eta \\ \sqrt{f^2 - 2(v + \eta)^2} \end{pmatrix}. \quad (48)$$

With the SM rotations and redefinitions of the fields (see figure 4), the kinetic Lagrangian can be written as

$$\mathcal{L}_{kin} = \frac{1}{2} \left[2(\partial_\mu\eta)^2 \left(1 + \frac{f^2}{f^2 - 2(v + \eta)^2} \right) + \right. \quad (49)$$

$$\left. + \frac{1}{2}(g_1^2 + g_2^2)Z_\mu Z^\mu (v + \eta)^2 + g_2^2 W_\mu^- W^{\mu+} (v + \eta)^2 \right]. \quad (50)$$

We get the SM masses for the W^\pm and Z -boson, plus we can include a massless photon. Until here everything is just as in the SM but there is an important change in the Higgs field.

The scalar field η is not normalized correctly. In order to be a physical field its propagator needs to have a term normalized like $\frac{1}{2}(\partial_\mu h)^2$. We have to make the substitution

$$\eta \rightarrow h = \frac{c_\alpha}{\sqrt{2}}\eta, \quad c_\alpha = \sqrt{1 - \frac{2v^2}{f^2}}. \quad (51)$$

The physical field h is related to the standard model Higgs field by the factor c_α . This redefinition comes from the global $SO(5)$ breaking, namely the fifth component of the five-plet ϕ . Notice that we get the SM in the limit of $f \rightarrow \infty$, where the strong sector has no effect at low energy scale.

As already stated in [1] the Higgs coupling to the gauge bosons and to the quarks is going to be reduced by the factor c_α . However for the quarks there is another effect coming from the rotation of the mass matrix which will be discussed in the following sections.

3.3 Yukawa-Lagrangian for the 3rd quark generation

We are now following [1]. We extend the left-handed top-bottom doublet q_L to an $SO(5)$ vector Ψ_L , which is the minimal fermionic content.

$$\Psi_L = (q, X, L)_L; \quad t_R, b_R, X_R, T_R, \quad (52)$$

where q_L, X_L, L are $SU(2)_L$ -doublets and all the other fields are singlets. The standard model quarks q_L, t_R, b_R have the usual quantum numbers, while the $SO(5)$ symmetry fixes the hypercharges for the new quarks. The quantum numbers are shown in figure 5.

The Yukawa Lagrangian consists of an $SO(5)$ symmetric mass term for the top and of two soft breaking mass terms.

$$\mathcal{L}_{top} = \lambda_1 \bar{\Psi}_L \phi t_R + \lambda_2 f \bar{T}_L T_R + \lambda_3 f \bar{T}_L t_R + m_X \bar{X}_L X_R + h.c. \quad (53)$$

Our goal is to determine the masses for the quarks, by rotating the mass matrix. And then find the Higgs couplings to the quarks. We start after EWSB and insert the VEV for ϕ

$$\langle \phi \rangle = \left\langle \begin{pmatrix} \vec{\phi} \\ \phi_5 \end{pmatrix} \right\rangle = \begin{pmatrix} v \\ 0 \\ 0 \\ v \\ f c_\alpha \end{pmatrix} \quad (54)$$

	Mass	Composition
W_μ^\pm	$\frac{1}{2}g_2^2 v^2$	$\sqrt{\frac{1}{2}}(W_\mu^1 \mp iW_\mu^2)$
Z_μ	$\frac{1}{2}(g_1^2 + g_2^2)v^2$	$\frac{1}{\sqrt{g_1^2 + g_2^2}}(g_2 W_\mu^3 + g_1 B_\mu)$

Figure 4: Mass and composition of the W and Z-Boson.

	Hypercharge	Isospin	Charge
$X^{5/3}$	7/6	1/2	5/3
$X^{2/3}$	7/6	-1/2	2/3
T	2/3	0	2/3

Figure 5: Quantum numbers for the new quarks.

into the the Yukawa Lagrangian, which then reads

$$\begin{aligned} \mathcal{L}_{top} = & \lambda_1 \overline{\begin{pmatrix} t_L \\ b_L \end{pmatrix}} \begin{pmatrix} v \\ 0 \end{pmatrix} t_R + \lambda_1 \overline{\begin{pmatrix} X_L^{5/3} \\ X_L^{2/3} \end{pmatrix}} \begin{pmatrix} 0 \\ v \end{pmatrix} t_R + \\ & + f(c_\alpha \lambda_1 + \lambda_3) \bar{T}_L t_R + \lambda_2 f \bar{T}_L T_R + m_X \overline{\begin{pmatrix} X_L^{5/3} \\ X_L^{2/3} \end{pmatrix}} \begin{pmatrix} X_R^{5/3} \\ X_R^{2/3} \end{pmatrix} \end{aligned} \quad (55)$$

or in matrix form the Yukawa Lagrangian is

$$\mathcal{L}_{top} = \overline{\begin{pmatrix} t_L \\ b_L \\ X_L^{5/3} \\ X_L^{2/3} \\ T_L \end{pmatrix}} \begin{pmatrix} \lambda_1 v & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_X & 0 & 0 \\ \lambda_1 & 0 & 0 & m_X & 0 \\ f(c_\alpha \lambda_1 + \lambda_3) & 0 & 0 & 0 & \lambda_2 f \end{pmatrix} \begin{pmatrix} t_R \\ b_R \\ X_R^{5/3} \\ X_R^{2/3} \\ T_R \end{pmatrix}. \quad (56)$$

3.4 The mass eigenstates

The mass matrix is not diagonal, which means that the states t, X, T are no mass eigenstates. To get the physical particles with a mass, we need to diagonalize this matrix. This means that we multiply from the left with a $SO(5)$ matrix L and from the right with another $SO(5)$ matrix R , so that LMR is diagonal. Notice that the Lagrangian itself does not change, since Ψ is an $SO(5)$ five-plet and thus we only do a gauge transformation which is a change of basis

$$\mathcal{L}_{top} = \bar{\Psi}_L M \Psi_R = \bar{\Psi}_L L M R \Psi_R. \quad (57)$$

Diagonalizing a Matrix is quite hard. We will demonstrate two ways to diagonalize this mass matrix: analytically in orders of v/f and numerically².

First we are only considering the analytical way. Looking at the matrix we see that there are 3 particles with charge 2/3 that can mix: $t, X^{2/3}$ and T . The other particles have charge 1/3 and 5/3. They can't mix with any other particles and therefore already are mass eigenstates $m_b = 0$ and $X^{5/3} = m_X$. Therefore we only need to consider a 3×3 Matrix when diagonalizing

$$M \equiv \begin{pmatrix} \lambda_1 v & 0 & 0 \\ \lambda_1 v & m_X & 0 \\ f(c_\alpha \lambda_1 + \lambda_3) & 0 & \lambda_2 f \end{pmatrix}. \quad (58)$$

This means that also the diagonalization matrices L and R are in $SO(3)$. The strategy is to diagonalize this matrix order by order in $\epsilon \equiv v/f$. First we apply a pre-rotation to make the matrix diagonal for $v = 0$.

$$T_R \rightarrow \cos(\chi) T_R - \sin(\chi) t_R, \quad t_R \rightarrow \cos(\chi) t_R + \sin(\chi) T_R \quad (59)$$

²Both ways have been done with MAPLE[©]. The numerical diagonalization will be discussed in section 3.7

The condition for the matrix to be diagonal at order ϵ^0 is then that the $M_{3,1}$ element has to vanish

$$M_{3,1} = f(\lambda_1 c_\alpha + \lambda_3) \cos(\chi) - \lambda_2 f \sin(\chi) = 0. \quad (60)$$

Substituting $\lambda'_1 \equiv (\lambda_1 c_\alpha + \lambda_3)$, we get

$$\tan(\chi) = \frac{\lambda'_1}{\lambda_2}. \quad (61)$$

The Lagrangian after the pre-rotation then reads

$$\mathcal{L}_{top} = \bar{t}_L v(\lambda_1 t_R + \lambda_T T_R) + \bar{X}_L v(\lambda_t t_R + \lambda_T T_R) + m_T \bar{T}_L T_R + m_X \bar{X}_L X_R, \quad (62)$$

where we have made the following substitutions

$$\lambda_t = \lambda_1 \cos(\chi) = \frac{\lambda_1 \lambda_2}{\sqrt{\lambda_1'^2 + \lambda_2^2}}, \quad \lambda_T = \lambda_1 \sin(\chi) = \frac{\lambda_1 \lambda'_1}{\sqrt{\lambda_1'^2 + \lambda_2^2}}, \quad (63)$$

$$m_T = \sqrt{\lambda_1'^2 + \lambda_2^2} f.$$

The matrix M can then be written as a part diagonal to order ϵ^0 plus and a part of order ϵ

$$M = \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & m_X & 0 \\ 0 & 0 & m_T \end{pmatrix}}_{\equiv M_0} + \epsilon \underbrace{\begin{pmatrix} \lambda_t f & 0 & \lambda_T f \\ \lambda_t f & 0 & \lambda_T f \\ 0 & 0 & 0 \end{pmatrix}}_{\equiv M_1}. \quad (64)$$

Now we can start with an iterative diagonalization procedure, where we expand the rotation matrices L and R also in orders of ϵ

$$L = L_0 + \epsilon L_1 + \epsilon^2 L_2 + \epsilon^3 L_3 + \mathcal{O}(\epsilon^4), \quad R = R_0 + \epsilon R_1 + \epsilon^2 R_2 + \epsilon^3 R_3 + \mathcal{O}(\epsilon^4) \quad (65)$$

diagonalize to every order and so find the rotation matrices. The unitarity of the rotation matrices gives us the following conditions

$$1 = L^\dagger L = 1 + \epsilon(L_1 + L_1^\dagger) + \epsilon^2(L_2 + L_1^\dagger L_1 + L_2^\dagger) + \mathcal{O}(\epsilon^3) \quad (66)$$

$$\Rightarrow L_1 + L_1^\dagger = 0, \quad L_2 + L_1^\dagger L_1 + L_2^\dagger = 0 \quad (67)$$

Note that we have set $L_0 = 0$ because we start with a matrix that is already diagonal at lowest order. The unitarity condition for R is of course exactly the same.

There is a nice trick to avoid having two different rotation matrices at the same time. Assume $LMR = D$, where D is diagonal then $(LMR)(LMR)^\dagger = LMM^\dagger L^\dagger = DD^\dagger$ is still diagonal. This means finding our rotation matrix L is equivalent to a standart diagonalization of a symmetric matrix³, MM^\dagger in our case⁴.

From Eq. (67) we see that L_1 is antisymmetric, this means it has 3 degrees of freedom. The order one term is a symmetric 3×3 matrix $Le1$

$$LMM^\dagger L^\dagger = M_0 M_0^\dagger + \epsilon \underbrace{(L_1 M_0 M_0^\dagger + M_1 M_0^\dagger + M_0 M_1^\dagger + M_0 M_0^\dagger L_1^\dagger)}_{\equiv Le1} + \mathcal{O}(\epsilon^2), \quad (68)$$

³A symmetric matrix A can be written in the form $A = QDQ^T$, where Q is orthogonal and D is diagonal. Furthermore the elements of D are the eigenvalues of A and the columns of Q are the eigenvectors of A .

