Institute for Theoretical Physics, ETH Zürich FS 2009

Exercises for "Phenomenology of Particle Physics II"

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Exercise 10 IR- and Collinear-safe Observables

The following event variables are used in practice:

Thrust

$$T_m = \max_{\vec{n}} \frac{\sum_{i=1}^{m} |\vec{p}_i \cdot \vec{n}|}{\sum_{i=1}^{m} |\vec{p}_i|}$$

C-Parameter

$$C_m = 3\left(\Theta_{11}\Theta_{22} + \Theta_{22}\Theta_{33} + \Theta_{33}\Theta_{11} - \Theta_{12}\Theta_{12} - \Theta_{23}\Theta_{23} - \Theta_{31}\Theta_{31}\right)$$

where

$$\Theta_{jk} = \frac{1}{\sum_{i=1}^{m} |\vec{p}_i|} \cdot \sum_{i=1}^{m} \frac{p_i^j p_i^k}{|\vec{p}_i|}$$

where p^j denotes the j-th component of the momentum $\vec{p}$.

Sphericity

$$S_m = \frac{3}{2} \min_{\vec{n}} \frac{\sum_{i=1}^{m} \left(\vec{p_i}^T\right)^2}{\sum_{i=1}^{m} \vec{p}_i^2}$$

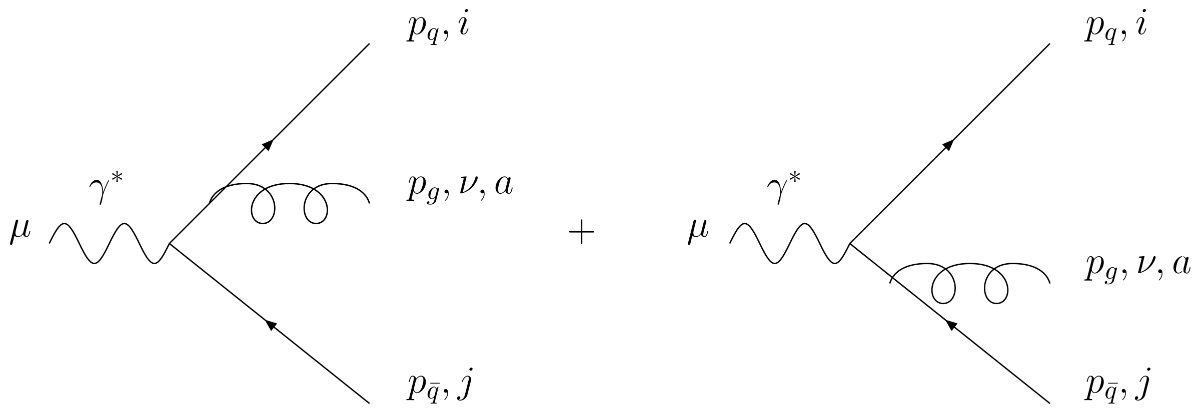
where $\vec{p_i}^T$ denotes the component of $\vec{p_i}$ orthogonal to $\vec{n}$.

Check whether these variables are infrared and collinear safe, means for $\varphi = T_m, C_m, S_m$

- collinear $\vartheta_n(p_1, p_2, \ldots, p_n) \xrightarrow{p_2 /\!/ p_1} \vartheta_{n-1}(p_1 + p_2, \ldots, p_n)$
- infrared $\vartheta_n(p_1, p_2, \ldots, p_{n-1}, p_n) \xrightarrow{E_n \to 0} \vartheta_{n-1}(p_1, p_2, \ldots, p_{n-1})$.

Exercise 11

Calculate the norm squared of the matrix element for $\gamma^* \to q\bar{q}g$ (summed over final states) in the limit of vanishing quark mass. Use $s_{ij} = 2p_ip_j (= (p_i + p_j)^2)$ as the variables.



- Read off the amplitudes from the two Feynman diagrams.
- Write down the two squared amplitudes and the interference terms.
- Use the following replacements for the polarisation sums of the photon and the gluon:

$$\sum_\lambda \epsilon_\mu(k,\lambda)\epsilon^*_\nu(k,\lambda) \to -g_{\mu\nu}$$

  Warning: In general you have to include the emission of ghosts except if you work in a physical gauge (for example the axial gauge).

- The fermionic traces can be written as a sum of f's and g's, where f and g are defined as follows:

$$\begin{aligned} f(a,b,c,d) &= \mathrm{Tr}\,(\not{a}\gamma^\mu \not{b}\gamma^\nu \not{c}\gamma_\nu \not{d}\gamma_\mu) = 16(a{\cdot}b\ c{\cdot}d - a{\cdot}c\ b{\cdot}d + a{\cdot}d\ b{\cdot}c) \\ g &= \mathrm{Tr}\,(\not{a}\gamma^\mu \not{b}\gamma^\nu \not{c}\gamma_\mu \not{d}\gamma_\nu) = -32 a{\cdot}c\ b{\cdot}d, \end{aligned}$$

  note that f vanishes if two massless momenta are next to each other:

$$f(a,a,c,d) = f(b,a,a,c) = f(b,c,a,a) = f(a,b,c,a) = 0, \qquad \text{if} \quad a^2 = 0$$

– please turn over –

Exercise 12 Limits for Soft or Collinear Particles

Investigate the limit of the matrix element norm squared in the preceding exercise

$$|M(\gamma^* \to q\bar{q}g)|^2 = 64\pi^2(e_q^2\alpha\alpha_s)\frac{N_C^2-1}{2}\left(\frac{s_{qg}}{s_{\bar{q}g}} + \frac{s_{\bar{q}g}}{s_{qg}} + 2\frac{s_{q\bar{q}}\left(s_{q\bar{q}} + s_{qg} + s_{\bar{q}g}\right)}{s_{qg}s_{\bar{q}g}}\right)$$

in the collinear limit

$$s_{qg} \to 0 \qquad p_q \to zP_{qg} \qquad p_g \to (1-z)P_{qg}.$$

and in the soft limit $s_{qg} \to 0, s_{\bar{q}g} \to 0$.