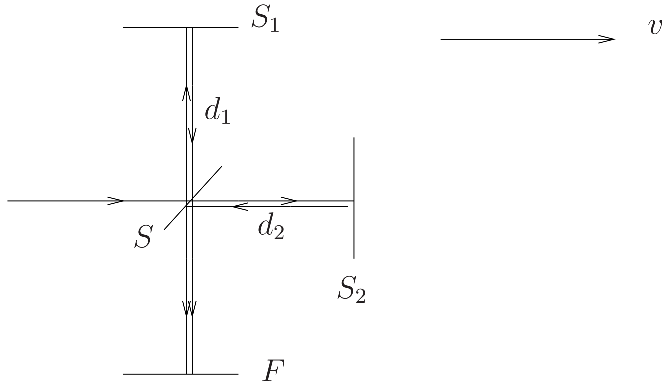
Theoretical Physics, Problem set 7.

FS15 *Hand in: 15.04.15*

1. The Michelson-Morley experiment

In the 19th century, light was regarded as an excitation of an aether. With respect to the aether, its velocity of propagation is isotropic. This is however not the case for a frame of reference which is moving with respect to the aether, as long as space and time coordinates are subject to Galilei transformations. In the course of a year, the Earth can not permanently be at rest with respect to the aether because of its motion around the Sun. Michelson and Morley were seeking for such an anisotropy in 1886 in vain. They were using the apparatus (interferometer) of the picture, where $\vec{v}$ is the velocity of the laboratory w.r.t. the aether.



A ray of light strikes a semipermeable mirror S which divides it into two perpendicular partial beams. Those are reaching, via the distance d_i $(i = 1, 2)$, the mirrors S_i and subsequently get back to S. From there, a part of each of them is reaching an observer telescope F, where an interference pattern in the from of stripes is visible. The condition is that $|d_2 - d_1|$ is small compared to the coherence length of the light and that the mirrors S_1 and S_2 are not arranged exactly perpendicular, such that variable path differences are the outcome. Displacements of the pattern can be measured within fractions of one wavelength.

i) Find the elapsed times t_1 and t_2 of the light along the two paths SS_iS, and $\Delta t = t_2 - t_1$ up to relative errors $O((v/c)^4)$ thereof. Then compute $\Delta t'$ for an arrangement rotated by 90°. The difference $\Delta t' - \Delta t$ determines the displacement of the pattern caused by the rotation.

ii) Numerical example: $v = 3 \cdot 10^4$m/s, $d_1 + d_2 = 3$m, wavelength of the light $\lambda \approx 3 \cdot 10^{-7}$m. By which part of the distance between the stripes is the pattern displaced?

Hint: The aether and the laboratory system should be assumed to be connected via a Galilei transformation, since the experiment is testing the classical notion of spacetime.

2.Applications of Lorentz transformations

(a) *Time dilation.* Two events A, B occur at the same place in the inertial frame K (e.g. time designation of a clock at rest w.r.t. K). Show by means of a boost that the time difference is larger in an inertial frame K' which is moving w.r.t. K.

(b) *Length contraction.* Consider a rigid rod which has length L in its rest frame K. Show that the length of the rod is smaller in an inertial frame K' which is in motion longitudinal to it. *Hint:* The length is given by the coordinate difference of the end points of the rod at the same time.

(c) There are given two paraxial inertial systems K and K', where K' is moving with relative velocity v w.r.t. K in the 1-direction. The rod is again pointing in the 1-direction, but now is moving with velocity w in the 2-direction. Determine its angle θ to the 1-direction w.r.t. K'.

(d) A 4-vector ξ is called *timelike* if $(\xi, \xi) > 0$ and *spacelike* if $(\xi, \xi) < 0$. Show: two events x, y are at the same time in an appropriate inertial system if and only if $x - y$ is spacelike. They occur at the same place in an appropriate inertial system if and only if $x - y$ is timelike.