⁴For the right rotation we get a very similar condition $(LMR)^\dagger(LMR) = R^\dagger M^\dagger MR = D^\dagger D$, which reduces the problem to the rotation of the symmetric matrix $M^\dagger M$

where we set the off-diagonal terms to zero, which gives us three equations and thus fully determines L_1 . The same can be done for R_1 which gives us the mass matrix diagonal to order ϵ . This can now be repeated order by order since the unitarity condition gives us always three parameters for every L_n and the matrices we want to make diagonal are always symmetric⁵. The full calculation can be found in appendix B. The final result diagonal to order three gives us the masses

$$m'_t = \lambda_t v \left(1 - \frac{1}{2}\epsilon_L^2 - \frac{1}{2}\epsilon_R^2 \right), \quad (69)$$

$$m'_X = m_X \left(1 + \frac{\epsilon_R^2}{2} + \frac{1}{2} \frac{(\lambda_T v)^2}{m_X^2 + m_T^2} \right), \quad (70)$$

$$m'_T = m_T \left(1 + \frac{\epsilon_L^2}{2} - \frac{1}{2} \frac{(\lambda_T v)^2}{m_X^2 - m_T^2} \right), \quad (71)$$

where we have defined $\epsilon_R \equiv \frac{\lambda_t v}{m_X}$ and $\epsilon_L \equiv \frac{\lambda_T v}{m_T}$. The mixing of the quarks, up to order one in ϵ_R and ϵ_L , are

$$\begin{aligned} \bar{t}'_L &\simeq \bar{t}_L - \epsilon_L \bar{T}_L, & t_{R'} &\simeq t_R - \epsilon_R X_R \\ X_R^{2/3'} &\simeq X_R^{2/3} - \epsilon_L \frac{m_T^2}{m_T^2 - m_X^2} \bar{T}_L, & X_R^{2/3'} &\simeq X_R^{2/3} + \epsilon_R t_R - \epsilon_L \frac{m_T m_X}{m_T^2 - m_X^2} T_R \\ \bar{T}'_L &\simeq \bar{T}_L + \epsilon_L \bar{t}_L + \epsilon_L \frac{m_T^2}{m_T^2 - m_X^2} \bar{X}_L, & T_{R'} &\simeq T_R + \epsilon_L \frac{m_T m_X}{m_T^2 - m_X^2} X_R \end{aligned} \quad (72)$$

This means that the top quark we see in accelerator experiments is actually a composite state of other quarks. The measured top mass is then the composite top mass, i.e. $m'_t \simeq 172.4$ GeV. At high enough energies, above m'_X and m'_T it should also be possible to produce the two other quarks. We will discuss later on at what scales these masses are.

3.5 The Higgs couplings

Now that we have diagonalized the matrix, we will investigate how the Higgs couplings change. The procedure is pretty straight forward. We start with the initial Lagrangian (53) and insert the VEV of ϕ plus we add some fluctuation η as in Eq. (48) and get

$$\begin{aligned} \mathcal{L}'_{top} = & \lambda_1 \overline{\begin{pmatrix} t_L \\ b_L \end{pmatrix}} \begin{pmatrix} v + \eta \\ 0 \end{pmatrix} t_R + \lambda_1 \overline{\begin{pmatrix} X_L^{5/3} \\ X_L^{2/3} \end{pmatrix}} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix} t_R + \lambda_2 f \bar{T}_L T_R + \\ & + f \left(\lambda_1 \sqrt{1 - \frac{2(v + \eta)^2}{f^2}} + \lambda_3 \right) \bar{T}_L t_R + m_X \overline{\begin{pmatrix} X_L^{5/3} \\ X_L^{2/3} \end{pmatrix}} \begin{pmatrix} X_R^{5/3} \\ X_R^{2/3} \end{pmatrix} \end{aligned} \quad (73)$$

In order to get the couplings we need to expand the squareroot around $\eta = 0$

$$\sqrt{1 - \frac{2(v + \eta)^2}{f^2}} = c_\alpha - \frac{2v}{f^2 c_\alpha} \eta + \mathcal{O}(\eta^2). \quad (74)$$

Here we only look at terms in η of order one⁶. We can now split up the Lagrangian into a mass-part that is exactly the same as \mathcal{L}_{top} in Eq. (56) and an interaction part which

⁵There are matrices with additional symmetries where this algorithm does not work in its easiest form, luckily the matrix in this model is well behaving.

⁶Terms with η^2 , i.e. vertices with two Higgs particles will be highly suppressed and not measurable at the LHC.

contains all the interactions with the Higgs particle.

$$\mathcal{L}_{top}' = \mathcal{L}_{top} + \mathcal{L}_{Higgs} + \mathcal{O}(\eta^2), \quad (75)$$

where

$$\mathcal{L}_{Higgs} = \lambda_1 \bar{t}_L \eta t_R + \lambda_1 \bar{X}_L^{2/3} \eta t_R - \lambda_1 \left(\frac{v}{f} \frac{2}{c_\alpha} \bar{T}_L \eta t_R \right) \quad (76)$$

However the particles in this Lagrangian are not physical. To get physical particles we have to rotate this Lagrangian into the mass eigenbasis. We first apply the pre-rotation (59), which gives us

$$\mathcal{L}_{Higgs} = \begin{pmatrix} \bar{t}_L \\ \bar{X}_L^{2/3} \\ \bar{T}_L \end{pmatrix} \underbrace{\begin{pmatrix} \lambda_t \eta & 0 & \lambda_T \eta \\ \lambda_t \eta & 0 & \lambda_T \eta \\ \frac{v}{f} \frac{2}{c_\alpha} \lambda_t \eta & 0 & \frac{v}{f} \frac{2}{c_\alpha} \lambda_T \eta \end{pmatrix}}_{\equiv H} \begin{pmatrix} t_R \\ X_R^{2/3} \\ T_R \end{pmatrix}. \quad (77)$$

To get the couplings of the physical particles to the Higgs we need to apply the left and right rotations found in (72) and make the substitution (51) in order to get the physical Higgs. The analytical results are quite messy. In the next chapter we will therefore only investigate the top to Higgs coupling analytically and look at the other couplings later on when we plug in the numbers.

3.6 Changes in the gluon fusion cross section

We can now look at new contributions to gluon fusion. With this model we get couplings between two different quarks and the Higgs boson (flavour changing neutral current), which were not possible in the SM. However these couplings won't play a role in gluon fusion because the initial gluons can still only couple to a quark-antiquark pair and therefore the Higgs boson will also be produced by a quark-antiquark pair. This means that only the diagonal entries in the rotated matrix LHR are relevant for gluon fusion. We call them $(C_t, C_X, C_T) \equiv \text{diag}(LHR)$. This means that there are now **three** changes in the gluon fusion cross section compared to the SM:

- Factor c_α with respect to the SM Higgs.
- The top quark is a composite of other quarks and the Yukawa coupling is therefore no longer proportional to the top mass.
- There are two new quark loops entering gluon fusion.

We now want to implement these changes in the calculation of section 2. The standard model Yukawa coupling was proportional to the mass of the top quark over the VEV of the Higgs, $\frac{m_t'}{v}$. The analytical expression for top to Higgs coupling in LHR is

$$C_t = \lambda_t \left(1 - \frac{3}{2} \epsilon_R^2 - \frac{3}{2} \epsilon_L^2 + 2 \frac{v}{f} \epsilon_L + \mathcal{O}(\epsilon_L^3, \epsilon_R^3, (v/f)^2) \right). \quad (78)$$

Comparing this to the SM coupling with the the top mass from Eq. (69), we get

$$\frac{m_t'}{v} - C_t = \frac{\epsilon_R^2}{2} + \frac{\epsilon_L^2}{2} - 2 \frac{v}{f} \epsilon_L + \mathcal{O}(\epsilon_L^3, \epsilon_R^3, (v/f)^2). \quad (79)$$

So assuming that ϵ_R , ϵ_L and v/f are small this correction is not very substantial. It is not very hard to adapt the above changes. We apply them before averaging over the polarizations. Starting from Eq. (27), we need to sum over all three quarks and change $y_q \mapsto C_q c_\alpha$, where $q = t, X, T$. Keep in mind that $A(m_q)$ depends on the mass of the quark

$$\mathcal{M}^{\alpha\beta} = 4 \sum_{q=t,X,T} \frac{C_q c_\alpha}{\sqrt{2}} g_s^2 m_q \delta^{ab} \left(g^{\alpha\beta} - \frac{p_2^\alpha p_1^\beta}{(p_1 \cdot p_2)} \right) A(m_q) \quad (80)$$

The final result for the modified matrix element, analogous to Eq. (34) now reads

$$|M|^2 = \frac{(4\pi)^2}{2} \alpha_s^2(\mu) c_\alpha^2 \left| \sum_{q=t,X,T} C_q m_q A(m_q) \right|^2. \quad (81)$$

3.7 Gluon fusion cross section in composite Higgs model with minimal fermionic content

In this section we want to calculate the proton proton to Higgs cross section in the composite Higgs model discussed above. As stated in [1] the model with minimal fermionic content does not pass EWPT and is thus not realistic. Anyway we will plug in some numbers to get a qualitative feeling what effect the global $SO(5)$ symmetry has on the gluon fusion cross section. In section 4.5 we will calculate the cross section of a composite Higgs model, which is compatible with EWPT. We can then compare the results for both models.

For our further discussion we set $f = 500$ GeV, $v = 174$ GeV and $m_{top} = 172.4$ GeV. Now there are four undetermined parameters left λ_T , λ_t , m_X and m_T . We will use the result of the analytical diagonalization, Eq. (69) and set $m'_t = 172.4$ GeV to fix λ_t . It is not a priori clear that the analytical result for the top mass is a good approximation, to check this, we will only look at parameters that give $m_{top} = 172.4 \pm 3$ GeV⁷. The analytical result however is quite good and for most input parameters the top mass is within the 3 GeV range.

For the diagonalization and the data output we used a program generated with Maple[®]. In appendix C is a similar program used to calculate the cross section for the more realistic model of section 4. Here we will describe in words what the program does

1. Define all variables and the matrix M . The parameters m_X , m_T and λ_T are varied using for-loops.
2. To fix λ_t we use Eq. (69) and set $m'_t = 172.4$ GeV.
3. Find the eigenvectors of MM^T and $M^T M$. The eigenvectors are the rotation matrices L and R respectively. Note that the eigenvalues of both matrices MM^T and $M^T M$ are the same and must also be ordered in the same way.
4. Calculate the Higgs couplings, i.e. the diagonal entries of LHR and plug everything into Eq. (99).
5. The integration over the Pdfs of section 2.7 does not change at all. Multiplying $\frac{\pi}{m_h^2} |M|^2$ with the result of the integration (see Eq. (43)), using Cteq data and $m_h = 120$ GeV we get the total proton proton to Higgs cross section.

⁷The top mass has so far not been measured very accurately, a 3 GeV uncertainty is about what recent measurements yielded [11].

For the first analysis, we will vary one parameter, leave the others fixed and then look how the cross section changes. We want the masses of the physical quarks m'_X and m'_T to be above the scale f but not higher than the cutoff of the theory, which is $2\pi f \simeq 3$ TeV [1]. Looking at Eq. (70) and (71), we see the parameters m_X and m_T are very similar to the physical masses m'_X and m'_T . It is therefore appropriate to consider the values m_X and m_T between 500 GeV and 3000 GeV. For the coupling constant λ_T we took values between 0.2 and 2.5.

In the first plot (figure 6) we varied one of the input parameters m_X and m_T from 500 GeV to 3000 GeV, while leaving the other fixed at 1000 GeV. The value for λ_T was fixed at 1. We see that the cross section changes only very little between 14.52 pb and 14.68 pb, i.e. it is almost insensitive to the input parameters m_X and m_T .

