Maxwell's Demon

Bennett's argument for reconciliation using Landauer's erasure principle.

Proseminar 2009

Robin Jäger

Topics

- Introduction
- Landauer's Erasure Principle
    - Reasons for Irreversible Computation
    - Basic Storage Devices
    - Irreversible Computation
    - 2^{nd} Law of Thermodynamics
    - Logical Irreversibility and Entropy Generation
- Bennett's Arguments
    - Maxwell's Demons
    - Reversible Copying
    - Reversible Measurement
    - Restoring the Demons memory
- Summary

Landauer's Erasure Principle

“Erasure of information increases entropy”

or

“Logically irreversible computation is dissipative”

($\delta W \sim k_B T$ per erased bit)

Why irreversible computing?

- Landauer's Definition of a Computer:

  N binary elements, 2^N possible states

$$N \rightarrow N : n' = f(n_1, n_2), f \in \{ \text{ID, AND, OR, XOR, RESET, ...} \}$$

  - Some are irreversible, e.g. erasure

Claim: logical irreversibility implies physical irreversibility

Basic Storage Devices

- Bistable potential well

$$U = \frac{f}{2} N k_B T \Rightarrow V(0) \gg k_B T$$

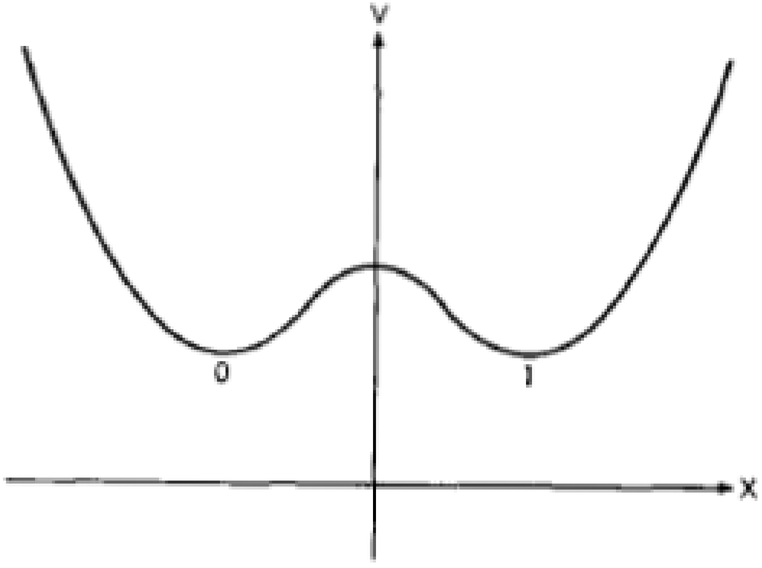

Figure 1 **Bistable potential well.**
x is a generalized coordinate representing quantity which is switched.

Irreversible Computation

- "RESTORE TO ONE" as basic example $f(0)=f(1)=1$
  - 2 routines: measure – operate – no dissipation of energy
  - 1 routine: F(t) should implement the function

Liouville's Theorem (for Hamiltonian Systems):

$$\frac{d\rho(q,p)}{dt} = 0$$

logical irreversibility → physical irreversibility

hence a damped system is required! → heat generation

2nd Law of Thermodynamics

"In a system, a process that occurs will tend to increase the total entropy of the universe."

$$dS = \frac{\delta Q}{T} + \frac{\delta W}{T}$$

or

$$\frac{dS}{dt} \geq 0$$

Logical Irreversibility and Entropy Generation

- Thermal relaxation: randomize stored data

Entropy

$$S \stackrel{\text{def}}{=} -k_B \sum_i p_i \ln(p_i)$$

$$H \stackrel{\text{def}}{=} -\sum_i p_i \ln(p_i)$$

$$S=0 \rightarrow S=k_B \ln(2) \approx 0.6931 k_B$$

$$H=0 \rightarrow H=\ln(2)$$

- Reverse process of thermalization: “RESTORE TO ONE”

