Exercises for "Phenomenology of Particle Physics II"

Prof. Dr. A. Gehrmann, Prof. Dr. U. D. Straumann
M. Ritzmann

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In this exercise, we are going to investigate the phase space corresponding to the decay of a massive particle with four-vector q and mass $\sqrt{M^2}$ into three massless particles in d dimensions.

Exercise 13

In the lecture, it was shown that

$$\int dR_3^{(d)}(M,p_1,p_2,p_3) = \frac{1}{4}\int dE_1 dE_2 d\theta_{12} d\Omega_{d-1} d\Omega_{d-2}\left(E_1 E_2 \sin(\theta_{12})\right)^{d-3}\delta(p_3^2)$$

with θ_{12}, $p_3 = q - p_1 - p_2$ the angle between the particles 1 and 2, $\int d\Omega_d = 2\pi^{d/2}/\Gamma(d/2)$. Change variables to the dimensionless $y_{ij} = \frac{s_{ij}}{M^2}$ and show that

$$\int dR_3^{(d)}(M,p_1,p_2,p_3) =$$
$$\frac{1}{4}\frac{1}{2^{d-1}}\int (y_{12}y_{13}y_{23})^{\frac{d-4}{2}} d\Omega_{d-1} d\Omega_{d-2}\delta(y_{12}+y_{13}+y_{23}-1)\left(M^2\right)^{d-3} dy_{12}dy_{13}dy_{23}$$

Hint: consider the center-of-mass frame in which $q^\mu = (M,0,0,0,\dots)$, $p_1^\mu = (E_1,E_1,0,0,\dots)$, $p_2^\mu = (E_2, E_2\cos\theta_{12}, E_2\sin\theta_{12},\dots)$. Using these explicit vectors, show first $s_{12}s_{13}s_{23} = 4M^2E_1^2E_2^2\sin^2\theta_{12}$, then calculate the determinant of $\frac{\partial s_{ij}}{\partial E_k/\theta_{12}}$ to show $dE_1dE_2d\theta_{12} = (16M^2 s_{12}s_{13}s_{23})^{1/2} ds_{12}ds_{13}ds_{23}$ (the vectors are d-dimensional).

Exercise 14

Use the parameterization of the three-particle phase space in the preceding exercise to calculate the integral of $S_{132} = \frac{s_{12}}{s_{13}s_{23}}$ over $0 < y_{13} < y_{min}, 0 < y_{23} < y_{min}$. In doing so, keep in mind $y_{min} \ll 1$ and neglect terms proportional to $y_{min}^a, a \geq 1$. Then calculate the Laurent series of your result around $\epsilon = 0$ up to order ϵ^0 ($\epsilon = -\frac{d-4}{2}$). (Consider only the y-integral for the sake of this exercise)