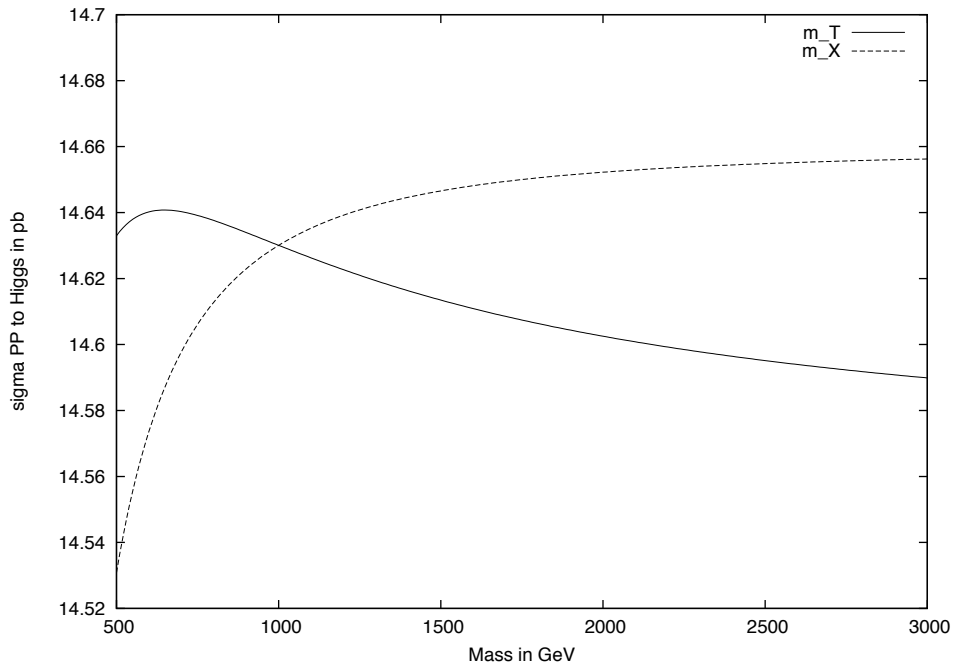


Figure 6: The effect of the input parameters m_X (dashed) and m_T (solid) on the cross section is negligible.

For the second plot (figure 7) we varied λ_T from 0.2 to 2.5, whereas the input parameters were fixed at $m_X = 1000$ GeV and $m_T = 1000$ GeV. We see that the change in the cross section ($\sigma = 14.57$ pb to 14.65 pb) is even less as in the first plot.

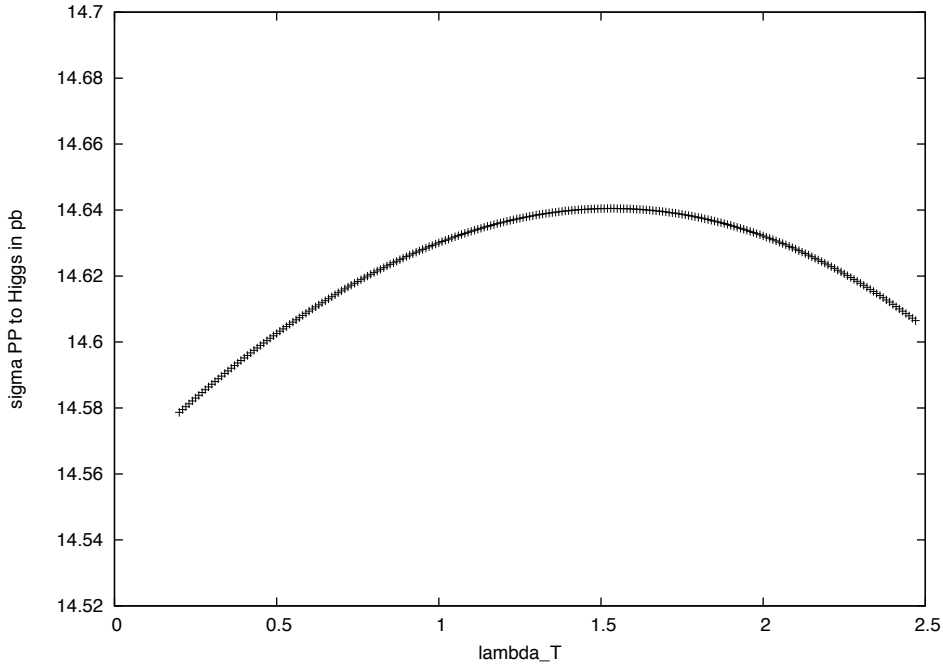


Figure 7: The effect of λ_T on the cross section is also negligible.

For the second analysis, we do a full parameter scan. We fix λ_t with the top mass and vary all the other parameters together: $\lambda_T \in [0.2, 4.0]$, $m_X \in [500, 3000]$ and $m_T \in [500, 3000]$. Just as in the first analysis the cross section stays in a very small region, between 14.45 pb and 14.70 pb.

This insensibility of the cross section with respect to the various parameters was already predictable from Eq. (78). There we saw that the top to Higgs coupling is very similar to the SM Yukawa coupling. Moreover the other quarks contribute only very little to the cross section independently of the input parameters. This means that the only effect that is left to change the gluon fusion cross section with respect to the SM is the factor c_α^2 . Indeed $\sigma_{SM} \cdot c_\alpha^2 = 14.58$ pb, which is exactly the mean value of our interval 14.45 pb to 14.70 pb. This is a remarkable result because the effect of the new fermionic sector on the cross section is totally negligible. In [1] only the c_α^2 -effect was mentioned. After the discussion for the model with minimal fermionic content this is totally sufficient. The deeper reason for this will be analyzed in section 5, after having discussed the model with non-minimal fermionic content.

4 A more realistic model

4.1 Non-minimal fermionic sector

This approach is analogous to [8]. Again we use a composite Higgs model with global $SO(5)$ symmetry as in section 3.1. But instead of extending the top sector to an $SO(5)$ five-plet (see section 3.3), we introduce a complete five-plet of fermionic composites, which span full representations of $SO(5)$

$$\Psi_{R,L} = (Q, X, T)_{R,L}, \quad (82)$$

where Q and X are doublets and T is a singlet. The quantum numbers are shown in figure 8.

	hypercharge	Isospin	Charge
Q^u	1/6	1/2	2/3
Q^b	1/6	-1/2	1/3
$X^{5/3}$	7/6	1/2	5/3
X^u	7/6	-1/2	2/3
T	2/3	0	2/3

Figure 8: Quantum numbers of the new quarks.

The top sector is thus not a priori included in $SO(5)$, but couples to these new quarks via the interaction Lagrangian

$$-\mathcal{L}_{int} = \lambda_{qi} f \bar{q}_L Q_R^i + \lambda_t f \bar{T}_L t_R + h.c.. \quad (83)$$

The new sector has an $SO(5)$ invariant mass Lagrangian, which looks like

$$-\mathcal{L}_{BSM} = m_\Psi \bar{\Psi}^i \Psi^i + \frac{\mu_{ij}}{f} (\bar{\Psi}_L^i \phi) (\phi^T \Psi_R^j). \quad (84)$$

Since we have only changed the fermionic content, the kinetic Lagrangian stays the same as in section 3.2. As before, the global $SO(5)$ symmetry is broken to $SO(4)$. This means we insert the VEV for ϕ before EWSB, in the Lagrangian. The VEV is identical to Eq. (54) with $v = 0$, i.e. $\langle \phi \rangle = (0, 0, 0, 0, f)$. The Lagrangian for the charge 2/3 particles then reads

$$\mathcal{L} = \begin{pmatrix} t_L \\ Q_L^u \\ X_L^u \\ T_L \end{pmatrix} \begin{pmatrix} 0 & \lambda_q^T f & 0 & 0 \\ 0 & m_\Psi & 0 & 0 \\ 0 & 0 & m_\Psi & 0 \\ \lambda_t f & 0 & 0 & m_\Psi + f\mu \end{pmatrix} \begin{pmatrix} t_R \\ Q_R^u \\ X_R^u \\ T_R \end{pmatrix}. \quad (85)$$

We see that this Lagrangian is not diagonal. To get the masses of the particles we need to perform the following rotations

$$q_L \rightarrow \cos \Theta_L q_L + \sin \Theta_L Q_L, \quad Q_L \rightarrow -\sin \Theta_L q_L + \cos \Theta_L Q_L, \quad (86)$$

$$t_R \rightarrow \cos \Theta_R t_R + \sin \Theta_R T_R, \quad T_R \rightarrow -\sin \Theta_R t_R + \cos \Theta_R T_R, \quad (87)$$

where $\tan \Theta_R = \frac{\lambda_t f}{m_T}$ and $\tan \Theta_L = \frac{\lambda_q^T f}{m_\Psi}$. Notice that in Eq. (86) we rotate the whole doublet, which is possible because both doublets have the same quantum numbers. Before EWSB the diagonal mass matrix then reads $diag(0, \frac{m_\Psi}{c_L}, m_\Psi, \frac{m_T}{c_R})$, where we have denoted $s_{L,R} \equiv \sin \Theta_{L,R}$, $c_{L,R} \equiv \cos \Theta_{L,R}$ and $m_T \equiv m_\Psi + f\mu$. This means that the new quarks

have non-degenerate mass before EWSB and that the top remains massless. After EWSB we insert the VEV for ϕ , analogous to Eq. (54)

$$\langle \phi \rangle \Psi_R^j = v Q_R^u + v X_R^u + f c_\alpha T_R \quad (88)$$

$$\bar{\Psi}_L^i \langle \phi \rangle = \bar{Q}_L^u v + \bar{X}_L^u v + \bar{T}_L f c_\alpha. \quad (89)$$

Before performing the pre-rotation Eq. (86), the Lagrangian is

$$\mathcal{L} = \overline{\begin{pmatrix} t_L \\ Q_L^u \\ X_L^u \\ T_L \end{pmatrix}} \begin{pmatrix} 0 & \lambda_q^T f & 0 & 0 \\ 0 & m_\Psi + \frac{s_\alpha^2}{2} f \mu & \frac{s_\alpha^2}{2} f \mu & c_\alpha v \mu \\ 0 & \frac{s_\alpha^2}{2} f \mu & m_\Psi + \frac{s_\alpha^2}{2} f \mu & c_\alpha v \mu \\ \lambda_t f & c_\alpha v \mu & c_\alpha v \mu & m_\Psi + c_\alpha^2 f \mu \end{pmatrix} \begin{pmatrix} t_R \\ Q_R^u \\ X_R^u \\ T_R \end{pmatrix} + h.c.. \quad (90)$$

After the pre-rotation Eq. (86) the Lagrangian is diagonal to zeroth order in $\epsilon \equiv \frac{v}{f}$ and reads

$$\mathcal{L} = \overline{\begin{pmatrix} t_L \\ Q_L^u \\ X_L^u \\ T_L \end{pmatrix}} \underbrace{\begin{pmatrix} s_L s_R c_\alpha v \mu & -s_L \frac{s_\alpha^2}{2} f \mu & -s_L \frac{s_\alpha^2}{2} f \mu & -s_L c_R c_\alpha v \mu \\ -s_R c_L c_\alpha v \mu & \frac{m_\Psi}{c_L} + c_L \frac{s_\alpha^2}{2} f \mu & c_L \frac{s_\alpha^2}{2} f \mu & c_L c_R c_\alpha v \mu \\ -s_R c_\alpha v \mu & \frac{s_\alpha^2}{2} f \mu & m_\Psi + \frac{s_\alpha^2}{2} f \mu & c_R c_\alpha v \mu \\ s_R c_\alpha v \mu & c_\alpha v \mu & c_\alpha v \mu & \frac{m_T}{c_R} - c_R s_\alpha^2 f \mu \end{pmatrix}}_{\equiv M} \begin{pmatrix} t_R \\ Q_R^u \\ X_R^u \\ T_R \end{pmatrix}. \quad (91)$$

Here one can already get a first impression about the masses of the Quarks. The diagonal elements of this matrix are exactly the same as of the matrix diagonalized to first order⁸. The top mass is $m_t \simeq s_L s_R c_\alpha v \mu$. To get a large top mass (172 GeV) we need $\mu s_L s_R \sim 1$. This means also that the top needs a substantial mixing with either the Q or the T quark.