2nd Law of thermodynamics: Entropy of a closed system cannot decrease → heat generation

$$\Delta Q = \Delta S \cdot T = k_B T \ln(2)$$

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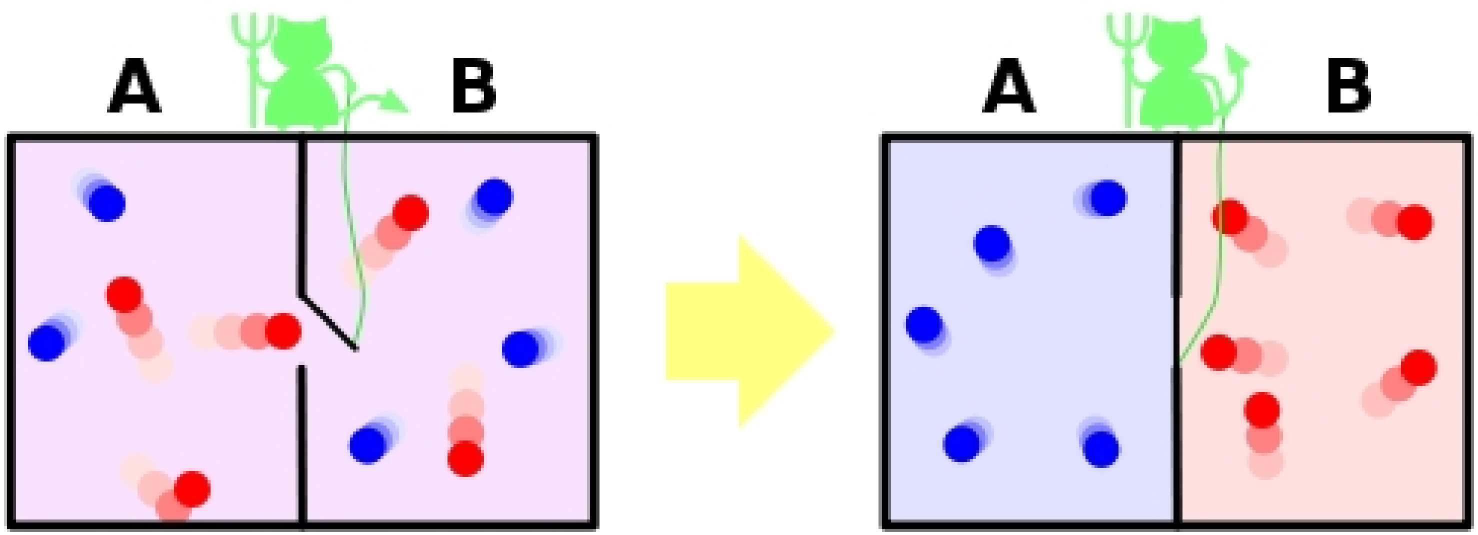
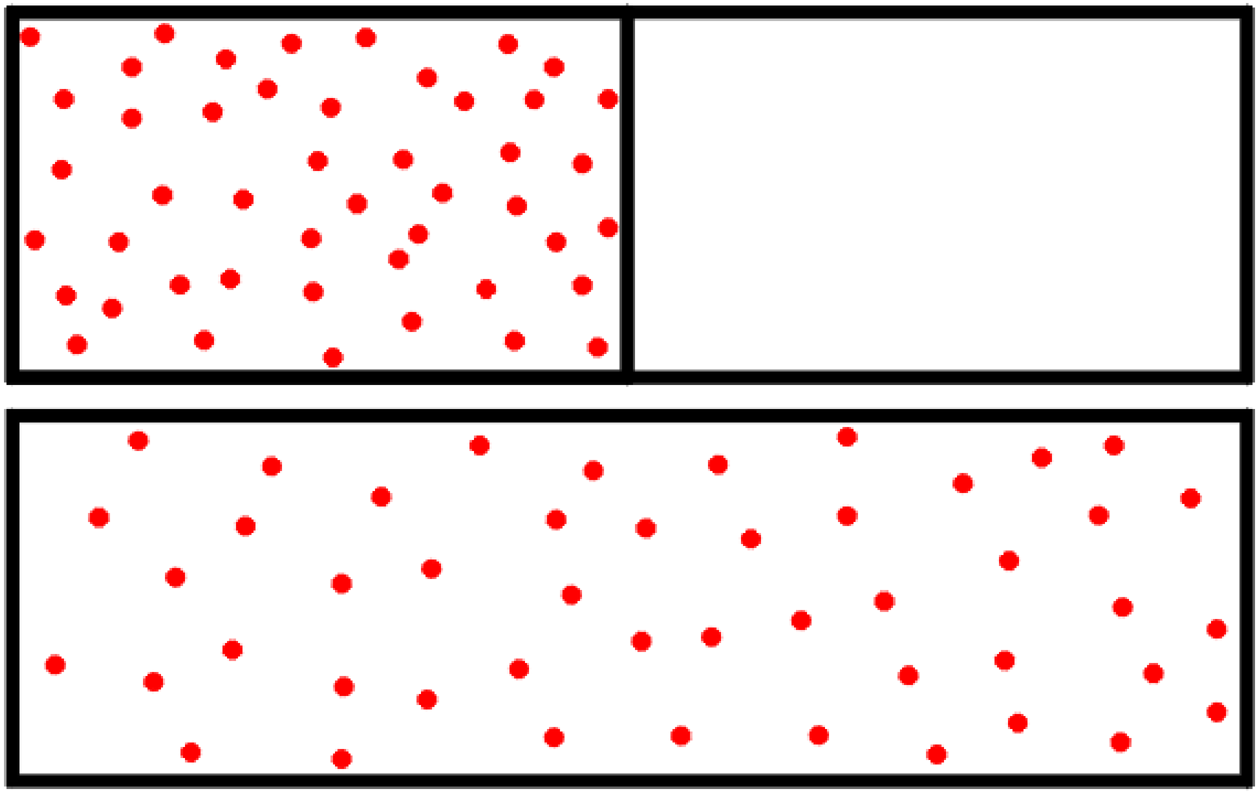
Maxwell's Demons



