Übungsblatt II

Rückgabe: 11.3.2008

Aufgabe 1 [*Ehrenfest's Theorem*]: Prove Ehrenfest's Theorem

$$m\frac{d^2}{dt^2}\langle x\rangle = -\langle V'(x)\rangle$$

for movement in a potential V.

Aufgabe 2 [*Probability current*]: Using the superposition principle or otherwise show that the wave-function

$$\Psi_k(x,t) = \left(Ae^{\frac{\imath kx}{\hbar}} + Be^{-\frac{\imath kx}{\hbar}}\right)e^{-\frac{\imath E_k}{\hbar}t},$$

where A, B and k are constants and $E_k = \frac{k^2}{2m}$, is a solution of the time-dependent Schrödinger equation with $V(x) = 0$. Show that the probability current

$$j(x,t) = \frac{\imath\hbar}{2m}\left(\Psi(x,t)\frac{\partial}{\partial x}\Psi(x,t)^* - \Psi(x,t)^*\frac{\partial}{\partial x}\Psi(x,t)\right)$$

corresponding to $\Psi_k(x,t)$ equals

$$j(x,t) = \left(|A|^2 - |B|^2\right)\frac{k}{m}\,.$$

Aufgabe 3 [*General Potentials*]: Let $V(x)$ be an arbitrary continuous potential in one dimension, with the property that $V(x) \to 0$ as $x \to \pm\infty$.

(i) Let $\chi(x)$ be a solution of the time-independent Schrödinger equation with the asymptotic behaviour

$$\chi(x) = e^{ikx} + Ae^{-ikx} \qquad \text{for} \quad x \ll 0$$

and

$$\chi(x) = Be^{ikx} \qquad \text{for} \quad x \gg 0\,.$$

Using the continuity equation show that the probability current is independent of x, and deduce that $|A|^2 + |B|^2 = 1$. [Later on we shall interpret $|A|^2$ as the reflection probability, and $|B|^2$ as the transmission probability of the corresponding scattering experiment.]

(ii) Show that, for any state, $\langle T\rangle \geq 0$, where $T = p^2/2m$. By considering $\langle T + V\rangle$ deduce that for a potential well V of any shape, the lowest energy state has energy greater than that of the bottom of the well.