4.2 The mass eigenstates

Now we can diagonalize using exactly the same methods as in section 3.4. Again we can split up the matrix of the Lagrangian into a diagonal part and a non-diagonal part of order ϵ

$$M \equiv \underbrace{\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{m_\Psi}{c_L} & 0 & 0 \\ 0 & 0 & m_\Psi & 0 \\ 0 & 0 & 0 & \frac{m_T}{c_R} \end{pmatrix}}_{\equiv M_0} + \epsilon \underbrace{\begin{pmatrix} s_L s_R c_\alpha f \mu & -s_L v \mu & -s_L v \mu & -s_L c_R c_\alpha f \mu \\ -s_R c_L c_\alpha f \mu & c_L v \mu & c_L v \mu & c_L c_R c_\alpha f \mu \\ -s_R c_\alpha f \mu & v \mu & v \mu & c_R c_\alpha f \mu \\ 2s_R v \mu & c_\alpha f \mu & c_\alpha f \mu & -2c_R v \mu \end{pmatrix}}_{\equiv M_1}. \quad (92)$$

The only difference to the other model is that now we have a 4×4 - matrix. This means that for each of our left and right rotation matrices L_i , R_i we get six free parameters but we also get six equations and thus the algorithm should work as before.

The analytical calculation has been made with the Maple[©] program of appendix B generalized to 4 dimensions. The results for the masses of the quarks up to terms of order $\mathcal{O}(\epsilon^2)$ are already quite messy. The simplest is the top mass, which reads

$$m_t' = \epsilon s_R s_L c_\alpha f \mu \left(1 + \epsilon \mu \left[\frac{2f c_R^2}{m_T} - \frac{v}{m_\Psi} (1 + c_L^2) \right] \right). \quad (93)$$

⁸The matrix diagonalized to first order is $M\epsilon 1 \equiv M_0 + \epsilon(L_1 M_0 + M_0 R_1 + M_1)$. One can see that $M_0 R_1$ and $L_1 M_0$ do not have any diagonal entries and therefore the diagonal entries of M are exactly the same as for $M\epsilon 1$.

4.3 Higgs couplings

To get the interactions between the quarks and the Higgs we can proceed exactly as in section 3. We insert the VEV with a small fluctuation η as in Eq. (48) into the mass Lagrangian Eq. (84) and get $\mathcal{L}_{BSM} = m_\Psi \bar{\Psi}^i \Psi^i + \mathcal{L}_c$, where

$$\mathcal{L}_c = \frac{\mu}{f} \overline{\begin{pmatrix} Q_L^u \\ X_L^u \\ T_L \end{pmatrix}} \cdot \begin{pmatrix} (v+\eta)^2 & (v+\eta)^2 & Y \\ (v+\eta)^2 & (v+\eta)^2 & Y \\ Y & Y & f^2 - 2(v+\eta)^2 \end{pmatrix} \begin{pmatrix} Q_R^u \\ X_R^u \\ T_R \end{pmatrix} \quad (94)$$

and $Y \equiv (v+\eta)\sqrt{f^2 - 2(v+\eta)^2}$. To get the couplings we need to expand around η

$$\begin{aligned} (v+\eta)\sqrt{f^2 - 2(v+\eta)^2} &= (v+\eta) \left(c_\alpha f - \frac{2v}{c_\alpha f} \eta + \mathcal{O}(\eta^2) \right) \\ &= v f c_\alpha + \eta \underbrace{\left(f c_\alpha - \frac{2v^2}{f c_\alpha} \right)}_{\equiv \rho} + \mathcal{O}(\eta^2). \end{aligned} \quad (95)$$

Again we get the normal Lagrangian (90) plus the Higgs interaction Lagrangian

$$\mathcal{L}' = \mathcal{L} + \mathcal{L}_{Higgs}, \quad (96)$$

where

$$\mathcal{L}_{Higgs} = \eta \cdot \left(\frac{\mu}{f} \right) \overline{\begin{pmatrix} Q_L^u \\ X_L^u \\ T_L \end{pmatrix}} \begin{pmatrix} 2v & 2v & \rho \\ 2v & 2v & \rho \\ \rho & \rho & -4v \end{pmatrix} \begin{pmatrix} Q_R^u \\ X_R^u \\ T_R \end{pmatrix}. \quad (97)$$

As before have to rotate to get physical states. First we include the top quark in the Higgs interaction Lagrangian above to get a 4×4 - matrix. Then we apply the pre-rotations (86) and (87) to get

$$H \equiv \frac{\mu}{f} \begin{pmatrix} s_R s_L \rho & -2v s_L & -2v s_L & -s_L c_R \rho \\ -s_R c_L \rho & 2v c_L & 2v c_L & c_R c_L \rho \\ -s_R \rho & 2v & 2v & c_R \rho \\ 4v s_R & \rho & \rho & -4v c_R \end{pmatrix}. \quad (98)$$

The first diagonal entry of H is the Higgs to top coupling to order ϵ , i.e. $\mu s_R s_L c_\alpha + \mathcal{O}(\epsilon^2)$. Comparing this to the top mass of order ϵ , Eq. (91), we see that the coupling is exactly $\frac{m_t}{v}$ as in section 3 and in the SM.

We could now apply the rotations found when diagonalizing in section 4.2. However, this time we are not going to look at the analytical results because they are too messy. We will instead apply a fully numerical diagonalization and thus find first the Higgs couplings and then the new gluon fusion cross section.

4.4 Gluon fusion cross section in composite Higgs model with non-minimal fermionic content

When discussing the changes in the gluon fusion cross section, we get exactly the same results as in section 3.6. The only thing that does change is that we have one more quark entering the loop. We can apply this directly to Eq. (81) and get

$$|M|^2 = \frac{(4\pi)^2}{2} \alpha_s^2(\mu) c_\alpha^2 \left| \sum_{q=t,Q,X,T} C_q m_q A(m_q) \right|^2. \quad (99)$$

In [8] composite Higgs model with non-minimal fermionic content has been shown to pass EWPT for certain data. Here we will apply the data points which passed EWPT, to gluon fusion and see how it changes. As before we take $v = 174$ GeV, $f = 500$ GeV and $m_{top} = 174.2$ GeV. There are four parameters left μ , s_L , s_R and m_ψ and one of them is constraint by the top mass. The datapoints are given as a vector containing s_L , s_R and m_T/m_ψ . Together with the top mass they fully determine our model.

Phases in Ψ_L and Ψ_R : There is a gauge transformation which we have not yet used, which is a phase in the five-plet Ψ . We will need this phase, because the value we get for μ is negative and therefore the first diagonal entry in the mass matrix, $M_{1,1}$, is also negative. However, the diagonalized $M_{1,1}$ value is the top mass, which must of course be positive. We will thus apply a rotation L' in $L'LMR = D$ such that D is a diagonal matrix with positive entries. Looking at Eq. (91) we see that for $\mu < 0$ and $s_L, s_R > 0$, $M_{1,1}$ and $M_{4,4}$ are negative. We can thus set $L' = \text{diag}(-1, 1, 1, -1)$ to make all the masses positive at first order.

As in section 3.7 the diagonalization procedure was done totally numerically with Maple[©]. The full program can be found in appendix C. It is very similar to the previous program but there are some changes. For completeness we put here a description of it.

1. Read in the data, define all variables and the matrix M . The only undetermined parameter is m_ψ .
2. Find the eigenvalues of MM^T and set the smallest eigenvalue equal to m_t . This gives us m_ψ and therefore the whole mass spectrum of the quarks.
3. Find the eigenvectors of MM^T and M^TM which are the rotation matrices L and R respectively. Note that the eigenvalues of both matrices MM^T and M^TM are the same and also must be ordered in the same way.
4. Check that LMR is diagonal and that its entries are the masses of the quarks. Here we use the freedom of a phase in Ψ to make all the diagonal entries of LMR positive.
5. Calculate the Higgs couplings, i.e. the diagonal entries of LHR and plug everything into Eq. (99).
6. The integration over the Pdfs of section 2.7 does not change at all. Multiplying $\frac{\pi}{m_h^2}|M|^2$ with the result of the integration (see Eq. (43)), using Cteq data and $m_h = 120$ GeV we get the total proton proton to Higgs cross section.

4.5 Discussion of the results

We analyzed 194 datapoints, i.e. vectors $[s_L, s_R, \frac{m_T}{m_\psi}]$ and calculated the proton proton to Higgs cross section. The first thing to notice is that the cross section for all datapoints is contained in a small interval between 6.3 pb and 6.8 pb, which corresponds to $\sim 0.34\sigma_{SM}$. We see that in this model the cross section is about a factor 2 smaller as for the model in the section 3.7. There we saw that the Higgs to top coupling did not change substantially with respect to the SM and that all the other quarks had only minor contribution. Now the case is totally different. We see in figure 9 that the Higgs to top coupling varies a lot (-0.15 to 0.75), whereas in the previous model as well as in the SM it was ~ 1 . This means that the other quarks must have a non-negligible contribution to the cross section.

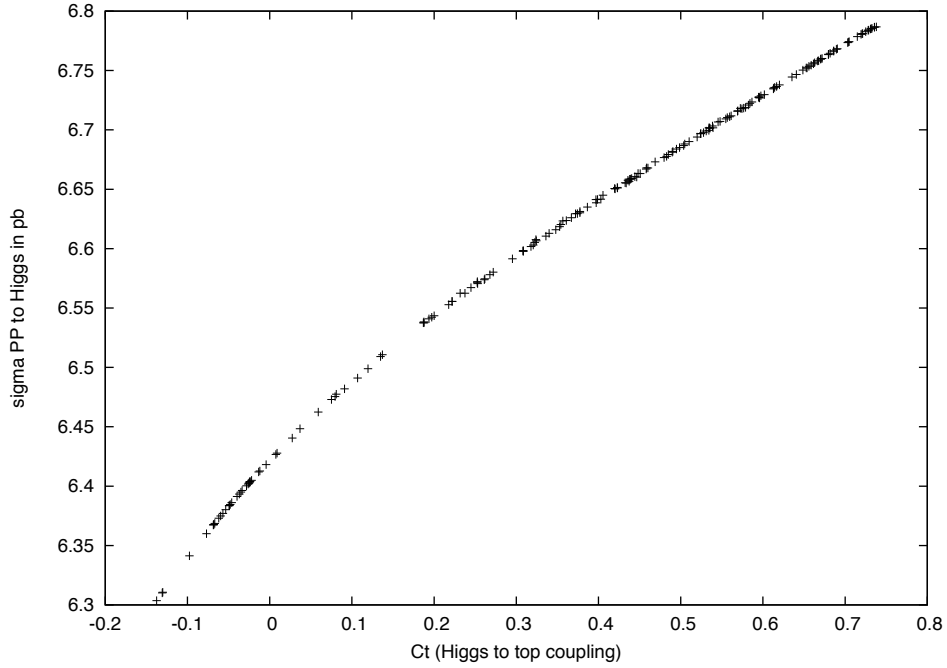


Figure 9: Higgs to top coupling versus cross section.

In the second diagram (figure 10) we analyze the data a little differently. We set the couplings of all but one quark to zero and calculate the contribution to the cross section of each quark separately. In the diagram we plot on the x-axis the mass of the quarks on the y-axis the cross section relative to the SM. We see that the Q and T quarks contribute only very little and the X quark contributes most of the cross section. The remarkable thing is that in spite of the large range of cross section the top and the X quark contribute alone, they always add up to a very small range of values ($0.325\sigma_{SM}$ to $0.355\sigma_{SM}$). Notice that the contribution to the cross section of a single quark is always positive. In figure 10 the X quark alone sometimes has a cross section that is much higher than the total cross section due to all quarks, so there must be a suppression coming from other quarks in order to get the total cross section of figure 9.

Another information contained in this diagram is the masses of the new quarks. We see that the X -quark is usually the lightest with a mass of 350 GeV to 600 GeV⁹. The T -quark has a mass of about 600 to 2000 GeV and the Q -quark has a mass of 3000 to 10000 GeV. The mass of the X -quark is low enough for the LHC to detect it. The T -quark will be only detectable if its mass is near the mass of the X -quark. The Q -quark is too heavy to be detected and masses of 10000 GeV are not good for the theory to be perturbative.

