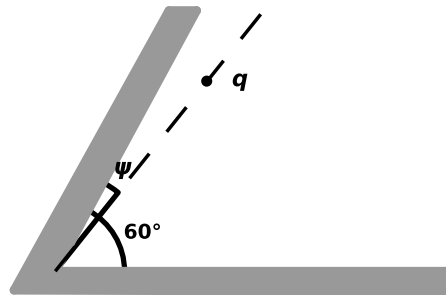


Exercise 1. Method of Images

Consider a point charge q located at any point between two grounded conductive metal plates that have 60° angle between them.



1. Calculate the electrostatic potential in the space between the plates using the method of images. In the figure sketch the needed mirror charges to determine the potential. Discuss how the result depends on the angle ψ .
2. Determine the direction and the magnitude of the force \vec{F} acting on the charge q .

Exercise 2. Solution of the Poisson Equation with Green's Function

The general solution for the potential is given by:

$$\Phi(\vec{y}) = \frac{1}{4\pi\epsilon_0} \int_V G(\vec{x}, \vec{y}) \rho(\vec{x}) d^3\vec{x} - \frac{1}{4\pi} \oint_S d\vec{S} \cdot (\Phi_s(\vec{x}) \nabla_{\vec{x}} G(\vec{x}, \vec{y}) - G(\vec{x}, \vec{y}) \nabla_{\vec{x}} \Phi_s(\vec{x})) \quad (1)$$

where Φ_s is the surface potential.

Now consider the figure below, where the $z < 0$ region is filled with a conductor and the $z > 0$ region is free. We choose the Green's function such that it is zero on the surface of the conductor: $G = 0$ at $z = 0$. Therefore in Equation (1) the last term vanishes.

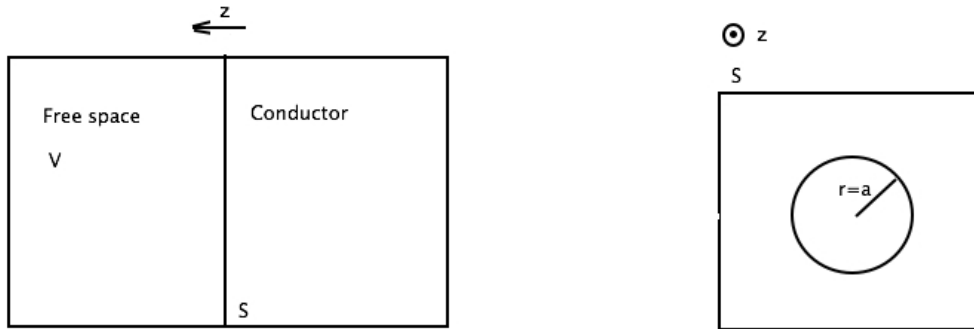


Figure 1: The conductor cross section and the surface of the conductor

1. Now consider that the surface potential is not constant but proportional to $1/r$:

$$\Phi_s = \frac{V}{r}$$

where $r = x_1^2 + x_2^2$ and V is a constant. Calculate the potential at a point on the z axis in the free part, i.e. $x_1 = x_2 = 0$. For this use the following Green's function:

$$G(\vec{x}, \vec{y}) = \left(\frac{1}{\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2}} - \frac{1}{\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 + y_3)^2}} \right) \quad (2)$$

Note that now x_3 points in the direction of the z axis. Also there are no charges $\rho(\vec{x}) = 0$ in the region we are interested so the first term in Equation (1) vanishes. Use the cylindrical coordinates.

2. Now assume that the surface of the conductor has the following potential:

$$\Phi_s = 0, r > a$$

$$\Phi_s = V, r < a$$

with $r = x_1^2 + x_2^2$, as seen in Figure 1. Hence the potential is everywhere zero on the $z = 0$ plane, except for the circle of the radius a around the coordinate beginning. Use the Green's function to calculate the potential on the z -axis.

3. Now imagine that the region $z > 0$ is not free, but includes a finite line charge of length L , as described in Sheet1, Question 3.2. This line charge is parallel to the surface of the conductor, and located at a distance b from it. Assume that the conductor is grounded so the surface potential is zero. How would you now calculate the potential in the arbitrary point in the region $z > 0$?

Exercise 3. Numerical solution of the Poisson equation

In this exercise we will see two simple examples of how electrostatic problems can be solved numerically.

- a) Consider a disk of radius 1 centered at (0,0) with dimensionless charge density given by $\rho(x, y) = (x - 0.1)^2$ as shown in the figure above.

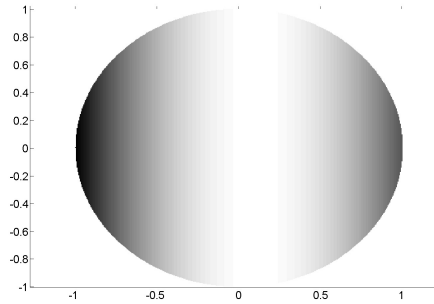
Matlab pdetool provides an intuitive interface to set up a configuration, solve differential equations and visualize the results. Using Matlab pdetool calculate the potential with the Poisson equation

$$\nabla^2 \Phi = -\rho$$

at any point of the disk, plot the equipotential lines and the electric field.

Assume Dirichlet boundary conditions (potential is zero at the domain boundary).

Note: Matlab can be found at all ETH publicly available computers. To start pdetool type pdetool in the Matlab command line. Choose Electrostatics mode. Use simple dimensionless units.



- b) Solve the one-dimensional Poisson equation

$$\frac{\partial^2 \Phi}{\partial x^2} = -\rho(x) \quad (3)$$

without using the Matlab PDE toolbox.

- i) Write eq. 3 in a discretized form, using finite differences with a spatial separation Δx . Φ is then only defined on discrete points $\Phi(x_n)$ ($n = 1, 2, \dots, N$)
- ii) Set the Dirichlet boundary conditions: $\Phi(x_0) = c_0$ and $\Phi(x_N) = c_N$ with two constants c_0 and c_N .
- iii) The discretized Poisson is a set of (N-1) coupled linear equations and can be written in matrix form as $A\Phi = \mathbf{b}$. Write out this system.
- iv) Set the initial conditions $\rho(x_n) = (x - 0.1)^2$ and solve the system on a computer, with B.C. $c_0 = c_N = 0$.
- v) Compare the result to the analytical solution.