Adiabatic Expansion by Gay-Lussac

Reverse process of Maxwell's Demon



$$\Delta S = k_B N \ln(2)$$

Reversible Measurement and Maxwell's Demon

Why does the Demon not work?

Quote: ”It is often supposed that *measurement* is an unavoidably irreversible act, requiring an entropy generation of at least kln2 per bit of information obtained, and that this is what prevents the demon from violating the second law.”

Quote: ”... attribute the entropy cost to logical irreversibility, rather than to measurement ...”

A Simple Maxwell Demon

Entropy decrease:

$\Delta S = -N k_B \ln(2)$

Work done:

$\delta W = \Delta S \cdot T = N T k_B \ln(2)$

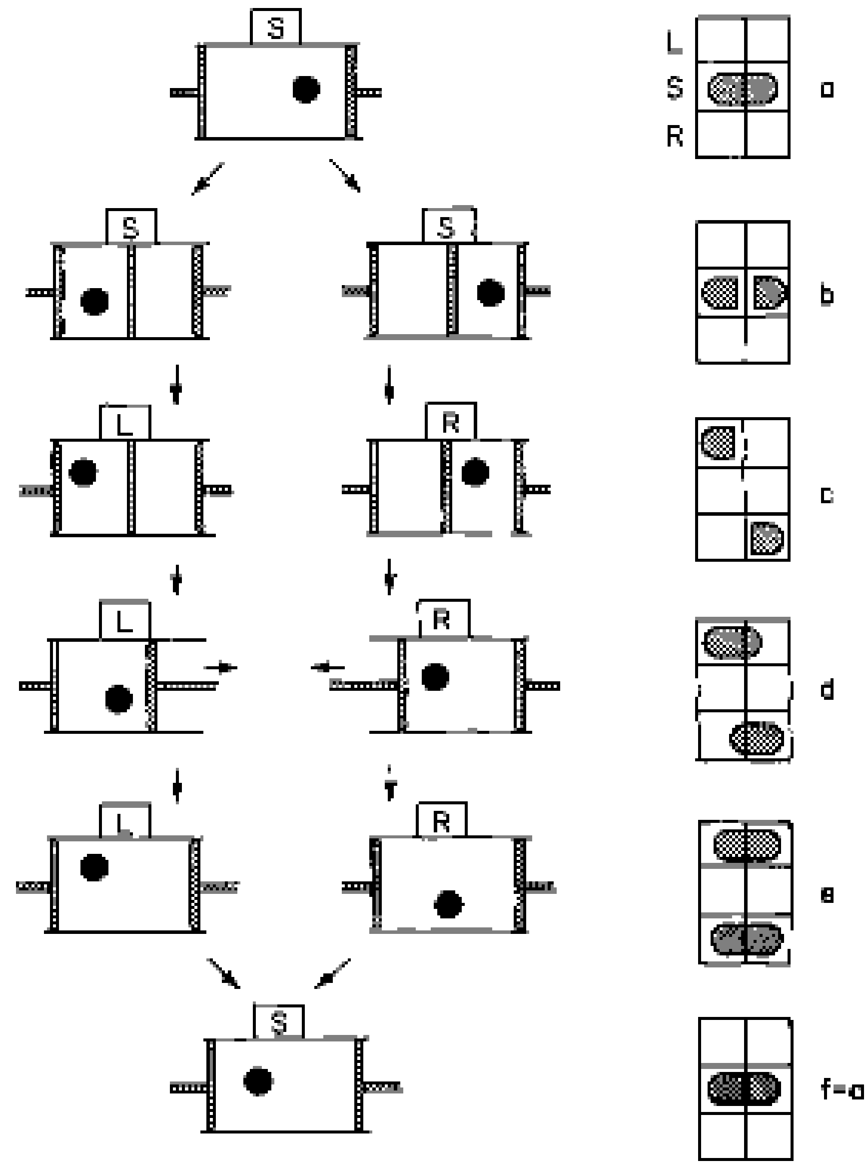
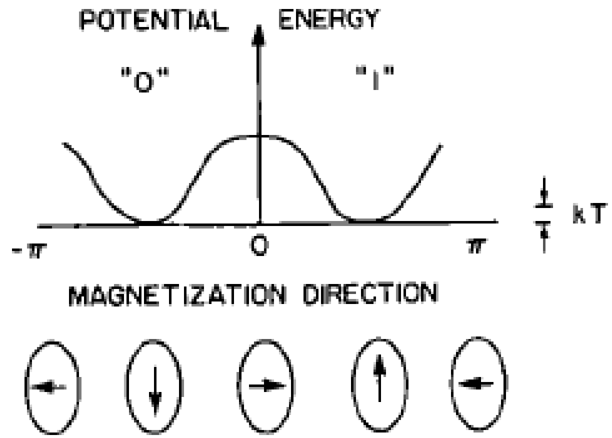
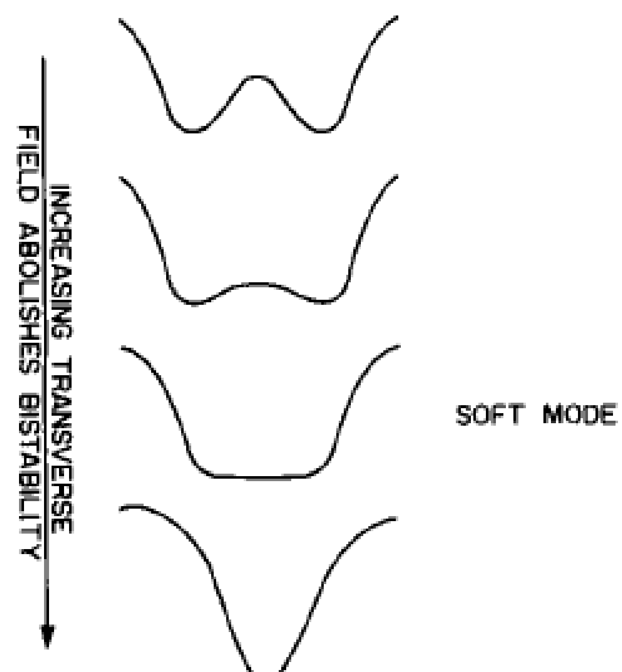