⁹Here we ignore outliers.

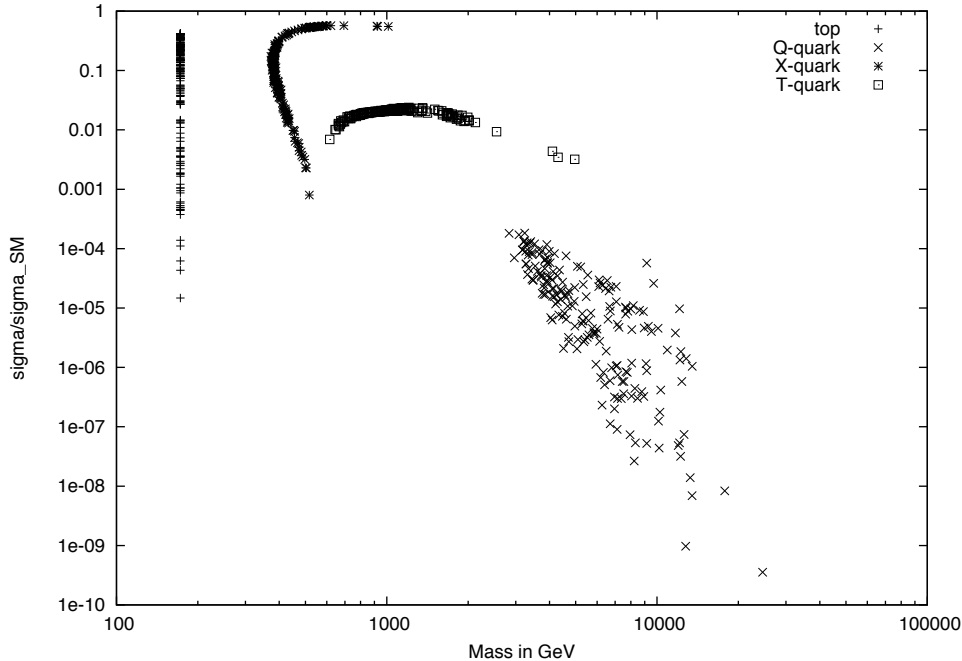


Figure 10: Contribution to the cross section of each quark type alone.

5 Conclusion

In the above discussion and from other sources of analyses [8], we saw that the composite Higgs model definitely is an appropriate candidate for an extension of the SM. First it can be made to agree fully with EWPT, and second it does solve the hierarchy problem. Moreover the compositeness of the top quark is able to explain why its mass is so much higher than the mass of the other SM quarks. We saw that with the global $SO(5)$ symmetry approach and the specifying of the fermionic content, we could indeed derive the implications to LO gluon fusion, and calculate the proton proton to Higgs cross section. The result depended on the fermionic content of the model. In the first case (with minimal fermionic content), the change in the cross section was determined exclusively by the energy scale f , and was a factor c_α^2 lower as in the SM. The reason for this was that the new quarks only had a minor effect on the cross section and that the Higgs to top coupling was very similar to the SM. With non-minimal fermionic content, and EWPT compatible data, we got a different result. The cross section was approximately three times lower as in the SM. Here the mixing of the quarks played a larger role and the top and X-quark contributed substantially to the cross section. In contrast to the other model it difficult to trace the source of this high suppression. However, there has been a very elegant analysis, predicting a gluon fusion cross section of $\sim 0.35\sigma_{SM}$ for five fermions and $f = 500$ GeV [14]. Moreover it predicts that for five fermions and $f = 1000$ GeV we get $0.82\sigma_{SM}$. Indeed setting $f = 1000$ GeV in the program in appendix C we get a cross section between $0.78\sigma_{SM}$ and $0.82\sigma_{SM}$, which is still in total agreement with EWPT.

The big question now is if this model could be tested at the LHC. The error for the gluon fusion cross section measurement at the LHC is $\sim 10\%$ [5]. So testing the model by measuring

the gluon fusion cross section will depend on the fermionic content and the energy scale f , but even for $\sim 0.8\sigma_{SM}$, it should be possible. Anyhow it will be much more difficult to detect the Higgs particle, if the cross section is suppressed by a factor of 0.3. However, the cross section for the Higgs production via vector boson fusion (VBF) can be measured with a precision of $\sim 5\%$, whereas the reduction in the $SO(5)$ model is also $\sim c_\alpha^2 \sim 0.75$ [1]. The VBF cross section could therefore be a fruitful testing ground for composite Higgs models. Another property which could be measured at the LHC is the new Quarks. In section 4.5 we saw that the X-quark is the lightest of the new quarks with a mass of 350 GeV to 600 GeV. New particles around this mass should be detectable, however, it will not be easy to determine what these new particles are. There are also flavour changing neutral currents in composite Higgs model, which were not present in the SM.

We should after all keep in mind that there is a large variety of different composite Higgs models, with different global symmetries and that it is not a priori clear that they will all lead to a similar phenomenology. It will therefore not be possible to disprove all composite Higgs models at once, if certain predictions cannot be experimentally verified. On the other hand it will not be easy to verify a particular model, even when there is experimental data in favour of it, because there are just too many possibilities such as fermionic content.

Even if this thesis only covered LO gluon fusion for two very specific models it can maybe help to get a feeling what composite Higgs phenomenology, especially gluon fusion looks like. Furthermore, I hope that this master's thesis showed that even for beyond standard models it is possible to calculate predictions for observables under certain assumptions.

Acknowledgements

I would like to thank Babis Anastasiou for his invaluable support and introducing me to the realm of Higgs physics. Moreover I would like to thank the whole crew of theoretical particle physics at the ETH, especially Elisabetta Furlan and Jose Santiago.

A Calculation for the gluon fusion

In this section we are going to compute the integral $A(m_t)$ from Eq. (28).

A.1 Master integrals

First we want to express our integral with master integrals. We have eliminated all Dirac indices of k , now we need to make the numerators of $A(m_t)$ independent of k . We have 6 different types of numerators: k^2 , $k \cdot p_1$, $k \cdot p_2$, $(k \cdot p_1)(k \cdot p_2)$, $(k \cdot p_1)^2$ and $(k \cdot p_2)^2$. For each of these we have to use a different method.

- $\boxed{k^2}$

$$\begin{aligned}
& \frac{k^2 + m_t^2 - m_t^2}{[2(p_1 \cdot k) + k^2 - m_t^2][k^2 - m_t^2][k^2 - 2(k \cdot p_2) - m_t^2]} \\
= & \frac{m_t^2}{[2(p_1 \cdot k) + k^2 - m_t^2][k^2 - m_t^2][k^2 - 2(k \cdot p_2) - m_t^2]} + \quad (100) \\
& + \frac{1}{[2(p_1 \cdot k) + k^2 - m_t^2][k^2 - 2(k \cdot p_2) - m_t^2]}
\end{aligned}$$

- $k \cdot p_1$

$$\begin{aligned}
& \frac{k \cdot p_1 + \frac{1}{2}(k^2 - k^2 + m_t^2 - m_t^2)}{[2(p_1 \cdot k) + k^2 - m_t^2][k^2 - m_t^2][k^2 - 2(k \cdot p_2) - m_t^2]} \\
&= \frac{\frac{1}{2}}{[k^2 - m_t^2][k^2 - 2(k \cdot p_2) - m_t^2]} - \\
& \quad - \frac{\frac{1}{2}}{[2(p_1 \cdot k) + k^2 - m_t^2][k^2 - 2(k \cdot p_2) - m_t^2]}
\end{aligned} \tag{101}$$

- $k \cdot p_2$

$$\begin{aligned}
& \frac{k \cdot p_2 + \frac{1}{2}(k^2 - k^2 + m_t^2 - m_t^2)}{[2(p_1 \cdot k) + k^2 - m_t^2][k^2 - m_t^2][k^2 - 2(k \cdot p_2) - m_t^2]} \\
&= \frac{-\frac{1}{2}}{[k^2 + 2(k \cdot p_1) - m_t^2][k^2 - m_t^2]} + \\
& \quad + \frac{\frac{1}{2}}{[2(p_1 \cdot k) + k^2 - m_t^2][k^2 - 2(k \cdot p_2) - m_t^2]}
\end{aligned} \tag{102}$$

- $(k \cdot p_1)(k \cdot p_2)$ Here we use the results of the $k \cdot p_1$ case.

$$\begin{aligned}
& \frac{1}{2} \int_{-\infty}^{\infty} d^d k \underbrace{\frac{p_2 \cdot k}{[k^2 - m_t^2][(k - p_2)^2 - m_t^2]}}_{\equiv I_1} + \\
& + \frac{1}{2} \int_{-\infty}^{\infty} d^d k \underbrace{\frac{-p_2 \cdot k}{[(k + p_1)^2 - m_t^2][(k - p_2)^2 - m_t^2]}}_{\equiv I_2} = \frac{1}{2}(I_1 + I_2)
\end{aligned} \tag{103}$$

For I_1 substitute $k \mapsto k + \frac{p_2}{2}$

$$I_1 = \int_{-\infty}^{\infty} d^d k \underbrace{\frac{p_2 \cdot k}{[(k + \frac{p_2}{2})^2 - m_t^2][(k - \frac{p_2}{2})^2 - m_t^2]}}_{\equiv 0} + \underbrace{\frac{p_2^2}{0}}_0 (\dots) = 0. \tag{104}$$

The first term in the sum vanishes because it is an odd function. For I_2 we substitute $k \mapsto k - \frac{p_1}{2} + \frac{p_2}{2}$

$$\begin{aligned}
I_2 &= \int_{-\infty}^{\infty} d^d k \underbrace{\frac{-p_2 \cdot k}{[(k + \frac{p_1}{2} + \frac{p_2}{2})^2 - m_t^2][(k - \frac{p_1}{2} - \frac{p_2}{2})^2 - m_t^2]}}_{\equiv 0} + \\
& \quad + \frac{1}{2} \int_{-\infty}^{\infty} d^d k \frac{p_1 \cdot p_2}{[(k + \frac{p_1}{2} + \frac{p_2}{2})^2 - m_t^2][(k - \frac{p_1}{2} - \frac{p_2}{2})^2 - m_t^2]}
\end{aligned} \tag{105}$$

$$\Rightarrow \frac{1}{2}(I_1 + I_2) = \frac{1}{4} \int_{-\infty}^{\infty} d^d k \frac{p_1 \cdot p_2}{[(k + \frac{p_1}{2} + \frac{p_2}{2})^2 - m_t^2][(k - \frac{p_1}{2} - \frac{p_2}{2})^2 - m_t^2]}. \tag{106}$$

- $\boxed{(k \cdot p_1)^2}$ Again we use the Results of $k \cdot p_1$. With a very similar procedure as above we get

$$\begin{aligned} & \frac{1}{4} \int_{\infty}^{\infty} d^d k \frac{p_1 \cdot p_2}{[(k + \frac{p_1}{2})^2 - m_t^2][(k - \frac{p_2}{2})^2 - m_t^2]} - \\ & - \frac{1}{4} \int_{\infty}^{\infty} d^d k \frac{p_1 \cdot p_2}{[(k + \frac{p_1}{2} + \frac{p_2}{2})^2 - m_t^2][(k - \frac{p_1}{2} - \frac{p_2}{2})^2 - m_t^2]}. \end{aligned} \quad (107)$$

- $\boxed{(k \cdot p_2)^2}$ This case is analogous to before and we get exactly the same result as for $(k \cdot p_1)^2$.

