ETH
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Statistical Physics
Exercise Sheet 4

HS 07
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Exercise 4.1 The Diffusion Equation

Consider particles of type A (e.g. radioactive molecules) mixed among particles of type B, whereby the concentration of the particles A is assumed to be much smaller than the concentration of the particles B. This justifies the use of the relaxation time approximation for the distribution function $f(\vec{r}, \vec{p}, t)$ of the particles A. Assuming that there are no external forces, derive *Fick's diffusion law*

$$\vec{j}(\vec{r}, t) \;=\; -D\, \vec{\nabla} n(\vec{r}, t)\,,$$

as well as an expression for the diffusion constant D. To this end use the relaxation time approximation for the local Maxwell-Boltzmann distribution

$$f_0(\vec{r}, \vec{p}, t) \;=\; n(\vec{r}, t)\, (2\pi m k_B T)^{-3/2}\, e^{-\frac{\vec{p}^2}{2mk_BT}}\,.$$

Use this reult and the continuity equation to derive a diffusion equation for $n(\vec{r}, t)$.

Exercise 4.2 Heat generation in viscous flow

Consider the static flow of an incompressible, viscous fluid through an infinteley long circular tube of radius R. The tube is assumed to be coupled to a heat bath of temperature T_0.

(a) Assuming no external forces, write down the Navier-Stokes equations, as well as the boundary conditions, for this situation.

(b) Determine the velocity field $\vec{c}(\vec{r})$. Hints: exploit the cylindrical symmetry of the problem; show first that $p(r, \phi, z) = p_0 + p'z$, for some constants p_0, p'.

(c) Determine the Temperature profile $T(\vec{r})$. Where is the heat generation at its strongest?