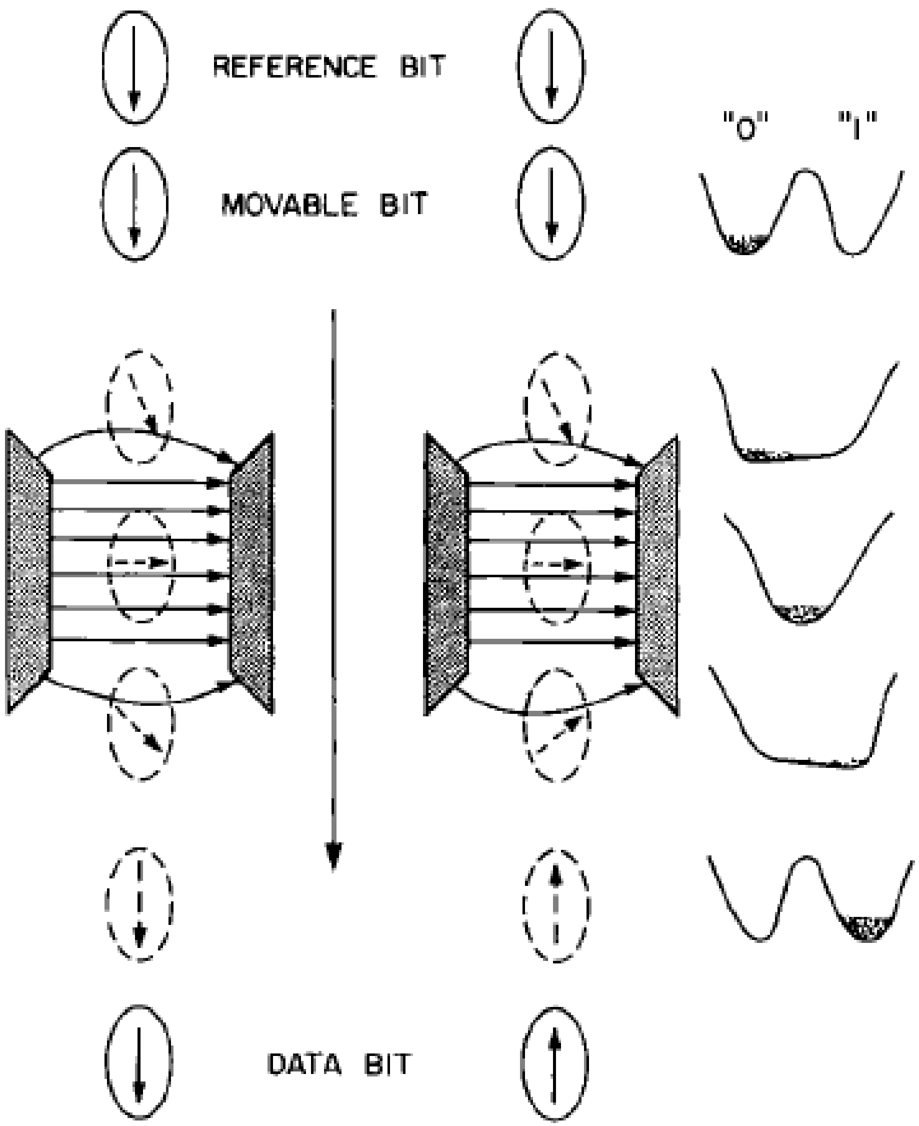
Fig. 12. A one-molecule Maxwell's demon apparatus.