All integrals can now be brought to two basic forms, which we call “the master integrals for LO gluon fusion”

$$J(a) \equiv \int \frac{d^d k}{(2\pi)^d} \frac{1}{[k^2 - m_t^2][(k+a)^2 - m_t^2]} \quad (108)$$

and

$$I(a, b) \equiv \int \frac{d^d k}{(2\pi)^d} \frac{1}{[k^2 - m_t^2][(k+a)^2 - m_t^2][(k+b)^2 - m_t^2]}, \quad a, b \in \mathbb{R}. \quad (109)$$

We can now express $A(m_t)$ with these master integrals

$$A(m_t) = \underbrace{I(p_1, -p_2) \left(\frac{4}{d-2} m_t^2 - p_1 \cdot p_2 \right)}_{\equiv A_I} + \underbrace{J(p_1 + p_2) \frac{4-d}{d-2}}_{\equiv A_J} \quad (110)$$

A.2 Feynman parameter and dimensional regularization

To solve the two master integrals $J(a)$ and $I(a, b)$ we need Feynman-parameters. These can be found in [3] on page 190.

The integral $J(a)$: We have two factors in the denominator, so we use the following Feynman parameter

$$\frac{1}{P \cdot Q} = \int_0^1 dx dy \frac{\delta(x+y-1)}{(xP+yQ)^2} = \int_0^1 dx \frac{1}{(xP+(1-x)Q)^2}. \quad (111)$$

In the last step we have evaluated the δ -function. Applied to $J(a)$ this is

$$J(a) = \int \frac{d^d k}{(2\pi)^d} \int_0^1 dx \frac{1}{[(1-x)(k^2 - m_t^2) + x((k+a)^2 - m_t^2)]^2}. \quad (112)$$

If we make the substitution $k \mapsto k - ax$, we get

$$J(a) = \int_0^1 dx \int \frac{d^d k}{(2\pi)^d} \frac{1}{[k^2 + a^2 x(1-x) - m_t^2]^2}. \quad (113)$$

Now we can apply the following formula found in [3], page 250

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - \Delta)^n} = \frac{(-1)^n i \Gamma(n - \frac{d}{2})}{(4\pi)^{\frac{d}{2}} \Gamma(n)} \left(\frac{1}{\Delta} \right)^{n - \frac{d}{2}}, \quad (114)$$

where Γ is the Gamma-function, defined by $\int_0^\infty x^{y-1}e^{-x}dx$. Especially $\Gamma(n+1) = n!$ for $n \in \mathcal{N}^0$. This gives us

$$J(a) = \int_0^1 dx \frac{i\Gamma(2 - \frac{d}{2})}{(4\pi)^{\frac{d}{2}} \underbrace{\Gamma(2)}_{=1}} (-xa^2 + m^2 + a^2x^2)^{\frac{d}{2}-n}. \quad (115)$$

The next step is to use *dimensional regularization*, i.e. we substitute $d = 4 - 2\epsilon$, where $\epsilon > 0$

$$J(a) = \int_0^1 dx \frac{i\Gamma(\epsilon)}{(4\pi)^{2-\epsilon}} (m^2 + a^2x^2 - xa^2)^{-\epsilon}. \quad (116)$$

Finally we can use the approximation $\Gamma(\epsilon) = \frac{1}{\epsilon} - \gamma + \mathcal{O}(\epsilon)$, where $\gamma \sim 0.5772$ is the Euler-Mascheroni constant. Using

$$\frac{x^{-\epsilon}}{\epsilon} = \frac{e^{-\epsilon \ln x}}{\epsilon} \simeq \frac{1 - \epsilon \ln x}{\epsilon} = \frac{1}{\epsilon} - \ln x, \quad (117)$$

we get the final result, expanded in powers of ϵ

$$J(a) = \frac{1}{\epsilon} \frac{i}{(4\pi)^2} - \frac{i\gamma}{(4\pi)^2} - \int_0^1 dx \frac{i\gamma}{(4\pi)^2} \ln[m^2 + a^2x^2 - xa^2]. \quad (118)$$

We now insert $J(a)$ into Eq.(110). Looking at the second term in the sum, A_J with $d = 4 - 2\epsilon$, we get

$$\begin{aligned} A_J &= J(p_1 + p_2) \frac{4-d}{d-2} \simeq \frac{i}{(4\pi)^2} \left[1 - \epsilon \left(1 + \int_0^1 dx \ln[m_t^2 + x(p_1 + p_2)^2(x-1)] \right) \right] \\ &\simeq \frac{i}{(4\pi)^2} + \mathcal{O}(\epsilon). \end{aligned} \quad (119)$$

We see that it is absolutely necessary to keep the number of dimensions open. At the end we can take the limes $\epsilon \rightarrow 0$ and thus get that the contribution from A_J is simply 1.

The integral $I(a, b)$: Here we have three factors in the denominator, so we use the Feynman parameter

$$\frac{1}{P \cdot Q \cdot R} = \int_0^1 dx dy dz \frac{2\delta(x+y+z-1)}{(Px + Qy + Rz)^3} = \quad (120)$$

$$= \int_0^1 dx dy \frac{2}{(Px + Qy + R(1-x-y))^3}, \quad (121)$$

to get

$$I(a, b) = 2 \int_0^1 dx dy \int \frac{d^d k}{(2\pi)^d} \frac{1}{[x((k+a)^2 - m_t^2) + y((k+b)^2 - m_t^2) + (1-x-y)(k^2 - m_t^2)]^3}. \quad (122)$$

As before we want to bring the denominator of the integrand to the form $(k^2 - \Delta)^n$. We substitute $k \mapsto k - ax - by$ and get

$$I(a, b) = 2 \int_0^1 dx dy \int \frac{d^d k}{(2\pi)^d} \frac{1}{[a^2x(x-1) + b^2y(y-1) + 2abxy + m_t^2]^3}. \quad (123)$$

As before we apply (114) with $d = 4$, which gives us

$$2 \int_0^1 dx dy \frac{\overbrace{-i \Gamma(1)}^{=1}}{(4\pi)^{\frac{d}{2}} \underbrace{\Gamma(3)}_{=2}} \frac{1}{a^2 x(x-1) + b^2 y(y-1) + 2abxy + m_t^2}. \quad (124)$$

A.3 Expressing A_I with dilogarithms

We insert our result for $I(a, b)$ into Eq. (110) and look at the first term in the sum, A_I

$$\begin{aligned} A_I &= (2m_t^2 - p_1 \cdot p_2) I(p_1, -p_2) \\ &= \frac{-i}{(4\pi)^2} \int_0^1 dx dy \frac{2m_t^2 - p_1 \cdot p_2}{\underbrace{p_1^2}_{=0} x(x-1) + \underbrace{p_2^2}_{=0} y(y-1) - 2(p_1 \cdot p_2)xy + m_t^2} \end{aligned} \quad (125)$$

In the denominator of the integrand we add a small imaginary part

$$A_I = \frac{i}{(4\pi)^2} \frac{2m_t^2 - p_1 \cdot p_2}{2(p_1 \cdot p_2)} \underbrace{\int_0^1 dy \int_0^{1-y} dx \left(\frac{1}{xy - \frac{m_t^2}{2(p_1 \cdot p_2)} + i\epsilon} \right)}_{\equiv Int} \quad (126)$$

For an on-shell Higgs boson we have that $q^2 = m_h^2$ and as we can see in the Feynman graph (figure 1) $q = p_1 + p_2$. Therefore $2(p_1 \cdot p_2)^2 = m_h^2$. We substitute $R \equiv (\frac{m_t}{m_h})^2$. Eq. (110) now reads

$$A(m_t) = \frac{i}{(4\pi)^2} \left[\left(2R - \frac{1}{2} \right) Int + 1 \right]. \quad (127)$$

Polylogarithms: So for our relevant quantity A , the only thing that is left to calculate is Int . First we want to express this in terms of *dilogarithms*. The *polylogarithm* is a function $\text{Li}_s(z)$, for all complex numbers s and $|z| < 1$, defined by

$$\text{Li}_s(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^s}. \quad (128)$$

We can see directly that the polylogarithm for $s = 1$ is $\text{Li}_1(z) = -\log(1 - z)$. Another way is to define the polylogarithms recursively

$$\text{Li}_{s+1}(z) = \int_0^z dt \frac{\text{Li}_s(t)}{t}. \quad (129)$$

The dilogarithm is the polylogarithm for $s = 2$. Using the above we can see quite easily that the dilogarithm can be written as

$$\text{Li}_2(z) = \int_0^z dt \frac{-\log(1-t)}{t}. \quad (130)$$

With this knowledge we can go on

$$Int = \int_0^1 \frac{dy}{y} \int_0^{1-y} \frac{1}{x - \frac{R}{y} + i\epsilon} dx = \int_0^1 \frac{dy}{y} \log \left(\frac{1}{R} (y^2 - y + R + i\epsilon) \right). \quad (131)$$

We factorize

$$(y^2 - y + R + i\epsilon) = \left(y - \underbrace{\frac{1}{2} + \sqrt{\frac{1}{4} - R}}_{\equiv \Lambda_1} + i\epsilon \right) \left(y - \underbrace{\frac{1}{2} - \sqrt{\frac{1}{4} - R}}_{\equiv \Lambda_2} - i\epsilon \right). \quad (132)$$

Now we need to do a case study.

Case a) For $R > \frac{1}{4} \Rightarrow \sqrt{\frac{1}{4} - R}$ is imaginary. So we don't need the $i\epsilon$. Now we can use $\log(ab) = \log a + \log b$ if $\text{sgn}(\Im a) = -\text{sgn}(\Im b)$.

$$\begin{aligned} \text{Int} &= \int_0^1 \frac{dy}{y} \left[\log \left(\frac{1}{\sqrt{R}}(y - \Lambda_1) \right) + \log \left(\frac{1}{\sqrt{R}}(y - \Lambda_2) \right) \right] \\ &= \int_0^1 \frac{dy}{y} \left[\underbrace{\log \left(\frac{\Lambda_1}{\sqrt{R}} \right) + \log \left(\frac{\Lambda_2}{\sqrt{R}} \right)}_{=0} + \log \left(\frac{y}{\Lambda_1} - 1 \right) + \log \left(\frac{y}{\Lambda_2} - 1 \right) \right]. \end{aligned} \quad (133)$$

Using the dilogarithms above we can write this as

$$\text{Int} = -\text{Li}_2 \left(\frac{\Lambda_2}{R} \right) - \text{Li}_2 \left(\frac{\Lambda_1}{R} \right), \quad (134)$$

where we have used that $\frac{1}{\Lambda_1} = \frac{\Lambda_2}{R}$ and $\frac{1}{\Lambda_2} = \frac{\Lambda_1}{R}$.

Case b) For $R < \frac{1}{4} \Rightarrow \sqrt{\frac{1}{4} - R}$ is real. We need to keep the $i\epsilon$. So we have

$$\text{Int} = \int_0^1 \frac{dy}{y} \log \left(\frac{1}{R}(y - \Lambda_1 + i\epsilon)(y - \Lambda_2 - i\epsilon) \right) \quad (135)$$

$$= - \left\{ \text{Li}_2 \left(\frac{\Lambda_1}{R} + i\epsilon \right) + \text{Li}_2 \left(\frac{\Lambda_2}{R} - i\epsilon \right) \right\}. \quad (136)$$

Claim: a)

$$f(\tau) = \arcsin^2 \left(\frac{1}{\sqrt{\tau}} \right) = \frac{1}{2} \left\{ \text{Li}_2 \left(\frac{\Lambda_1}{R} \right) \text{Li}_2 \left(\frac{\Lambda_2}{R} \right) \right\}, \quad \tau \geq 1, \quad (137)$$

where $\tau \equiv 4R$.

