Exercises for "Phenomenology of Particle Physics II"

Prof. Dr. A. Gehrmann, Prof. Dr. U. D. Straumann sheet 3 handed out: 11.3.2009
M. Ritzmann handed in: 18.3.2009
http://www.itp.phys.ethz.ch/education/lectures_fs09/PPPII returned: 25.3.2009

Exercise 5 Higgs-strahlung

In Exercise 4 we used the covariant derivative $D_\mu = \partial_\mu - igT^a W^a_\mu - ig' \frac{Y}{2} B_\mu$ with $Y = 1$ and $T^a = \frac{1}{2}\sigma^a$ to find the physical gauge bosons and their couplings to the higgs. Now we include (multiplets of) fermions. Our Lagrangian should have a term

$$\overline{\psi} i D^\mu \gamma_\mu \psi.$$

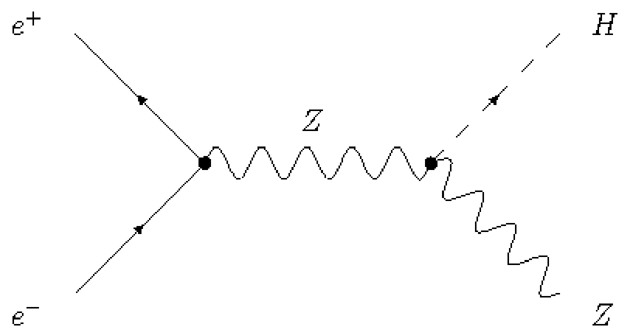
The multiplet of dirac fields is in some representation of our gauge groups $SU(2)_W$ and $U(1)_Y$, for this exercise we only need to consider T^3 and Y. The dirac term does then read

$$\overline{\psi}(gT^3 W^3_\mu + g' \frac{Y}{2} B_\mu)\gamma^\mu \psi.$$

From the requirement that the massless physical gauge boson should be the photon, i.e. the QED interaction term $-eQ\overline{\psi}A^\mu \gamma_\mu \psi$ should be reproduced for every fermion in the multiplet, determine the Z-boson-fermion coupling term eliminating Y in favor of the electromagnetic charge Q (assume that T^3 is diagonal and denote its value for a given fermion by I^3_f). The rotation to mass eigenstates we used in the preceding exercise was

$$\begin{aligned} Z_\mu &= \frac{gW^3_\mu - g' B_\mu}{\sqrt{g^2 + g'^2}} \\ A_\mu &= \frac{g' W^3_\mu + g B_\mu}{\sqrt{g^2 + g'^2}}. \end{aligned}$$

In the standard model, the right-handed fermions are assigned $T^a = 0$ (they are singlets), for the left-handed leptons we have $T^a = \sigma^a/2$ for the doublet $\binom{\nu_e}{e_L}$. We want to calculate the production of a Z together with a Higgs in electron-positron annihilation. The relevant diagram at tree level is



We consider the electron as massless throughout this exercise. From the preceding exercise we know that the Feynman rule for a $Z\mu Z\nu H$ interaction is $i\frac{g^2+g'^2 v}{2} g_{\mu\nu}$ (a factor of two enters because of symmetry). The propagator of the Z is given by

$$-i\frac{g^{\mu\nu} - p^\mu p^\nu / M^2}{p^2 - M^2 + i\epsilon}.$$

Use the following momenta and polarisation vectors in the center-of-mass frame: $p^\mu_\pm = E(1, 0, 0, \pm 1)$ for the incoming electron/positron, $k_{Z,H} = E(1 \pm \frac{M_Z^2 - M_H^2)}{s}, \pm\beta \sin\theta, 0, \pm\beta\cos\theta)$ for the momenta of Z and H, (calculate β, E is the beam energy and $s = 4E^2$ the center-of-masss energy squared, θ is the scattering angle between e^+ and Z), the polarisation vectors for the Z are given by $\epsilon^{*\mu}(\pm) = \frac{1}{\sqrt{2}}(0, \cos\theta, \mp i, -\sin\theta)$, $\epsilon^{*\mu}(0) = \frac{E}{M_Z}(\beta, \frac{s+M_Z^2-M_H^2}{s}\sin\theta, 0, \frac{s+M_Z^2-M_H^2}{s}\cos\theta)$.
Use $\frac{1\pm\gamma_5}{2}$ to project onto one helicity, don't insert the couplings but keep them symbolic, mind the following trace identities including γ^5: $\mathrm{Tr}(\gamma^5) = \mathrm{Tr}(\gamma^\mu\gamma^5) = \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^5) = 0$, $\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5) = -4i\epsilon^{\mu\nu\rho\sigma}$.