The Demons Memory

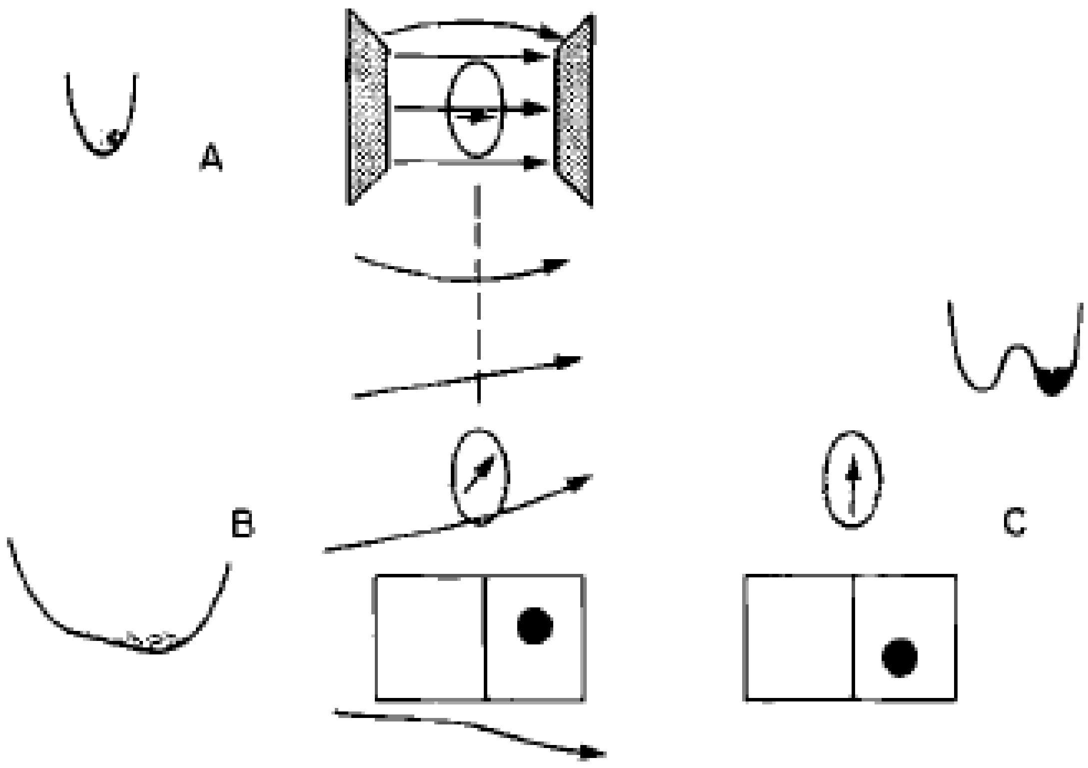




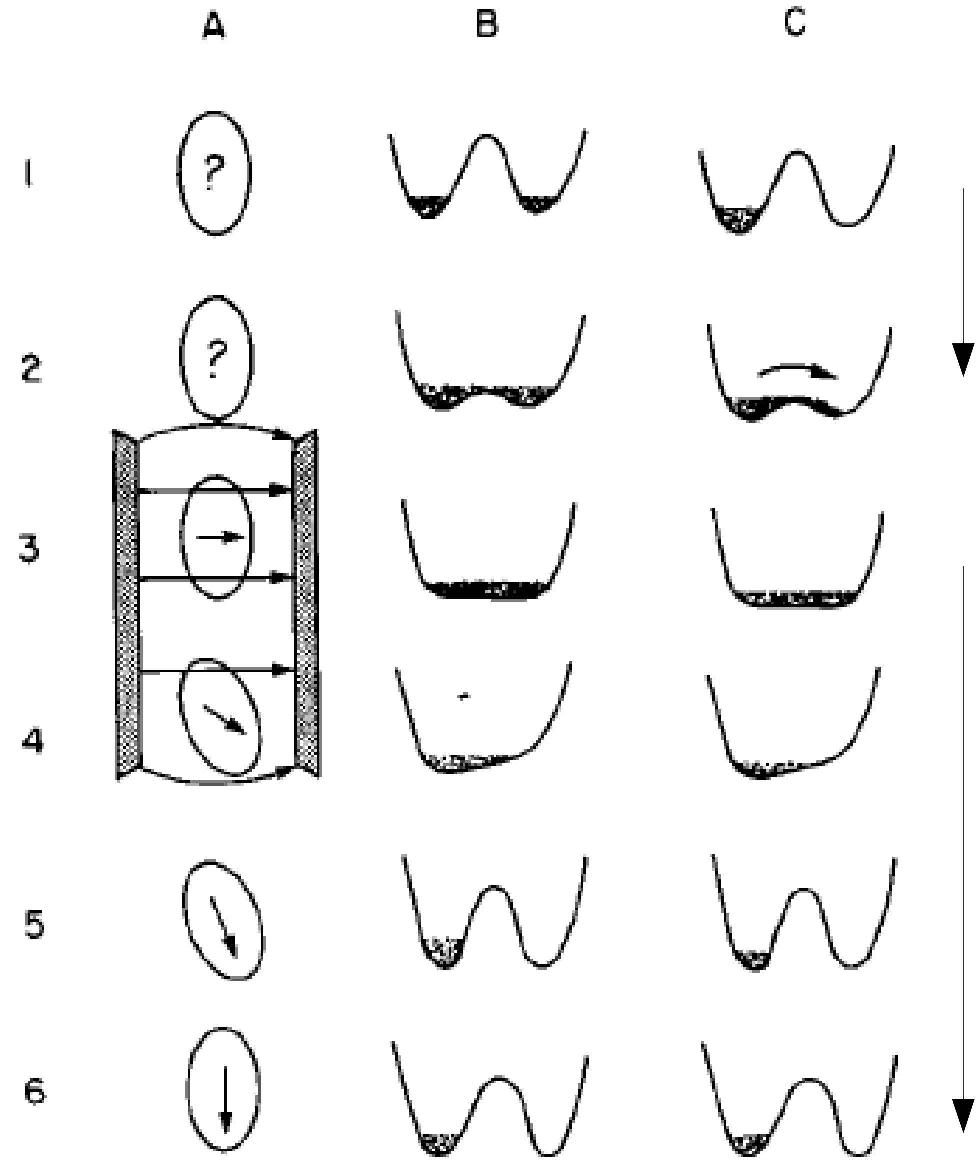
Reversible Copying



Reversible Measurement



Restoring Standard State - Erasure



$$B: S = +k_B \ln(2)$$
$$C: S = 0 \rightarrow S = +k_B \ln(2)$$

$$\Delta S = -k_B \ln(2)$$
$$\Rightarrow \delta W = k_B T \ln(2)$$

Summary

- “Erasure of information increases entropy”

  $$\sim \Delta S = k_B \ln(2)\, per\, bit$$

- Measurement can be done reversible

- Reseting the demons memory is what keeps him from violating the 2^{nd} Law

Sources

- Bennett, C.H. The Thermodynamics of Computation - a Review. International Journal of Theoretical Physics, Vol21, No 12, (1982).
- Landauer, R. Irreversibility and Heat Generation in the Computing Process. IBM Journal of Research and Development, 3, 183-191.
- Bennett, C.H. Notes on Landauer's principle, Reversible Computation and Maxwell's Demon. Studies in History and Philosophy of Modern Physics vol. 34 pp. 501-510 (2003)