Proof: a) The following formulas are useful:

1.

$$\arcsin x = -i \log \left(ix + \sqrt{1 - x^2} \right) \quad (138)$$

2.

$$-\frac{1}{2} \log^2(x) = \text{Li}_2(1 - x) + \text{Li}_2 \left(1 - \frac{1}{x} \right) \quad (139)$$

3.

$$\text{Li}_2(x) = - \int_0^1 \frac{dy}{y} \log(1 - x \cdot y) \quad (140)$$

With these the proof is straightforward.

$$2f(\tau) = 2 \arcsin^2\left(\frac{1}{\sqrt{\tau}}\right) \quad (141)$$

$$= -\frac{1}{2} \log^2 \left[\left(\frac{i}{\sqrt{\tau}} + \sqrt{1 - \frac{1}{\tau}} \right)^2 \right] \quad (142)$$

$$= \text{Li}_2 \left[1 - \left(\frac{i + \sqrt{\tau - 1}}{\sqrt{\tau}} \right)^2 \right] + \text{Li}_2 \left[1 - \left(\frac{\sqrt{\tau}}{i + \sqrt{\tau - 1}} \right)^2 \right] \quad (143)$$

$$= \text{Li}_2 \left[2 \left(\frac{1 - i\sqrt{\tau - 1}}{\tau} \right) \right] + \text{Li}_2 \left[2 \left(\frac{1 + i\sqrt{\tau - 1}}{\tau} \right) \right] \quad (144)$$

$$= \frac{1}{2} \left\{ \text{Li}_2 \left[\frac{1}{R} \left(-\frac{1}{2} - \sqrt{\frac{1}{4}} \right) \right] + \text{Li}_2 \left[\frac{1}{R} \left(-\frac{1}{2} + \sqrt{\frac{1}{4}} \right) \right] \right\} \quad (145)$$

Claim: b)

$$\frac{1}{2} \left\{ \text{Li}_2 \left(\frac{\Lambda_1}{R} + i\epsilon \right) + \text{Li}_2 \left(\frac{\Lambda_2}{R} - i\epsilon \right) \right\} = -\frac{1}{4} \left(\log \left[\frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}} \right] - i\pi \right)^2 \quad (146)$$

Proof: b) Using Eq. (139) with

$$x = \frac{\tau - 2 - 2\sqrt{1 - \tau}}{\tau} - i\epsilon, \quad (147)$$

we get

$$\begin{aligned} \left\{ \text{Li}_2 \left(\frac{\Lambda_1}{R} + i\epsilon \right) + \text{Li}_2 \left(\frac{\Lambda_2}{R} - i\epsilon \right) \right\} &= -\frac{1}{2} \left(\log \left[\frac{\tau - 2 - 2\sqrt{1 - \tau}}{\tau} - i\epsilon \right] \right)^2 \\ &= -\frac{1}{2} \left(\log \left| \frac{\tau - 2 - 2\sqrt{1 - \tau}}{\tau} \right| - i\pi \right)^2 \\ &= -\frac{1}{2} \left(\log \left[\frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}} \right] - i\pi \right)^2 \end{aligned} \quad (148)$$

Putting everything together the final result is

$$A = \frac{1}{(4\pi)^4} |1 + (1 - \tau)f(\tau)|, \quad (149)$$

$$f(\tau) = \begin{cases} \arcsin^2 \frac{1}{\sqrt{\tau}}, & \tau \geq 1, \\ -\frac{1}{4} \left(\log \left[\frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}} \right] - i\pi \right)^2, & \tau < 1, \end{cases} \quad (150)$$

$$\tau = 4 \left(\frac{m_t}{m_h} \right)^2 \quad (151)$$

B Diagonalization with Maple

The input for the diagonalization of the mass matrix from Eq. (64) with Maple is shown below. The expansion of LMR in terms of ϵ has been done manually. There would be a more elegant way if we would collect all the ϵ and then let Maple automatically spot terms of every order. Lines that were too long have been broken, indicated by a backslash \backslash .

```

M_0:=Matrix([[0,0,0],[0,m_x,0],[0,0,m_T]]);
M_1:=Matrix([[lambda_t*f,0,lambda_T*f],[lambda_t*f,0,lambda_T*f]\
,[0,0,0]]);
L_1:=Matrix([[0,l_11,l_12],[-l_11,0,l_13],[-l_12,-l_13,0]]);

with(LinearAlgebra):
ML1:=simplify(M_0.Transpose(M_0)+e*(L_1.M_0.Transpose(M_0)+M_1.T\
ranspose(M_0)+M_0.Transpose(M_1)+Transpose(M_0).(M_0).Transpose(\
L_1)));

l_11:=simplify(solve(ML1[1,2]=0,l_11));
l_12:=simplify(solve(ML1[1,3]=0,l_12));
l_13:=simplify(solve(ML1[2,3]=0,l_13));

R_1:=Matrix([[0,r_11,r_12],[-r_11,0,r_13],[-r_12,-r_13,0]]);
MR1:=simplify(Transpose(M_0).M_0+e*(Transpose(R_1).Transpose(M_0)\
.M_0+Transpose(M_1).(M_0)+Transpose(M_0).M_1+Transpose(M_0).(M_0)\
.(R_1)));

r_11:=simplify(solve(MR1[1,2]=0,r_11));
r_12:=simplify(solve(MR1[1,3]=0,r_12));
r_13:=simplify(solve(MR1[2,3]=0,r_13));

Me1:=simplify(M_0+e*(L_1.M_0+M_1+M_0.R_1));

L2:=-L_1.Transpose(L_1);

L_2:=1/2*Matrix([[L2[1,1], 0, 0], [0, L2[2,2], 0], [0, 0, L2[3,3]\
]])+Matrix([[0,L2[1,2]+l_21,L2[1,3]+l_22],[-l_21,0,L2[2,3]+l_23],\
[-l_22,-l_23,0]]);

simplify(L_2+Transpose(L_2)+L_1.Transpose(L_1));
ML2:=simplify(simplify(ML1)+e^2*(L_2.M_0.Transpose(M_0)+L_1.M_1.T\
ranspose(M_0)+L_1.M_0.Transpose(M_1)+L_1.M_0.Transpose(M_0).Trans\
pose(L_1)+M_1.Transpose(M_1)+M_1.Transpose(M_0).Transpose(L_1)+M_\
0.Transpose(M_0).Transpose(L_2)+M_0.Transpose(M_1).Transpose(L_1)\
));

l_21:=simplify(solve(ML2[1,2]=0,l_21));
l_22:=simplify(solve(ML2[1,3]=0,l_22));
l_23:=simplify(solve(ML2[2,3]=0,l_23));

R2:=-R_1.Transpose(R_1);

```

```

R_2:=1/2*Matrix([[R2[1,1], 0, 0], [0, R2[2,2], 0], [0, 0, R2[3,3]\
]])+Matrix([[0,R2[1,2]+r_21,R2[1,3]+r_22], [-r_21,0,R2[2,3]+r_23],\
[-r_22,-r_23,0]]);

R_2+Transpose(R_2)+R_1.Transpose(R_1);
MR2:=simplify(simplify(MR1)+e^2*(Transpose(R_2).Transpose(M_0).M_0\
+Transpose(R_1).Transpose(M_1).M_0+Transpose(R_1).Transpose(M_0)\
.M_1+Transpose(R_1).Transpose(M_0).M_0.R_1+Transpose(M_1).M_1+Tra\
nspose(M_1).M_0.R_1+Transpose(M_0).M_1.R_1+Transpose(M_0).M_0.R_2\
));

r_21:=simplify(solve(MR2[1,2]=0,r_21));
r_22:=simplify(solve(MR2[1,3]=0,r_22));
r_23:=simplify(solve(MR2[2,3]=0,r_23));

Me2:=simplify(Me1+e^2*(L_2.M_0+L_1.M_1+L_1.M_0.R_1+M_1.R_1+M_0.R_1\
2));

L3:=simplify(-Transpose(L_2).(L_1)-Transpose(L_1).(L_2));

L_3:=1/2*Matrix([[L3[1,1], 0, 0], [0, L3[2,2], 0], [0, 0, L3[3,3]\
]])+Matrix([[0,L3[1,2]+l_31,L3[1,3]+l_32], [-l_31,0,L3[2,3]+l_33],\
[-l_32,-l_33,0]]);

simplify(L_3+Transpose(L_3)+Transpose(L_2).(L_1)+Transpose(L_1).(L_2));
ML3:=simplify((L_3.M_0.Transpose(M_0)+L_2.M_1.Transpose(M_0)+L_2.M_0\
.Transpose(M_1)+L_2.M_0.Transpose(M_0).Transpose(L_1)+L_1.M_1.\
Transpose(M_1)+L_1.M_1.Transpose(M_0).Transpose(L_1)+L_1.M_0.Tran\
spose(M_1).Transpose(L_1)+L_1.M_0.Transpose(M_0).Transpose(L_2)+M_0\
.Transpose(M_0).Transpose(L_3)+M_1.Transpose(M_1).Transpose(L_1)\
)+M_0.Transpose(M_1).Transpose(L_2)+M_1.Transpose(M_0).Transpose(L_2));

l_31:=simplify(solve(ML3[1,2]=0,l_31));
l_32:=simplify(solve(ML3[1,3]=0,l_32));
l_33:=simplify(solve(ML3[2,3]=0,l_33));

R3:=-Transpose(R_2).R_1-Transpose(R_1).R_2;

R_3:=1/2*Matrix([[R3[1,1], 0, 0], [0, R3[2,2], 0], [0, 0, R3[3,3]\
]])+Matrix([[0,R3[1,2]+r_31,R3[1,3]+r_32], [-r_31,0,R3[2,3]+r_33],\
[-r_32,-r_33,0]]);

R_3+Transpose(R_3)+Transpose(R_2).R_1+Transpose(R_1).R_2;
MR3:=simplify(Transpose(R_3).Transpose(M_0).M_0+Transpose(R_2).Tr\
anspose(M_1).M_0+Transpose(R_2).Transpose(M_0).M_1+Transpose(R_2)\
.Transpose(M_0).M_0.R_1+Transpose(R_1).Transpose(M_1).M_1+Transpo\
se(R_1).Transpose(M_1).M_0.R_1+Transpose(R_1).Transpose(M_0).M_1.\

```

```

R_1+Transpose(R_1).Transpose(M_0).M_0.R_2+Transpose(M_0).M_0.R_3+\
Transpose(M_1).M_1.R_1+Transpose(M_0).M_1.R_2+Transpose(M_1).M_0.\
R_2);

r_31:=simplify(solve(MR3[1,2]=0,r_31));
r_32:=simplify(solve(MR3[1,3]=0,r_32));
r_33:=simplify(solve(MR3[2,3]=0,r_33));

Me3:=simplify(Me2+e^3*(M_0.R_3+L_1.M_0.R_2+L_1.M_1.R_1+L_2.M_0.R_\
1+L_2.M_1+L_3.M_0+M_1.R_2));
Le3:=simplify(Matrix([[1,0,0],[0,1,0],[0,0,1]])+e*L_1+e^2*L_2+e^3\
*L_3);
Re3:=simplify(Matrix([[1,0,0],[0,1,0],[0,0,1]])+e*R_1+e^2*R_2+e^3\
*R_3);

```

C Program for the gluon fusion cross section with 4 fermion composite Higgs model

This is the program described in section 4.4. At the beginning of the file there is a list with the various parameters, which is not included in order to save trees. At the end there is the 'fprintf' command which generates a file with all chosen data in it. For the plots in section 4.5 we also put the matrix elements of L and R in the output file.

```

Datalist:=[[.,.,.],...]:
f := 500;
v := 174;
c[alpha] := evalf(sqrt(f^2-2*v^2)/f);
e := v/f;
mh:=120;
conv:=3.894*10^8;
sigmasm:= 19.24933;
sig:=sigmasm*c[alpha]^2;
integ:=0.02770763;
alphas:=evalf(0.1176/(1-23/(6*Pi)*0.1176*log(91.187/mh)));
con:=evalf(c[alpha]^2*alphas^2*(1/mh)^2/(32*Pi));

for n from 1 to 194 do
N := n;
b := true;
Ra := Datalist[N, 3];
s[L] := Datalist[N, 1];
s[R] := Datalist[N, 2];
c[R] := sqrt(1-s[R]^2);
c[L] := sqrt(1-s[L]^2);
m[T] := m[psi]*Ra;
mu := (Ra-1)*m[psi]/f;
M0 := Matrix([[0, 0, 0, 0], [0, m[psi]/c[L], 0, 0], [0, 0, m[psi]\
], 0], [0, 0, 0, m[T]/c[R]]);
M1 := Matrix([[s[R]*s[L]*c[alpha]*f*mu, -s[L]*v*mu, -s[L]*v*mu, \

