Solid State Theory
Exercise 12

Exercise 12.1 Critical temperature in the Stoner model

We consider three types of dispersion relations:

- $\epsilon_{\vec{k}} = \epsilon_0 \pm \frac{\hbar^2 \vec{k}^2}{2m}$ (3D) and
- $\epsilon_k = \epsilon_0 + \alpha k$ (1D).

Plot the critical temperature of the Stoner model for fixed interaction strength U depending on the chemical potential μ.

Exercise 12.2 Stoner instability

In the lecture, it was shown that a system described by the mean-field Hamiltonian

$$\mathcal{H}_{\mathrm{MF}} = \frac{1}{\Omega} \sum_{\vec{k},s} (\epsilon_{\vec{k}} + U n_{-s}) c^{\dagger}_{\vec{k}s} c_{\vec{k}s} - U n_{\uparrow} n_{\downarrow} \tag{1}$$

shows an instability towards a magnetically ordered state at $N(\epsilon_F)U_C = 1$ (note that here, $N(\epsilon)$ is the density of states per spin).
Show for the case of a parabolic dispersion and $T = 0$ that there are actually three distinct states:

- a paramagnetic state: $N(\epsilon_F)U < 1$,
- an imperfect ferromagnetic state: $3/2^{4/3} > N(\epsilon_F)U > 1$ and
- a perfect ferromagnetic state: $N(\epsilon_F)U > 3/2^{4/3}$.

Hint: Introduce a variable for the magnitude of the polarization

$$\frac{N_{\uparrow}}{N_e} = \frac{1}{2}(1+x) \qquad \frac{N_{\downarrow}}{N_e} = \frac{1}{2}(1-x) \tag{2}$$

where $N_{\uparrow(\downarrow)}$ is the total number of up-spins (down-spins) and N_e is the total number of electrons. Minimize the total energy of the system with respect to x.
Plot the polarization of the system x as a function of $N(\epsilon_F)U$.

Exercise 12.3 Particle-Hole Excitations in Itinerant Ferromagnets

In section 6.3 of the lecture notes the low-energy spectrum of (magnons) spin-waves in itinerant ferromagnets was derived. It is crucial for the existence of well-defined magnons that elementary particle-hole excitations are gapped. Try to explain (without detailed calculations) why there is such a gap and why it is important for the observability of magnons!