```

```

-s[L]*c[R]*c[alpha]*f*mu, [-s[R]*c[L]*c[alpha]*f*mu, c[L]*v*mu,\
  c[L]*v*mu, c[L]*c[R]*c[alpha]*f*mu, [-s[R]*c[alpha]*f*mu, v*mu\
, v*mu, c[R]*c[alpha]*f*mu, [2*s[R]*v*mu, c[alpha]*f*mu, c[alph\
a]*f*mu, -2*c[R]*v*mu]]);
M := M0+e*M1;
MMT := evalf(M.Transpose(M))/m[psi]^2;
JJ := evalf(Eigenvalues(MMT));
KK := sort([Re(sqrt(JJ[1])), Re(sqrt(JJ[2])), Re(sqrt(JJ[3])), R\
e(sqrt(JJ[4]))]);
m[psi] := solve(KK[1]*m[psi] = 172.4, m[psi]);
mt := KK[1]*m[psi];
mX := KK[2]*m[psi];
mT := KK[3]*m[psi];
mQ := KK[4]*m[psi];
LL := Matrix([[0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0\
, 0]]);
if Re(sqrt(JJ[1]))*m[psi] = mt then LL[1, 1] := 1; LL[2, 1] := 0;
  LL[3, 1] := 0; LL[4, 1] := 0 elif Re(sqrt(JJ[1]))*m[psi] = mQ t\
hen LL[1, 1] := 0; LL[2, 1] := 1; LL[3, 1] := 0; LL[4, 1] := 0 e\
lif Re(sqrt(JJ[1]))*m[psi] = mX then LL[1, 1] := 0; LL[2, 1] := \
0; LL[3, 1] := 1; LL[4, 1] := 0 else LL[1, 1] := 0; LL[2, 1] := \
0; LL[3, 1] := 0; LL[4, 1] := 1 end if;
if Re(sqrt(JJ[2]))*m[psi] = mt then LL[1, 2] := 1; LL[2, 2] := 0\
; LL[3, 2] := 0; LL[4, 2] := 0 elif Re(sqrt(JJ[2]))*m[psi] = mQ \
then LL[1, 2] := 0; LL[2, 2] := 1; LL[3, 2] := 0; LL[4, 2] := 0 \
elif Re(sqrt(JJ[2]))*m[psi] = mX then LL[1, 2] := 0; LL[2, 2] :=\
0; LL[3, 2] := 1; LL[4, 2] := 0 else LL[1, 2] := 0; LL[2, 2] :=\
0; LL[3, 2] := 0; LL[4, 2] := 1 end if;
if Re(sqrt(JJ[3]))*m[psi] = mt then LL[1, 3] := 1; LL[2, 3] := 0\
; LL[3, 3] := 0; LL[4, 3] := 0 elif Re(sqrt(JJ[3]))*m[psi] = mQ \
then LL[1, 3] := 0; LL[2, 3] := 1; LL[3, 3] := 0; LL[4, 3] := 0 \
elif Re(sqrt(JJ[3]))*m[psi] = mX then LL[1, 3] := 0; LL[2, 3] :=\
0; LL[3, 3] := 1; LL[4, 3] := 0 else LL[1, 3] := 0; LL[2, 3] :=\
0; LL[3, 3] := 0; LL[4, 3] := 1 end if;
if Re(sqrt(JJ[4]))*m[psi] = mt then LL[1, 4] := 1; LL[2, 4] := 0\
; LL[3, 4] := 0; LL[4, 4] := 0 elif Re(sqrt(JJ[4]))*m[psi] = mQ \
then LL[1, 4] := 0; LL[2, 4] := 1; LL[3, 4] := 0; LL[4, 4] := 0 \
elif Re(sqrt(JJ[4]))*m[psi] = mX then LL[1, 4] := 0; LL[2, 4] :=\
0; LL[3, 4] := 1; LL[4, 4] := 0 else LL[1, 4] := 0; LL[2, 4] :=\
0; LL[3, 4] := 0; LL[4, 4] := 1 end if;
MTM := evalf(Transpose(M).M)/m[psi]^2;
PP := evalf(Eigenvalues(MTM));
QQ := sort([Re(sqrt(PP[1])), Re(sqrt(PP[2])), Re(sqrt(PP[3])), R\
e(sqrt(PP[4]))]);
mt := QQ[1]*m[psi];
mX := QQ[2]*m[psi];
mT := QQ[3]*m[psi];
mQ := QQ[4]*m[psi];
RR := Matrix([[0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0\
, 0]]);

```

```

if Re(sqrt(PP[1]))*m[psi] = mt then RR[1, 1] := 1; RR[2, 1] := 0;
  RR[3, 1] := 0; RR[4, 1] := 0 elif Re(sqrt(PP[1]))*m[psi] = mQ \
then RR[1, 1] := 0; RR[2, 1] := 1; RR[3, 1] := 0; RR[4, 1] := 0 \
elif Re(sqrt(PP[1]))*m[psi] = mX then RR[1, 1] := 0; RR[2, 1] :=\
  0; RR[3, 1] := 1; RR[4, 1] := 0 else RR[1, 1] := 0; RR[2, 1] :=\
  0; RR[3, 1] := 0; RR[4, 1] := 1 end if;
if Re(sqrt(PP[2]))*m[psi] = mt then RR[1, 2] := 1; RR[2, 2] := 0 \
; RR[3, 2] := 0; RR[4, 2] := 0 elif Re(sqrt(PP[2]))*m[psi] = mQ \
then RR[1, 2] := 0; RR[2, 2] := 1; RR[3, 2] := 0; RR[4, 2] := 0 \
elif Re(sqrt(PP[2]))*m[psi] = mX then RR[1, 2] := 0; RR[2, 2] :=\
  0; RR[3, 2] := 1; RR[4, 2] := 0 else RR[1, 2] := 0; RR[2, 2] :=\
  0; RR[3, 2] := 0; RR[4, 2] := 1 end if;
if Re(sqrt(PP[3]))*m[psi] = mt then RR[1, 3] := 1; RR[2, 3] := \
0; RR[3, 3] := 0; RR[4, 3] := 0 elif Re(sqrt(PP[3]))*m[psi] = mQ \
then RR[1, 3] := 0; RR[2, 3] := 1; RR[3, 3] := 0; RR[4, 3] := 0 \
elif Re(sqrt(PP[3]))*m[psi] = mX then RR[1, 3] := 0; RR[2, 3] := \
= 0; RR[3, 3] := 1; RR[4, 3] := 0 else RR[1, 3] := 0; RR[2, 3] := \
= 0; RR[3, 3] := 0; RR[4, 3] := 1 end if;
if Re(sqrt(PP[4]))*m[psi] = mt then RR[1, 4] := 1; RR[2, 4] := 0 \
; RR[3, 4] := 0; RR[4, 4] := 0 elif Re(sqrt(PP[4]))*m[psi] = mQ \
then RR[1, 4] := 0; RR[2, 4] := 1; RR[3, 4] := 0; RR[4, 4] := 0 \
elif Re(sqrt(PP[4]))*m[psi] = mX then RR[1, 4] := 0; RR[2, 4] :=\
  0; RR[3, 4] := 1; RR[4, 4] := 0 else RR[1, 4] := 0; RR[2, 4] :=\
  0; RR[3, 4] := 0; RR[4, 4] := 1 end if;
E[L], V[L] := evalf(EigenVectors(MMT)); E[R], V[R] := evalf(Eige\
n\ectors(MTM)); U[L] := V[L].Transpose(LL); U[R] := V[R].Transpo\
se(RR); U[L].LL.Matrix([[JJ[1], 0, 0, 0], [0, JJ[2], 0, 0], [0, \
0, JJ[3], 0], [0, 0, 0, JJ[4]]]).Transpose(LL).Transpose(U[L])-M\
MT;
U[R].RR.Matrix([[KK[1], 0, 0, 0], [0, KK[2], 0, 0], [0, 0, KK[3]\
, 0], [0, 0, 0, KK[4]]]).Transpose(RR).Transpose(U[R])-MTM;
prerot := Matrix([[-1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0,\
0, 0, -1]]);
Diag := Transpose(U[L]).M.U[R];
if Diag[1, 1] < 0 then lp1 := -1 else lp1 := 1 end if;
if Diag[2, 2] < 0 then lp2 := -1 else lp2 := 1 end if;
if Diag[3, 3] < 0 then lp3 := -1 else lp3 := 1 end if;
if Diag[4, 4] < 0 then lp4 := -1 else lp4 := 1 end if;
Lp := Matrix([[lp1, 0, 0, 0], [0, lp2, 0, 0], [0, 0, lp3, 0], [0\
, 0, 0, lp4]]);
UL := Lp.Transpose(U[L]);
diag := UL.M.U[R];
if diag[1, 1]-mt > .1 then b := false end if;
if diag[2, 2]-mQ > .1 then b := false end if;
if diag[3, 3]-mX > .1 then b := false end if;
if diag[4, 4]-mT > .1 then b := false end if;
  b;
rho := c[alpha]*f-2*v^2/(f*c[alpha]);
H := Matrix(4, 4, {(1, 1) = mu*s[R]*s[L]*rho/f, (1, 2) = -2*s[L]\
*v*mu/f, (1, 3) = -2*s[L]*v*mu/f, (1, 4) = -mu*s[L]*c[R]*rho/f, \

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(2, 1) = -mu*s[R]*c[L]*rho/f, (2, 2) = 2*c[L]*v*mu/f, (2, 3) = 2\
*c[L]*v*mu/f, (2, 4) = mu*c[R]*c[L]*rho/f, (3, 1) = -mu*s[R]*rho\
/f, (3, 2) = 2*v*mu/f, (3, 3) = 2*v*mu/f, (3, 4) = mu*c[R]*rho/f\
, (4, 1) = 4*s[R]*v*mu/f, (4, 2) = mu*rho/f, (4, 3) = mu*rho/f, \
(4, 4) = -4*c[R]*v*mu/f});
C := UL.H.U[R];
for j to 4 do
mass := [mt, mQ, mX, mT];
tau := 4*(mass[j]/mh)^2; if tau < 1 then ftau := -(1/4)*(log((1+\
sqrt(1-tau))/(1-sqrt(1-tau)))-i*Pi)^2 else ftau := arcsin(1/sqrt\
(tau))^2 end if; ta[j] := tau;
fta[j] := ftau;
end do;
sigma := con*conv*integ*((Re(C[1, 1])*mt*(1+(1-ta[1])*fta[1])+Re\
(C[2, 2])*mQ*(1+(1-ta[2])*fta[2])+Re(C[3, 3])*mX*(1+(1-ta[3])*ft\
a[3])+Re(C[4, 4])*mT*(1+(1-ta[4])*fta[4]))^2+(Re(C[1, 1])*mt*(1-\
ta[1])*Im(fta[1])+Re(C[2, 2])*mQ*(1-ta[2])*Im(fta[2])+Re(C[3, 3]\
)*mX*(1-ta[3])*Im(fta[3])+Re(C[4, 4])*mT*(1-ta[4])*Im(fta[4]))^2);
fd := fopen("stuff", APPEND);
fprintf(fd, "%d %e %e %e %e %e %e %e %e %e %e %e %e\n", N, s[\
L], s[R], Ra, mt, mQ, mX, mT, Re(C[1, 1]), Re(C[2, 2]), Re(C[3, \
3]), Re(C[4, 4]), sigma, sigma/sigmaasm);
fclose(fd);
unassign('m[psi]')
end do:

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